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Experimental Determination of the Velocity of Light

Made at the U.S. Naval Academy, Annapolis.

By

Albert A. Michelson,
Master U.S. Navy.

Note.

The probability that the most accurate method of determining the solar parallax now available is that resting on the measurement of the velocity of light, has led to the acceptance of the following paper as one of the series having in view the increase of our knowledge of the celestial motions. The researches described in it, having been made at the United States Naval Academy, though at private expense, were reported to the Honorable Secretary of the Navy, and referred by him to this Office. At the suggestion of the writer, the paper was reconstructed with a fuller general discussion of the processes, and with the omission of some of the details of individual experiments.

To prevent a possible confusion of this determination of the velocity of light with another now in progress under official auspices, it may be stated that the credit and responsibility for the present paper rests with Master Michelson.

Simon Newcomb,
Professor, U.S. Navy,
Superintendent Nautical Almanac.

Nautical Almanac Office,
Bureau of Navigation,
Navy Department,
Washington, February 20, 1880.

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Experimental Determination of the Velocity of Light.

By Albert A. Michelson, *Master, U.S.N.*

Introduction.

In Cornu's elaborate memoir upon the determination of the velocity of light, several objections are made to the plan followed by Foucault, which will be considered in the latter part of this work. It may, however, be stated that the most important among these was that the deflection was too small to be measured with the required degree of accuracy. In order to employ this method, therefore, it was absolutely necessary that the deflection should be increased.

In November, 1877, a modification of Foucault's arrangement suggested itself, by which this result could be accomplished. Between this time and March of the following year a number of preliminary experiments were performed in order to familiarize myself with the optical arrangements. The first experiment tried with the revolving mirror produced a deflection considerably greater than that obtained by Foucault. Thus far the only apparatus used was such as could be adapted from the apparatus in the laboratory of the Naval Academy.

At the expense of $10 a revolving mirror was made, which could execute 128 turns per second. The apparatus was installed in May, 1878, at the laboratory. The distance used was 500 feet, and the deflection was about twenty times that obtained by Foucault.[1]

[Footnote 1: See Proc. Am. Assoc. Adv. Science, Saint Louis meeting.]

These experiments, made with very crude apparatus and under great difficulties, gave the following table of results for the velocity of light in miles per second:

186730
188820
186330
185330
187900
184500
186770

185000
185800
187940

Mean 186500 ± 300 miles per second,
or 300140 kilometers per second.

In the following July the sum of $2,000 was placed at my disposal by a private gentleman for carrying out these experiments on a large scale. Before ordering any of the instruments, however, it was necessary to find whether or not it was practicable to use a large distance. With a distance (between the revolving and the fixed mirror) of 500 feet, in the preliminary experiments, the field of light in the eye-piece was somewhat limited, and there was considerable indistinctness in the image, due to atmospheric disturbances.

Accordingly, the same lens (39 feet focus) was employed, being placed, together with the other pieces of apparatus, along the north sea-wall of the Academy grounds, the distance being about 2,000 feet. The image of the slit, at noon, was so confused as not to be recognizable, but toward sunset it became clear and steady, and measurements were made of its position, which agreed within one one-hundredth of a millimeter. It was thus demonstrated that with this distance and a deflection of 100 millimeters this measurement could be made within the ten-thousandth part.

In order to obtain this deflection, it was sufficient to make the mirror revolve 250 times per second and to use a "radius" of about 30 feet. In order to use this large radius (distance from slit to revolving mirror), it was necessary that the mirror should be large and optically true; also, that the lens should be large and of great focal length. Accordingly the mirror was made 1¼ inches in diameter, and a new lens, 8 inches in diameter, with a focal length of 150 feet was procured.

In January, 1879, an observation was taken, using the old lens, the mirror making 128 turns per second. The deflection was about 43 millimeters. The micrometer eye-piece used was substantially the same as Foucault's, except that part of the inclined plate of glass was silvered, thus securing a much greater quantity of light. The deflection having reached 43 millimeters, the inclined plate of glass could be dispensed with, the light going past the observer's head through the slit, and returning 43 millimeters to the left of the slit, where it could be easily observed.

Thus the micrometer eye-piece is much simplified, and many possible sources of error are removed.

The field was quite limited, the diameter being, in fact, but little greater than the width of the slit. This would have proved a most serious objection to the new arrangement. With the new lens, however, this difficulty disappeared, the field being about twenty times the width of the slit. It was expected that, with the new lens, the image would be less distinct; but the difference, if any, was small, and was fully compensated by the greater size of the field.

The first observation with the new lens was made January 30, 1879. The deflection was 70 millimeters. The image was sufficiently bright to be observed without the slightest effort. The first observation with the new micrometer eye-piece was made April 2, the deflection being 115 millimeters.

The first of the final series of observations was made on June 5. All the observations previous to this, thirty sets in all, were rejected. After this time, no set of observations nor any single observation was omitted.

Theory of New Method.

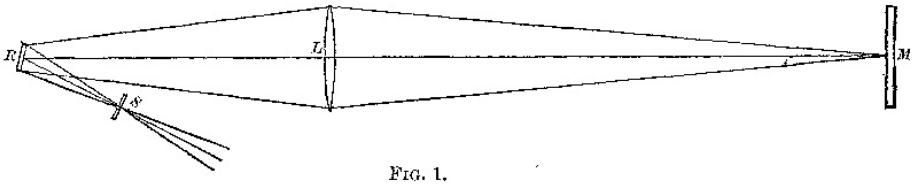

Fig. 1.

Let S, Fig. 1, be a slit, through which light passes, falling on R, a mirror free to rotate about an axis at right angles to the plane of the paper; L, a lens of great focal length, upon which the light falls which is reflected from R. Let M be a plane mirror whose surface is perpendicular to the line R, M, passing through the centers of R, L, and M, respectively. If L be so placed that an image of S is formed on the surface of M, then, this image acting as the object, its image will be formed at S, and will coincide, point for point, with S.

If, now, R be turned about the axis, so long as the light falls upon the lens, an image of the slit will still be formed on the surface of the mirror, though on a different part, and as long as the returning light falls on the lens an image of this image will be formed at S, notwithstanding the change of position of the first image at M. This result, namely, the production of a stationary image of an image in motion, is absolutely necessary in this method of experiment. It was first accomplished by Foucault, and in a manner differing apparently but little from the foregoing.

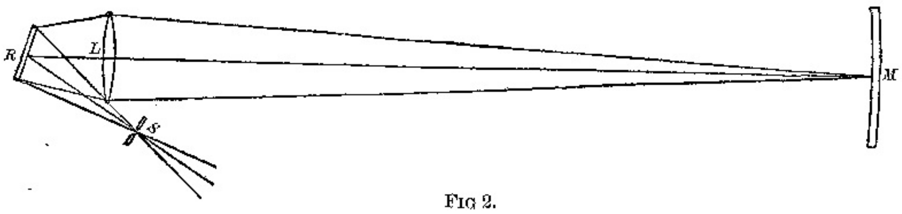

Fig 2.

In his experiments L, Fig. 2, served simply to form the image of S at M, and M, the returning mirror, was spherical, the center coinciding with the axis of R. The lens L was placed as near as possible to R. The light forming the return image lasts, in this case, while the first image is sweeping over the face of the mirror, M. Hence, the greater the distance RM, the larger must be the mirror in order that the same amount of light may be preserved, and its dimensions would soon become inordinate. The difficulty was partly met by Foucault, by using five concave reflectors instead of one, but even then the greatest distance he found it practicable to use was only 20 meters.

Returning to Fig. 1, suppose that R is in the principal focus of the lens L; then, if the plane mirror M have the same diameter as the lens, the first, or moving image, will remain upon M as long as the axis of the pencil of light remains on the lens, and *this will be the case no matter what the distance may be*.

When the rotation of the mirror R becomes sufficiently rapid, then the flashes of light which produce the second or stationary image become blended, so that the image appears to be continuous. But now it no longer coincides with the slit, but is *deflected* in the direction of rotation, and through twice the angular distance described by the mirror, during the time required for light to travel twice the distance between the mirrors. This displacement is measured by the tangent of the arc it subtends. To make this as large as possible, the distance between the mirrors, the radius, and the speed of rotation should be made as great as possible.

The second condition conflicts with the first, for the radius is the difference between the focal length for parallel rays, and that for rays at the distance of the fixed mirror. The greater the distance, therefore, the smaller will be the radius.

There are two ways of solving the difficulty: first, by using a lens of great focal length; and secondly, by placing the revolving mirror within the principal focus of the lens. Both means were employed. The focal length of the lens was 150 feet, and the mirror was placed about 15 feet within the principal focus. A limit is soon reached, however, for the quantity of light received diminishes very rapidly as the revolving mirror approaches the lens.

Arrangement and Description of Apparatus.

Site and Plan.

The site selected for the experiments was a clear, almost level, stretch along the north sea-wall of the Naval Academy. A frame building was erected at the western end of the line, a plan of which is represented in Fig. 3.

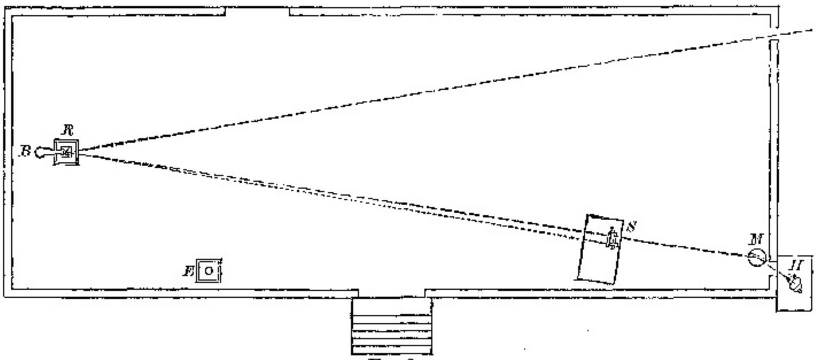

Fig 3.

The building was 45 feet long and 14 feet wide, and raised so that the line along which the light traveled was about 11 feet above the ground. A heliostat at H reflected the sun's rays through the slit at S to the revolving mirror R, thence through a hole in the shutter, through the lens, and to the distant mirror.

The Heliostat.

The heliostat was one kindly furnished by Dr. Woodward, of the Army Medical Museum, and was a modification of Foucault's form, designed by Keith. It was found to be accurate and easy to adjust. The light was reflected from the heliostat to a plane mirror, M, Fig. 3, so that the former need not be disturbed after being once adjusted.

The Revolving Mirror.

The revolving mirror was made by Fauth & Co., of Washington. It consists of a cast-iron frame resting on three leveling screws, one of which was connected by cords to the table at S, Fig. 3, so that the mirror could be inclined forward or backward while making the observations.

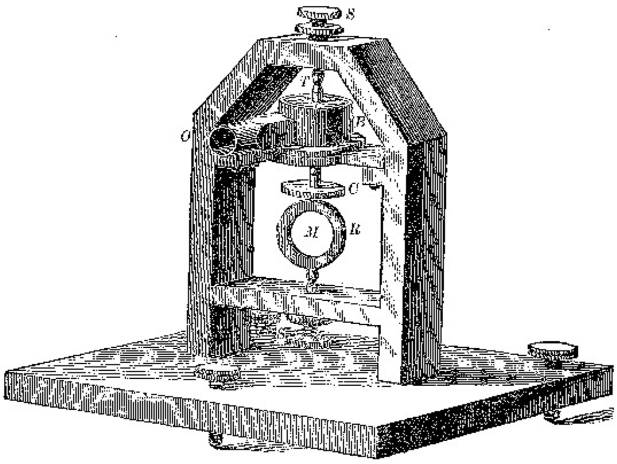

Fig. 4.

Two binding screws, S, S, Fig. 4, terminating in hardened steel conical sockets, hold the revolving part. This consists of a steel axle, X, Y, Figs. 4 and 5, the pivots being conical and hardened. The axle expands into a ring at R, which holds the mirror M. The latter was a disc of plane glass, made by Alvan Clark & Sons, about 1¼ inch in diameter and 0.2 inch thick. It was silvered on one side only, the reflection taking place from the outer or front surface. A species of turbine wheel, T, is held on the axle by friction. This wheel has six openings for the escape of air; a section of one of them is represented in Fig 6.

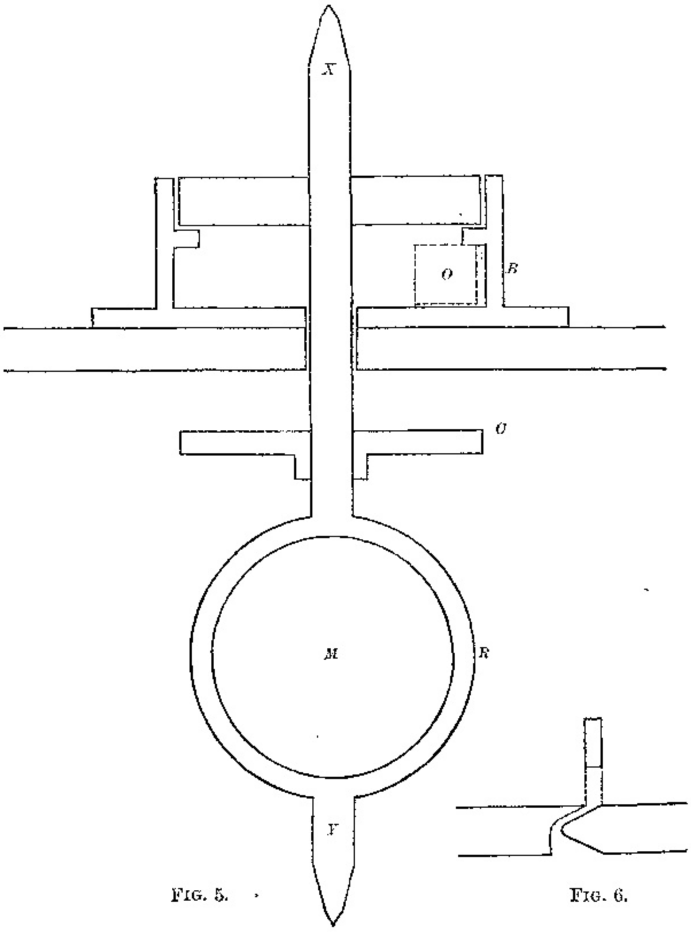

FIG. 5.

FIG. 6.

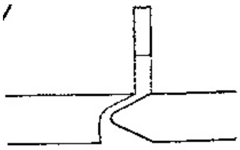

FIG. 6.

Adjustment of the Revolving Mirror.

The air entering on one side at O, Fig. 5, acquires a rotary motion in the box B, B, carrying the wheel with it, and this motion is assisted by the reaction of the air in escaping. The disc C serves the purpose of bringing the center of gravity in the axis of rotation. This was done, following Foucault's plan, by allowing the pivots to rest on two inclined planes of glass, allowing the arrangement to come to rest, and filing away the lowest part of the disc; trying again, and so on, till it would rest in indifferent equilibrium. The part corresponding to C, in Foucault's apparatus, was furnished with three vertical screws, by moving which the axis of figure was brought into coincidence with the axis of rotation. This adjustment was very troublesome. Fortunately, in this apparatus it was found to be unnecessary.

When the adjustment is perfect the apparatus revolves without giving any sound, and when this is accomplished, the motion is regular and the speed great. A slight deviation causes a sound due to the rattling of the pivots in the sockets, the speed is very much diminished, and the pivots begin to wear. In Foucault's apparatus oil was furnished to the pivots, through small holes running through the screws, by pressure of a column of mercury. In this apparatus it was found sufficient to touch the pivots occasionally with a drop of oil.

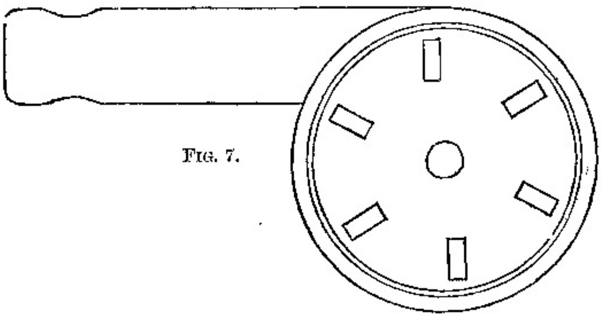

FIG. 7.

THE MICROMETER.

Fig. 7 is a view of the turbine, box, and supply-tube, from above. The quantity of air entering could be regulated by a valve to which was attached a cord leading to the observer's table.

The instrument was mounted on a brick pier.

The Micrometer.

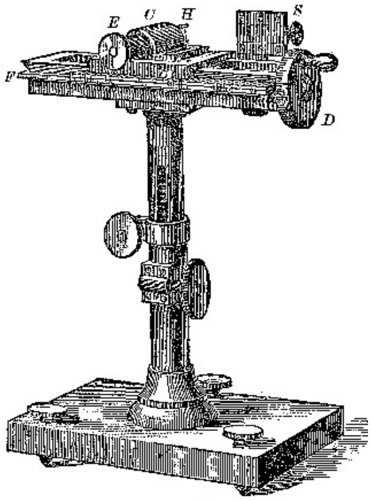

FIG. 8.

The apparatus for measuring the deflection was made by Grunow, of New York.

This instrument is shown in perspective in Fig. 8, and in plan by Fig. 9. The adjustable slit S is clamped to the frame F. A long millimeter-screw, not shown in Fig. 8, terminating in the divided head D, moves the carriage C, which supports the eye-piece E. The frame is furnished with a brass scale at F for counting revolutions, the head counting hundredths. The eye-piece consists of a single achromatic lens, whose focal length is about two inches. At its focus, in H, and in nearly the same plane as the face of the slit, is a single vertical silk fiber. The apparatus is furnished with a standard with rack and pinion, and the base furnished with leveling screws.

Manner of Using the Micrometer.

In measuring the deflection, the eye-piece is moved till the cross-hair bisects the slit, and the reading of the scale and divided head gives the position. This measurement need not be repeated unless the position or width of the slit is changed. Then the eye-piece is moved till the cross-hair bisects the deflected image of the slit; the reading of scale and head are again taken, and the difference in readings gives the deflection. The screw was found to have no lost motion, so that readings could be taken with the screw turned in either direction.

Measurement of Speed of Rotation.

To measure the speed of rotation, a tuning-fork, bearing on one prong a steel mirror, was used. This was kept in vibration by a current of electricity from five "gravity" cells. The fork was so placed that the light from the revolving mirror was reflected to a piece of plane glass, in front of the lens of the eye-piece of the micrometer, inclined at an angle of 45°, and thence to the eye. When fork and revolving mirror are both at rest, an image of the revolving mirror is seen. When the fork vibrates, this image is drawn out into a band of light.

When the mirror commences to revolve, this band breaks up into a number of moving images of the mirror; and when, finally, the mirror makes as many turns as the fork makes vibrations, these images are reduced to one, which is stationary. This is also the case when the number of turns is a submultiple. When it is a multiple or simple ratio, the only difference is that there are more images. Hence, to make the mirror execute a certain number of turns, it is simply necessary to pull the cord attached to the valve to the right or left till the images of the revolving mirror come to rest.

The electric fork made about 128 vibrations per second. No dependence was placed upon this rate, however, but at each set of observations it is compared with a standard Ut_3 fork, the temperature being noted at the same time. In making the comparison the sound-beats produced by the forks were counted for 60 seconds. It is interesting to note that the electric fork, as long as it remained untouched and at the same temperature, did not change its rate more than one or two hundredths vibrations per second.

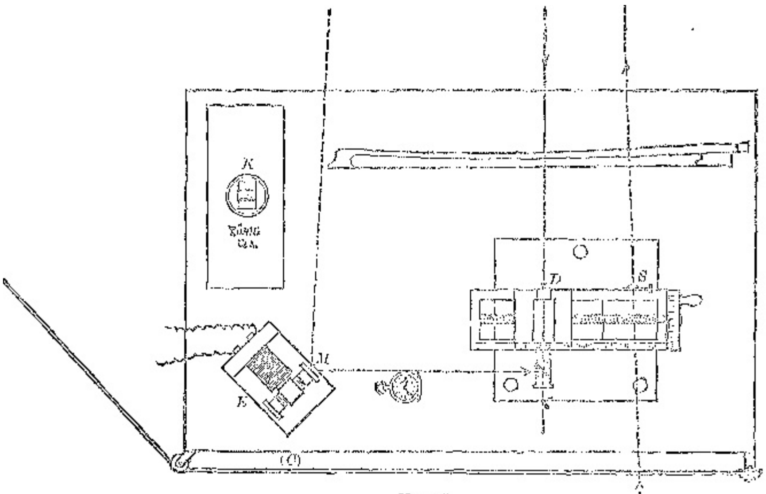

FIG. 9.

The Observer's Table.

Fig. 9 Represents The Table At Which The Observer Sits. The Light From The Heliostat Passes Through The Slit At S, Goes To The Revolving Mirror, &c., And, On Its Return, Forms An Image Of The Slit At D, Which Is Observed Through The Eye-piece. E Represents The Electric Fork (the Prongs Being Vertical) Bearing The Steel Mirror M. K Is The Standard Fork On Its Resonator. C Is The Cord Attached To The Valve Supplying Air To The Turbine.

The Lens.

The lens was made by Alvan Clark & Sons. It was 8 inches in diameter; focal length, 150 feet; not achromatic. It was mounted in a wooden frame, which was placed on a support moving on a slide, about 16 feet long, placed about 80 feet from the building. As the diameter of the lens was so small in comparison with its focal length, its want of achromatism was inappreciable. For the same reason, the effect of "parallax" (due to want of coincidence in the plane of the image with that of the silk fiber in the eye-piece) was too small to be noticed.

The Fixed Mirror.

The fixed mirror was one of those used in taking photographs of the transit of Venus. It was about 7 inches in diameter, mounted in a brass frame capable of adjustment in a vertical and a horizontal plane by screw motion. Being wedge-shaped, it had to be silvered on the front surface. To facilitate adjustment, a small telescope furnished with cross-hairs was attached to the mirror by a universal joint. The heavy frame was mounted on a brick pier, and the whole surrounded by a wooden case to protect it from the sun.

Adjustment of the Fixed Mirror.

The adjustment was effected as follows: A theodolite was placed at about 100 feet in front of the mirror, and the latter was moved about by the screws till the observer at the theodolite saw the image of his telescope reflected in the center of the mirror. Then the telescope attached to the mirror was pointed (without moving the mirror itself) at a mark on a piece of card-board attached to the theodolite. Thus the line of collimation of the telescope was placed at right angles to the surface of the mirror. The theodolite was then moved to 1,000 feet, and, if found necessary, the adjustment was repeated. Then the mirror was moved by the screws till its telescope pointed at the hole in the shutter of the building. The adjustment was completed by moving the mirror, by signals, till the observer, looking through the hole in the shutter, through a good spy-glass, saw the image of the spy-glass reflected centrally in the mirror.

The whole operation was completed in a little over an hour.

Notwithstanding the wooden case about the pier, the mirror would change its position between morning and evening; so that the last adjustment had to be repeated before every series of experiments.

Apparatus for Supplying and Regulating the Blast of Air.

Fig. 10 represents a plan of the lower floor of the building. E is a three-horse power Lovegrove engine and boiler, resting on a stone foundation; B, a small Roots' blower; G, an automatic regulator. From this the air goes to a delivery-pipe, up through the floor, and to the turbine. The engine made about 4 turns per second and the blower about 15. At this speed the pressure of the air was about half a pound per square inch.

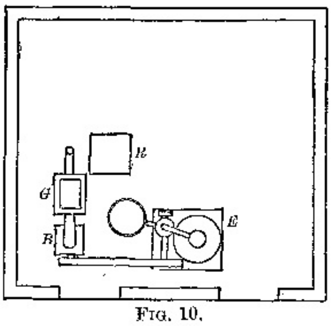

Fig. 10.

The regulator, Fig. 11, consists of a strong bellows supporting a weight of 370 pounds, partly counterpoised by 80 pounds in order to prevent the bellows from sagging. When the pressure of air from the blower exceeds the weight, the bellows commences to rise, and, in so doing, closes the valve V.

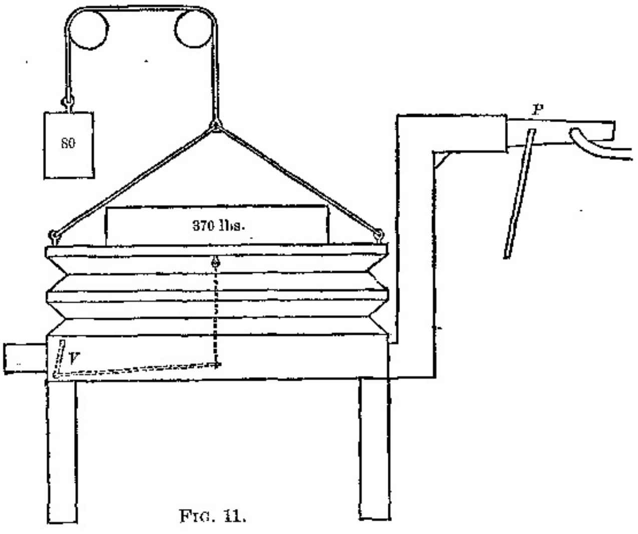

Fig. 11.

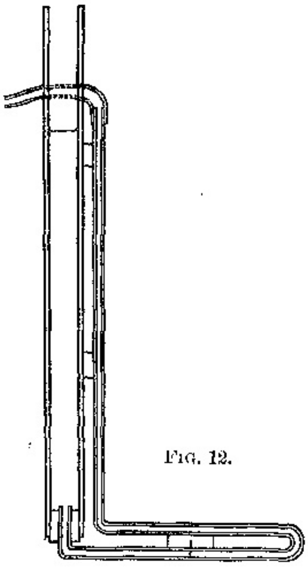

Fig. 12.

This arrangement was found in practice to be insufficient, and the following addition was made: A valve was placed at P, and the pipe was tapped a little farther on, and a rubber tube led to a water-gauge, Fig 12. The column of water in the smaller tube is depressed, and, when it reaches the horizontal part of the tube, the slightest variation of pressure sends the column from one end to the other. This is checked by an assistant at the valve; so that the column of water is kept at about the same place, and the pressure thus rendered very nearly constant. The result was satisfactory, though not in the degree anticipated. It was possible to keep the mirror at a constant speed for three or four seconds at a time, and this was sufficient for an observation. Still it would have been more convenient to keep it so for a longer time.

I am inclined to think that the variations were due to changes in the friction of the pivots rather than to changes of pressure of the blast of air.

It may be mentioned that the test of uniformity was very delicate, as a change of speed of one or two hundredths of a turn per second could easily be detected.

Method Followed in Experiment.

It was found that the only time during the day when the atmosphere was sufficiently quiet to get a distinct image was during the hour after sunrise, or during the hour before sunset. At other times the image was "boiling" so as not to be recognizable. In one experiment the electric light was used at night, but the image was no more distinct than at sunset, and the light was not steady.

The method followed in experiment was as follows: The fire was started half an hour before, and by the time everything was ready the gauge would show 40 or 50 pounds of steam. The mirror was adjusted by signals, as before described. The heliostat was placed and adjusted. The revolving mirror was inclined to the right or left, so that the *direct* reflection of light from the slit, which otherwise would flash into the eye-piece at every revolution, fell either above or below the eye-piece.[2]

[Footnote 2: Otherwise this light would overpower that which forms the image to be observed. As far as I am aware, Foucault does not speak of this difficulty. If he allowed this light to interfere with the brightness of the image, he neglected a most obvious advantage. If he did incline the axis of the mirror to the right or left, he makes no allowance for the error thus introduced.]

The revolving mirror was then adjusted by being moved about, and inclined forward and backward, till the light was seen reflected back from the distant mirror. This light was easily seen through the coat of silver on the mirror.

The distance between the front face of the revolving mirror and the cross-hair of the eye-piece was then measured by stretching from the one to the other a steel tape, making the drop of the catenary about an inch, as then the error caused by the stretch of the tape and that due to the curve just counterbalance each other.

The position of the slit, if not determined before, was then found as before described. The electric fork was started, the temperature noted, and the sound-beats between it and the standard fork counted for 60 seconds. This was repeated two or three times before every set of observations.

The eye-piece of the micrometer was then set approximately[3] and the revolving mirror started. If the image did not appear, the mirror was inclined forward or backward till it came in sight.

[Footnote 3: The deflection being measured by its tangent, it was necessary that the scale should be at right angles to the radius (the radius drawn from the mirror to one or the other end of that part of the scale which represents this tangent). This was done by setting the eye-piece approximately to the expected deflection, and turning the whole micrometer about a vertical axis till the cross-hair bisected the circular field of light reflected from the revolving mirror. The axis of the eye-piece being at right angles to the scale, the latter would be at right angles to radius drawn to the cross-hair.]

The cord connected with the valve was pulled right or left till the images of the revolving mirror, represented by the two bright round spots to the left of the cross-hair, came to rest. Then the screw was turned till the cross-hair bisected the deflected image of the slit. This was repeated till ten observations were taken, when the mirror was stopped, temperature noted, and beats counted. This was called a set of observations. Usually five such sets were taken morning and evening.

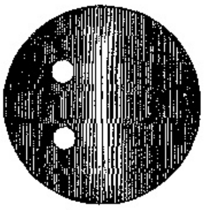

Fig. 13.

Fig. 13 represents the appearance of the image of the slit as seen in the eye-piece magnified about five times.

Determination of The Constants.

Comparison of the Steel Tape with the Standard Yard.

The steel tape used was one of Chesterman's, 100 feet long. It was compared with Wurdeman's copy of the standard yard, as follows:

Temperature was 55° Fahr.

The standard yard was brought under the microscopes of the comparator; the cross-hair of the unmarked microscope was made to bisect the division marked o, and the cross-hair of the microscope, marked I, was made to bisect the division marked 36. The reading of microscope I was taken, and the other microscope was not touched during the experiment. The standard was then removed and the steel tape brought under the microscopes and moved along till the division marked 0.1 (feet) was bisected by the cross-hair of the unmarked microscope. The screw of microscope I was then turned till its cross-hair bisected the division marked 3.1 (feet), and the reading of the screw taken. The difference between the original reading and that of each measurement was noted, care being taken to regard the direction in which the screw was turned, and this gave the difference in length between the standard and each succesive portion of the steel tape in terms of turns of the micrometer-screw.

To find the value of one turn, the cross-hair was moved over a millimeter scale, and the following were the values obtained:

Turns of screw of microscope I in 1^{mm}—

7.68	7.73	7.60	7.67
7.68	7.62	7.65	7.57
7.72	7.70	7.64	7.69
7.65	7.59	7.63	7.64
7.55	7.65	7.61	7.63

Mean =7.65

Hence one turn = 0.1307^{mm}.

or = 0.0051 inch.

The length of the steel tape from 0.1 to 99.1 was found to be greater than 33 yards, by 7.4 turns $=.96^{mm}$	+.003 feet.
Correction for temperature	+.003 feet.
Length	100.000 feet.

Corrected length	100.006 feet.

Determination of the Value of Micrometer.

Two pairs of lines were scratched on one slide of the slit, about 38^{mm} apart, i.e., from the center of first pair to center of second pair. This distance was measured at intervals of 1^{mm} through the whole length of the screw, by bisecting the interval between each two pairs by the vertical silk fiber at the end of the eye-piece. With these values a curve was constructed which gave the following values for this distance, which we shall call D′:

Turns of screw.		
At 0 of scale	D′ =	38.155
10 of scale	D′	38.155
20 of scale	D′	38.150

30 of scale D′	38 150
40 of scale D′	38.145
50 of scale D′	38.140
60 of scale D′	38.140
70 of scale D′	38.130
80 of scale D′	38.130
90 of scale D′	38.125
100 of scale D′	38.120
110 of scale D′	38.110
120 of scale D′	38.105
130 of scale D′	38.100
140 of scale D′	38.100

Changing the form of this table, we find that,—

For the *first*

10 turns the *average* value of D′ is	38.155
20 turns	38.153
30 turns	38.152
40 turns	38.151
50 turns	38.149
60 turns	38.148
70 turns	38.146
80 turns	38.144
90 turns	38.142
100 turns	38.140
110 turns	38.138
120 turns	38.135
130 turns	38.132
140 turns	38.130

On comparing the scale with the standard meter, the temperature being 16°.5 C., 140 divisions were found to = 139.462^{mm}. This multiplied by (1 + .0000188 × 16.5) = 139.505^{mm}.

One hundred and forty divisions were found to be equal to 140.022 turns of the screw, whence 140 turns of the screw = 139.483^{mm}, or 1 turn of the screw = 0.996305^{mm}.

This is the *average* value of one turn in 140.

But the average value of D, for 140 turns is, from the preceding table, 38.130.

Therefore, the true value of D, is 38.130 × .996305^{mm}, and the average value of one turn for 10, 20, 30, etc., turns, is found by dividing 38.130 × .996305 by the values of D;, given in the table.

This gives the value of a turn—

		mm.
For the first	10 turns	0.99570
	20 turns	0.99570
	30 turns	0.99573
	40 turns	0.99577
	50 turns	0.99580
	60 turns	0.99583
	70 turns	0.99589
	80 turns	0.99596
	90 turns	0.99601
	100 turns	0.99606
	110 turns	0.99612
	120 turns	0.99618
	130 turns	0.99625
	140 turns	0.99630

Note.—The micrometer has been sent to Professor Mayer, of Hoboken, to test the screw again, and to find its value. The steel tape has been sent to Professor Rogers, of Cambridge, to find its length again. (See page 145.)

Measurement of the Distance between the Mirrors.

Square lead weights were placed along the line, and measurements taken from the forward side of one to forward side of the next. The tape rested on the ground (which was very nearly level), and was stretched by a constant force of 10 pounds.

The correction for length of the tape (100.006) was +0.12 of a foot.

To correct for the stretch of the tape, the latter was stretched with a force of 15 pounds, and the stretch at intervals of 20 feet measured by a millimeter scale.

	mm.
At 100 feet the stretch was	8.0
80 feet the stretch was	5.0
60 feet the stretch was	5.0
40 feet the stretch was	3.5
20 feet the stretch was	1.5
---	---
300	23.00

Weighted mean = 7.7 mm.
For 10 pounds, stretch = 5.1 mm.
= 0.0167 feet.
Correction for whole distance = +0.33 feet.

The following are the values obtained from five separate measurements of the distance between the caps of the piers supporting the revolving mirror and the distant reflector; allowance made in each case for effect of temperature:

	1985.13 feet.
	1985.17 feet.
	1984.93 feet.
	1985.09 feet.
	1985.09 feet.

Mean =	1985.082 feet.
+.70.	Cap of pier to revolving mirror.
+.33.	Correction for stretch of tape.
+.12.	Correction for length of tape.

1986.23.	True distance between mirrors.

Rate of Standard Ut_3 Fork.

The rate of the standard Ut_3 fork was found at the Naval Academy, but as so much depended on its accuracy, another series of determinations of its rate was made, together with Professor Mayer, at the Hoboken Institute of Technology.

Set of determinations made at Naval Academy.

The fork was armed with a tip of copper foil, which was lost during the experiments and replaced by one of platinum having the same weight, 4.6 mgr. The fork, on its resonator, was placed horizontally, the platinum tip just touching the lampblacked cylinder of a Schultze chronoscope. The time was given either by a sidereal break-circuit chronometer or by the break-circuit pendulum of a mean-time clock. In the former case the break-circuit worked a relay which interrupted the current from three Grove cells. The spark from the secondary coil of an inductorium was delivered from a wire near the tip of the fork. Frequently two sparks near together were given, in which case the first alone was used. The rate of the chronometer, the record of which was kept at the Observatory, was very regular, and was found by observations of transits of stars during the week to be +1.3 seconds per day, which is the same as the recorded rate.

Specimen of a Determination of Rate of Ut_3 Fork.

Temp.=27° C. Column 1 gives the number of the spark or the number of the second. Column 2 gives the number of sinuosities or vibrations at the corresponding second. Column 3 gives the difference between 1 and 11, 2 and 12, 3 and 13, etc.

July 4, 1879.

1.	0.1	2552.0	
2.	255.3	2551.7	
3.	510.5	2551.9	
4.	765.6	2551.9	
5.	1020.7	2552.1	
6.	1275.7	2552.0	
7.	1530.7	2551.8	
8.	1786.5	2551.4	
9.	2041.6	2551.7	
10.	2297.0	2551.5	

11.	2552.1	255.180	= mean ÷ 10.
12.	2807.0	+ .699	= reduction for mean time.
13.	3062.4	+ .003	= correction for rate.
14.	3317.5	+ .187	= correction for temperature.

15.	3572.8	256.069	= number of vibrations per second at 65° Fahr.
16.	3827.7		
17.	4082.5		
18.	4335.9		
19.	4593.3		
20.	4848.5		

The correction for temperature was found by Professor Mayer by counting the sound-beats between the standard and another Ut_3 fork, at different temperatures. His result is +.012 vibrations per second for a diminution of 1° Fahr. Using the same method, I arrived at the result +.0125. Adopted +.012.

Résumé of determinations made at Naval Academy.

In the following table the first column gives the date, the second gives the total number of seconds, the third gives the result uncorrected for temperature, the fourth gives the temperature (centigrade), the fifth gives the final result, and the sixth the difference between the greatest and least values obtained in the several determinations for intervals of ten seconds:

July 4	20	255.882	27.0	256.069	0.07
5	19	255.915	26.4	256.089	0.05
5	18	255.911	26.0	256.077	0.02
6	21	255.874	24.7	256.012	0.13
6	9	255.948	24.8	256.087	0.24
7	22	255.938	24.6	256.074	0.05
7	21	255.911	25.3	256.061	0.04
8	20	255.921	26.6	256.100	0.02
8	20	255.905	26.6	256.084	0.06
8	20	255.887	26.6	256.066	0.03

			Mean =	256.072	

In one of the preceding experiments, I compared the two Vt_3 forks while the standard was tracing its record on the cylinder, and also when it was in position as for use in the observations. The difference, if any, was less than .01 vibration per second.

Second determination.

(Joint work with Professor A.M. Mayer, Stevens Institute, Hoboken.)

The fork was wedged into a wooden support, and the platinum tip allowed to rest on lampblacked paper, wound about a metal cylinder, which was rotated by hand Time was given by a break-circuit clock, the rate of which was ascertained, by comparisons with Western Union time-ball, to be 9.87 seconds. The spark from secondary coil of the inductorium passed from the platinum tip, piercing the paper. The size of the spark was regulated by resistances in primary circuit.

The following is a specimen determination:

Column 1 gives the number of the spark or the number of seconds. Column 2 gives the corresponding number of sinuosities or vibrations. Column 3 gives the difference between the 1st and 7th ÷ 6, 2nd and 8th ÷ 6, etc.

1	0.3	255.83	
2	256.1	255.90	
3	511.7	255.90	
4	767.9	255.93	
5	1023.5	255.92	
6	1289.2	256.01	
7	1535.3	255.95	

8	1791.5	255.920	= mean.
9	2047.1	- .028	= correction for rate.

10	2303.5	255.892	
11	2559.0	+ .180	= correction for temperature.

12	2825.3	256.072	= number of vibrations per second at 65° Fahr.
13	3071.0		

In the following *résumé*, column 1 gives the number of the experiments. Column 2 gives the total number of seconds. Column 3 gives the result not corrected for temperature. Column 4 gives the temperature Fahrenheit. Column 5 gives the final result. Column 6 gives the difference between the greatest and least values:

1	13	255.892	80	256.072	0.18
2	11	255.934	81	256.126	0.17
3	13	255.899	81	256.091	0.12
4	13	255.988	75	256.108	0.13
5	11	255.948	75	256.068	0.05
6	12	255.970	75	256.090	0.05
7	12	255.992	75	256.112	0.20
8	11	255.992	76	256.124	0.03
9	11	255.888	81	256.080	0.13
10	13	255.878	81	256.070	0.13

			Mean =	256.094	

Effect of Support and of Scraping.

The standard Vt_3 fork held in its wooden support was compared with another fork on a resonator loaded with wax and making with standard about five beats per second. The standard was free from the cylinder. The beats were counted by coincidences with the ⅕ second beats of a watch.

Specimen.

Coincidences were marked—

At 32 seconds.
37 seconds.
43.5 seconds.
49 seconds.
54.5 seconds.
61.5 seconds.
61.5 - 32 = 29.5.
29.5 ÷ 5 = 5.9 = time of one interval.

Résumé.

1	5.9
2	6.2
3	6.2
4	6.2

Mean = 6.13 = time of one interval between coincidences.

In this time the watch makes 6.13×5 = 30.65 beats, and the forks make 30.65 + 1 = 31.65 beats.

Hence the number of beats per second is 31.65 ÷ 6.13 = 5.163.

Specimen.

Circumstances the same as in last case, except that standard Vt_3 fork was allowed to trace its record on the lampblacked paper, as in finding its rate of vibration.

Coincidences were marked at—

59 seconds.
04 seconds.
10.5 seconds.
17 seconds.

77 - 59 = 18.
18 ÷ 3 = 6.0 = time of one interval.

Résumé.

No.	1	6.0 seconds.	6.31 × 5 = 31.55
	2	6.0 seconds.	+ 1.00
	3	6.7 seconds.	----

4	6.3 seconds.	
5	6.5 seconds.	32.55
6	6.7 seconds.	32.55 ÷ 6.31 = 5.159
7	6.0 seconds.	With fork free 5.163
	----	-----
Mean =	6.31 seconds	Effect of scrape = - .044

Specimen.

Circumstances as in first case, except that both forks were on their resonators.

Coincidences were observed at—

21	seconds.
28	seconds.
36	seconds.
44	seconds.
51	seconds.
60	seconds.

60 - 21 = 39

39 ÷ 5 = 7.8 = time of one interval.

Résumé.

No. 1	7.8	seconds.	7.42 × 5 =	37.10
2	7.1	seconds.	+	1.00
3	7.6	seconds.		-----
4	7.4	seconds.		38.10
5	7.2	seconds.	38.10 ÷ 7.42 =	5.133
	----		(Above)	5.159

Mean = 7.42 seconds. Effect of support and scrape = - .026

Mean of second determination was	256.094
Applying correction (scrape, etc.)	- .026

Corrected mean	256.068
Result of first determination	256.072

Final value	256.070

Note—The result of first determination excludes all work except the series commencing July 4. If previous work is included, and also the result first obtained by Professor Mayer, the result would be 256.089.

256.180
256.036
256.072
256.068

Mean = 256.089

The previous work was omitted on account of various inaccuracies and want of practice, which made the separate results differ widely from each other.

The Formulæ.

The formulæ employed are—

$$(1)\ \tan \varphi = \frac{d}{r}$$

$$(2)\ V = \frac{2592000'' \times D \times n}{\varphi''}$$

φ = angle of deflection.
d = corrected displacement (linear).
r = radius of measurement.
D = twice the distance between the mirrors.
n = number of revolutions per second.
α = inclination of plane of rotation
d = deflection as read from micrometer.
B = number of beats per second between electric Vt_2 fork and standard Vt_3
Cor = correction for temperature of standard Vt3.
V = velocity of light.
T = value of one turn of screw. (Table, page 126.)

Substituting for d, its value or d×T×sec α (log sec α = .00008), and for D its value 3972.46, and reducing to kilometers, the formulæ become—

$$(3)\ \tan \varphi = c' \frac{dT}{r}; \quad \log c' = .51607$$

n
log c = .49670

(4) V = c ---;

φ

D and r are expressed in feet and d′ in millimeters.

Vt_3 fork makes 256.070 vibrations per second at 65° Fahr.

D = 3972.46 feet.

tan α = tangent of angle of inclination of plane of rotation = 0.02 in all but the last twelve observations, in which it was 0.015.

log c′ = .51607 (.51603 in last twelve observations.).

log c = .49670.

The electric fork makes ½(256.070 + B + cor.) vibrations per second, and n is a multiple, submultiple, or simple ratio of this.

Observations.

Specimen Observation.

June 17. sunset. Image good; best in column (4).

The columns are sets of readings of the micrometer for the deflected image of slit.

	112.81	112.80	112.83	112.74	112.79
	81	81	81	76	78
	79	78	78	74	74
	80	75	74	76	74
	79	77	74	76	77
	82	79	72	78	81
	82	73	76	78	77
	76	78	81	79	75
	83	79	74	83	82
	73	73	76	78	82
	-------	-------	-------	-------	-------
Mean =	112.801	112.773	112.769	112.772	112.779
Zero =	0.260	0.260	0.260	0.260	0.260
	-------	-------	-------	-------	-------
d =	112.451	112.513	112.509	112.512	112.519
Temp =	77°	77°	77°	77°	77°
B =	+ 1.500				
Corr =	- .144				

	+ 1.365				
	256.070				
	-------				
n =	257.426	257.43	257.43	257.43	257.43
r =	28.157	28.157	28.157	28.157	28.157

The above specimen was selected because in it the readings were all taken by another and noted down without divulging them till the whole five sets were completed.

The following is the calculation for V:

		1st set.	2d, 3d, and 4th sets.	5th set.
log	c′ =	51607	51607	51607
"	T =	99832	99832	99832
"	d =	05131	05119	05123
		-------	-------	-------
		56570	56558	56562
"	r =	44958	44958	44958
		-------	-------	-------
"	tan φ =	11612	11600	11604
	φ =	2694″.7	2694″.1	2694″.3
"	c =	49670	49670	49670
"	n =	41066	41066	41066
		-------	-------	-------
		90736	90736	90736
"	φ =	43052	43042	43046
		-------	-------	-------
"	V =	47684	47694	47690
	V =	299800	299880	299850

In the following table, the numbers in the column headed "Distinctness of Image" are thus translated: 3, good; 2, fair; 1, poor. These numbers do not, however, show the relative weights of the observations.

The numbers contained in the columns headed "Position of Deflected Image," "Position of Slit," and displacement of image in divisions were obtained as described in the paragraph headed "Micrometer," page 120.

The column headed "B" contains the number of "beats" per second between the electric Vt_2 fork and the standard Vt_3 as explained in the paragraph headed "Measurement of the Speed of Rotation." The column headed "Cor." contains the correction of the rate of the standard fork for the difference in temperature of experiment and 65° Fahr., for which temperature the rate was found. The numbers in the column headed "Number of revolutions per second" were found by applying the corrections in the two preceding columns to the rate of the standard, as explained in the same paragraph.

The "radius of measurement" is the distance between the front face of the revolving mirror and the cross-hair of the micrometer.

The numbers in the column headed "Value of one turn of the screw" were taken from the table, page 127.

Distinctness	Temperature,	Position of	Position	Displacement	Difference between	Number of revolutions	Radius of	Value of one	Velocity of light in air,

Date.	of image.	Fahr.	deflected image.	of slit.	of image in divisions.	greatest and least values.	B.	Cor.	per second.	measurement, in feet.	turn of the screw.	in kilometers.	Remarks.
June 5	3	76	114.85	0.300	114.55	0.17	1.423	-0.132	257.36	28.672	0.99614	299850	Electric light.
June 7	2	72	114.64	0.074	114.56	0.10	1.533	-0.084	257.52	28.655	0.99614	299740	P.M. Frame inclined at various angles
June 7	2	72	114.58	0.074	114.50	0.08	1.533	-0.084	257.52	28.647	0.99614	299900	P.M. Frame inclined at various angles
June 7	2	72	85.91	0.074	85.84	0.12	1.533	-0.084	193.14	28.647	0.99598	300070	P.M. Frame inclined at various angles
June 7	2	72	85.97	0.074	85.89	O.07	1.533	-0.084	193.14	28.650	0.99598	299930	P.M. Frame inclined at various angles
June 7	2	72	114.61	0.074	114-53	0.07	1.533	-0.084	257.42	28.650	0.99614	299850	P.M. Frame inclined at various angles
June 9	3	83	114.54	0.074	114.47	0.07	1.533	-0.216	257.39	28.658	0.99614	299950	P.M. Frame inclined at various angles
June 9	3	83	114.54	0.074	114.46	0.10	1.533	-0.216	257.39	28.658	0.99614	299980	P.M. Frame inclined at various angles
June 9	3	83	114.57	0.074	114.47	0.08	1.533	-0.216	257.39	28.662	0.99614	299980	P.M. Frame inclined at various angles
June 9	3	83	114.57	0.074	114.50	0.06	1.533	-0.216	257.39	28.660	0.99614	299880	P.M. Frame inclined at various angles
June 9	2	83	114.61	0.074	114.53	0.13	1.533	-0.216	257.39	28.678	0.99614	300000	P.M. Frame inclined at various angles
June 10	2	90	114.60	0.074	114.52	0.11	1.517	-0.300	257.29	28.685	0.99614	299980	P.M.
June 10	2	90	114.62	0.074	114.54	0.08	1.517	-0.300	257.29	28.685	0.99614	299930	P.M.
June 12	2	71	114.81	0.074	114.74	0.09	1.450	-0.072	257.45	28.690	0.99614	299650	A.M.
June 12	2	71	114.78	0.074	114.70	0.05	1.450	-0.072	257.45	28.690	0.99614	299760	A.M.
June 12	1	71	114.76	0.074	114.68	0.09	1.450	-0.072	257.45	28.690	0.99614	299810	A.M.
June 13	3	72	112.64	0.074	112.56	0.09	1.500	-0.084	257.49	28.172	0.99614	300000	A.M.
June 13	3	72	112.63	0.074	112.56	0.10	1.500	-0.084	257.49	28.172	0.99614	300000	A.M.
June 13	2	72	112.65	0.074	112.57	0.08	1.500	-0.084	257.49	28.172	0.99614	299960	A.M.
June 13	3	79	112.82	0.260	112.56	0.06	1.517	-0.168	257.42	28.178	0.99614	299960	P.M.
June 13	3	79	112.82	0.260	112.56	0.13	1.517	-0.168	257.42	28.178	0.99614	299960	P.M.
June 13	3	79	112.83	0.260	112.57	0.07	1.517	-0.168	257.42	28.178	0.99614	299940	P.M.
June 13	3	79	112.82	0.260	112.56	0.06	1.517	-0.168	257.42	28.178	0.99614	299960	P.M.
June 13	3	79	112.83	0.260	112.57	0.11	1.517	-0.168	257.42	28.178	0.99614	299940	P.M.
June 13	3	79	113.41	0.260	113.15	11	1.517	-0.168	258.70	28.152	0.99614	299880	P.M. Set micrometer and counted oscillations.
June 13	3	79	112.14	0.260	111.88	6	1.517	-0.168	255.69	28.152	0.99614	299800	Oscillations of image of revolving mirror.
June 14	1	64	112.83	0.260	112.57	0.12	1.500	+0.012	257.58	28.152	0.99614	299850	A.M.
June 14	1	64	112.83	0.260	112.57	0.05	1.517	+0.012	257.60	28.152	0.99614	299880	A.M.
June 14	1	65	112.81	0.260	112.55	0.11	1.517	0.000	257.59	28.152	0.99614	299900	A.M.
June 14	1	66	112.83	0.260	112.57	0.09	1.517	-0.012	257.57	28.152	0.99614	299840	A.M.
June 14	1	67	112.83	0.260	112.57	0.12	1.517	-0.024	257.56	28.152	0.99614	299830	A.M.
June 14	1	84	112.78	0.260	112.52	0.06	1.517	-0.228	257.36	28.159	0.99614	299790	P.M. Readings taken by Lieut. Nazro.
June 14	1	85	112.76	0.260	112.50	0.08	1.500	-0.240	257.33	28.159	0.99614	299810	P.M. Readings taken by Lieut. Nazro.
June 14	1	84	112.72	0.260	112.46	0.08	1.483	-0.228	257.32	28.159	0.99614	299880	P.M. Readings taken by Lieut. Nazro.
June 14	1	84	112.73	0.260	112.47	0.09	1.483	-0.228	257.32	28.159	0.99614	299880	P.M.
June 14	1	84	112.75	0.260	112.49	0.09	1.483	-0.228	257.32	28.129	0.99614	299830	P.M.
June 17	2	62	112.85	0.260	112.59	0.09	1.517	+0.036	257.62	28.149	0.99614	299800	A.M.
June 17	2	63	112.84	0.260	112.58	0.06	1.500	+0.024	257.59	28.149	0.99614	299790	A.M.
June 17	1	64	112.85	0.260	112.59	0.07	1.500	+0.012	257.58	28.149	0.99614	299760	A.M.

June 17	3	77	112.80	0.260	112.54	0.07	1.500	-0.144	257-43	28.157	0.99614	299800	P.M. Readings taken by Mr. Clason.
June 17	3	77	112.77	0.260	112.51	0.08	1.500	-0.144	257.43	28.157	0.99614	299880	P.M. Readings taken by Mr. Clason.
June 17	3	77	112.77	0.260	112.51	0.11	1.500	-0.144	257.43	28.157	0.99614	299880	P.M. Readings taken by Mr. Clason.
June 17	3	77	112.77	0.260	112.51	0.09	1.500	-0.144	257.43	28.157	0.99614	299880	P.M. Readings taken by Mr. Clason.
June 17	3	77	112.78	0.260	112.52	0.08	1.500	-0.144	257 43	28.157	0.99614	299860	P.M. Readings taken by Mr. Clason.
June 18	1	58	112.90	0.265	112.64	0.07	1.500	+0.084	257.65	28.150	0.99614	299720	A.M.
June 18	1	58	112.90	0.265	112.64	0.10	1.500	+0.084	257.65	28.150	0.99614	299720	A.M.
June 18	1	59	112.92	0.265	112.66	0.07	1.483	+0.072	257.62	28.150	0.99614	299620	A.M.
June 18	2	75	112.79	0.265	112.52	0.09	1.483	-0.120	257-43	28.158	0.99614	299860	P.M.
June 18	2	75	112.75	0.265	112.48	0.10	1.483	-0.120	257-43	28.158	0.99614	299970	P.M.
June 18	2	75	112.76	0.265	112.49	0.08	1.483	-0.120	257-43	28.158	0.99614	299950	P.M.
June 20	3	60	112.94	0.265	112.67	0.07	1.517	+0.063	257.65	28.172	0.99614	299880	A.M.
June 20	3	61	112.92	0.265	112.65	0.09	1.517	+0.048	257.63	28.172	0.99614	299910	A.M.
June 20	2	62	112.94	0.265	112.67	0.07	1.517	+0.036	257.62	28.172	0.99614	299850	A.M.
June 20	2	63	112.93	0.265	112.66	0.03	1.517	+0.024	257.61	28.172	0.99614	299870	A.M.
June 20	2	78	133.48	0.265	133.21	0.13	1.450	-0.156	257.36	33.345	0.99627	299840	P.M.
June 20	2	79	133.49	0.265	133.23	0.09	1.500	-0.168	257.40	33.345	0.99627	299840	P.M.
June 20	2	80	133.49	0.265	133.22	0.07	1.500	-0.180	257.39	33.345	0.99627	299850	P.M.
June 20	2	79	133.50	0.265	133.24	0.13	1.483	-0.168	257.39	33.345	0.99627	299840	P.M.
June 20	2	79	133.49	0.265	133.22	0.06	1.483	-0.168	257.38	33.345	0.99627	299840	P.M.
June 20	2	79	133.49	0.265	133.22	0.10	1.483	-0.168	257.38	33.345	0.99627	299840	P.M.
June 21	2	61	133.56	0.265	133.29	0.12	1.533	+0.048	257.65	33.332	0.99627	299890	A.M.
June 21	2	62	133.58	0.265	133.31	0.08	1.533	+0.036	257.64	33.332	0.99627	299810	A.M.
June 21	2	63	133.57	0.265	133.31	0.09	1.533	+0.024	257.63	33.332	0.99627	299810	A.M.
June 21	2	64	133.57	0.265	133.30	0.11	1.533	+0.012	257.61	33.332	0.99627	299820	A.M.
June 21	2	65	133.56	0.265	133.30	0.13	1.533	0.000	257.60	33.332	0.99627	299800	A.M.
June 21	3	80	133.48	0.265	133.21	0.06	1.533	-0.180	257.42	33.330	0.99627	299770	P.M.
June 21	3	81	133.46	0.265	133.19	0.10	1.500	-0.192	257.38	33.330	0.99627	299760	P.M.
June 21	3	82	133.46	0.265	133.20	0.05	1.500	-0.204	257.37	33.330	0.99627	299740	P.M.
June 21	3	82	133.46	0.265	133.20	0.08	1.517	-0.204	257.38	33.330	0.99627	299750	P.M.
June 21	3	81	133.46	0.265	133.19	0.08	1.500	-0.192	257.38	33.330	0.99627	299760	P.M.
June 23	3	89	133.43	0.265	133.16	0.08	1.542	-0.288	257.32	33.345	0.99627	299910	P.M.
June 23	3	89	133.42	0.265	133.15	0.06	1.550	-0.288	257.33	33.345	0.99627	299920	P.M.
June 23	3	90	133.43	0.265	133.17	0.09	1.550	-0.300	257.32	33.345	0.99627	299890	P.M.
June 23	3	90	133.43	0.265	133.16	0.07	1.533	-0.300	257.30	33.345	0.99627	299860	P.M.
June 23	3	90	133.42	0.265	133.16	0.07	1.517	-0.300	257.29	33.345	0.99627	299880	P.M.
June 24	3	72	133.47	0.265	133.20	0.15	1.517	-0.084	257.50	33.319	0.99627	299720	A.M.
June 24	3	73	133.44	0.265	133.17	0.04	1.517	-0.096	257.49	33.319	0.99627	299840	A.M.
June 24	3	74	133.42	0.265	133.16	0.11	1.517	-0.108	257.48	33.319	0.99627	299850	A.M.
June 24	3	75	133.42	0.265	133.16	0.06	1.517	-0.120	257.47	33.319	0.99627	299850	A.M.
June 24	3	76	133.44	0.265	133.18	0.10	1.517	-0.132	257.45	33.319	0.99627	299780	A.M.
June 26	2	86	133.42	0.265	133.15	0.05	1.508	-0.252	257.33	33.339	0.99627	299890	P.M.
June 26	2	86	133.44	0.265	133.17	0.08	1.508	-0.252	257.33	33.339	0.99627	299840	P.M.
June 27	3	73	133.49	0.265	133.22	0.11	1.483	-0.096	257.46	33.328	0.99627	299780	A.M.
June 27	3	74	133.47	0.265	133.20	0.06	1.483	-0.108	257.44	33.328	0.99627	299810	A.M.
June 27	3	75	133.47	0.265	133.21	0.09	1.483	-0.120	257.43	33.328	0.99627	299760	A.M.
June 27	3	75	133.45	0.265	133.19	0.09	1.467	-0.120	257.42	33.328	0.99627	299810	A.M.
June 27	3	76	133.47	0.265	133.20	0.08	1.483	-0.132	257.42	33.328	0.99627	299790	A.M.
June 27	3	76	133.45	0.265	133.19	0.10	1.483	-0.132	257.42	33.328	0.99627	299810	A.M.
June 30	2	85	35.32	135.00	99.68	0.05	1.500	-0.240	193.00	33.274	0.99645	299820	P.M. Mirror inverted.
June 30	2	86	35.34	135.00	99.67	0.06	1.508	-0.252	193.00	33.274	0.99645	299850	P.M. Mirror inverted.
June 30	2	86	35.34	135.00	99.66	0.10	1.508	-0.252	193.00	33.274	0.99645	299870	P.M. Mirror inverted.
June 30	2	86	35.34	135.00	99.66	0.09	1.517	-0.252	193.00	33.274	0.99645	299870	P.M. Mirror inverted.
July 1	2	83	02.17	135.145	132.98	0.07	1.500	-0.216	257.35	33.282	0.99627	299810	P.M. Mirror inverted.
July 1	2	84	02.15	135.145	133.00	0.09	1.500	-0.228	257.34	33.282	0.99627	299740	P.M. Mirror inverted.
July 1	2	86	02.14	135.145	133.01	0.06	1.467	-0.252	257.28	33.311	0.99627	299810	P.M. Mirror inverted.
July 1	2	86	02.14	135.145	133.00	0.08	1.467	-0.252	257.28	33.311	0.99627	299940	P.M. Mirror inverted.
July 2	3	86	99.85	0.400	99.45	0.05	1.450	-0.252	192.95	33.205	0.99606	299950	P.M. Mirror erect.
July 2	3	86	66.74	0.400	66.34	0.03	1.450	-0.252	128.63	33.205	0.99586	299800	P.M. Mirror erect.
July 2	3	86	50.16	0.400	47.96	0.07	1.467	-0.252	96.48	33.205	0.99580	299810	P.M. Mirror erect.
													P.M. Mirror

July 2	3	85	33.57	0.400	33.17	0.06	1.450	-0.240	64.32	33.205	0.99574	299870 erect.

In the last two sets of June 13, the micrometer was fixed at 113.41 and 112.14 respectively. The image was bisected by the cross-hair, and kept as nearly as possible in this place, meantime counting the number of seconds required for the image of the revolving mirror to complete 60 oscillations. In other words, instead of measuring the deflection, the speed of rotation was measured. In column 7 for these two sets, the numbers 11 and 6 are the differences between the greatest and the smallest number of seconds observed.

In finding the mean value of V from the table, the sets are all given the same weight. The difference between the result thus obtained and that from any system of weights is small, and may be neglected.

The following table gives the result of different groupings of sets of observations. Necessarily some of the groups include others:

Electric light (1 set)	299850
Set micrometer counting oscillations (2)	299840
Readings taken by Lieutenant Nazro (3)	299830
Readings taken by Mr. Clason (5)	299860
Mirror inverted (8)	299840
Speed of rotation, 192 (7)	299990
Speed of rotation, 128 (1)	299800
Speed of rotation, 96 (1)	299810
Speed of rotation, 64 (1)	299870
Radius, 28.5 feet (54)	299870
Radius, 33.3 feet (46)	299830
Highest temperature, 90° Fahr. (5)	299910
Mean of lowest temperatures, 60° Fahr. (7)	299800
Image, good (46)	299860
Image, fair (39)	299860
Image, poor (15)	299810
Frame, inclined (5)	299960
Greatest value	300070
Least value	299650
Mean value	299852
Average difference from mean	60
Value found for π	3.26
Probable error	± 5

Discussion of Errors.

The value of V depends on three quantities D, n, and φ. These will now be considered in detail.

The Distance.

The distance between the two mirrors may be in error, either by an erroneous determination of the length of the steel tape used, or by a mistake in the measurement of the distance by the tape.

The first may be caused by an error in the copy of the standard yard, or in the comparison between the standard and the tape. An error in this copy, of .00036 inch, which, for such a copy, would be considered large, would produce an error of only .00001 in the final result. Supposing that the bisections of the divisions are correct to .0005 inch, which is a liberal estimate, the error caused by supposing the error in each yard to be in the same direction would be only .000014; or the total error of the tape, if both errors were in the same direction, would be 000024 of the whole length.

The calculated probable error of the five measurements of the distance was ±.000015; hence the total error due to D would be at most .00004. The tape has been sent to Professor Rogers, of Cambridge, for comparison, to confirm the result.

The Speed of Rotation.

This quantity depends on three conditions. It is affected, first, by an error in the rate of the standard; second, by an error in the count of the sound beats between the forks; and third, by a false estimate of the moment when the image of the revolving mirror is at rest, at which moment the deflection is measured.

The calculated probable error of the rate is .000016. If this rate should be questioned, the fork can be again rated and a simple correction applied. The fork is carefully kept at the Stevens Institute, Hoboken, and comparisons were made with two other forks, in case it was lost or injured.

In counting the sound beats, experiments were tried to find if the vibrations of the standard were affected by the other fork, but no such effect could be detected. In each case the number of beats was counted correctly to .02, or less than .0001 part, and in the great number of comparisons made this source of error could be neglected.

The error due to an incorrect estimate of the exact time when the images of the revolving mirror came to rest was eliminated by making the measurement sometimes when the speed was slowly increasing, and sometimes when slowly decreasing. Further, this error would form part of the probable error deduced from the results of observations.

We may then conclude that the error, in the measurement of *n*, was less than .00002.

The Deflection.

The angle of deflection φ was measured by its tangent, tan φ = d/r; d was measured by the steel screw and brass scale, and r by the steel tape.

The value of one turn of the screw was found by comparison with the standard meter for all parts of the screw. This measurement, including the possible error of the copy of the standard meter, I estimate to be correct to .00005 part. The instrument is at the Stevens Institute, where it is to be compared with a millimeter scale made by Professor Rogers, of Cambridge.

The deflection was read to within three or four hundredths of a turn at each observation, and this error appears in the probable error of the result.

The deflection is also affected by the inclination of the plane of rotation to the horizon. This inclination was small, and its secant varies slowly, so that any slight error in this angle would not appreciably affect the result.

The measurement of r is affected in the same way as D, so that we may call the greatest error of this measurement .00004. It would probably be less than this, as the mistakes in the individual measurements would also appear in the probable error of the result.

The measurement of φ was not corrected for temperature. As the corrections would be small they may be applied to the final result. For an increase of 1° F. the correction to be applied to the screw for unit length would be -.0000066. The correction for the brass scale would be +.0000105, or the whole correction for the micrometer would be +.000004. The correction for the steel tape used to measure r would be +.0000066. Hence the correction for tan. φ would be -.000003 t. The average temperature of the experiments is 75°.6 F. 75.6-62.5 = 13.1. -.000003×13.1 = -.00004

Hence φ should be divided by 1.00004, or the final result should be multiplied by 1.00004. This would correspond to a correction of +12 kilometers.

The greatest error, excluding the one just mentioned, would probably be less than .00009 in the measurement of φ.

Summing up the various errors, we find, then, that the total constant error, in the most unfavorable case, where the errors are all in the same direction, would be .00015. Adding to this the probable error of the result, .00002, we have for the limiting value of the error of the final result ±.00017. This corresponds to an error of ±51 kilometers.

The correction for the velocity of light in vacuo is found by multiplying the speed in air by the index of refraction of air, at the temperature of the experiments. The error due to neglecting the barometric height is exceedingly small. This correction, in kilometers, is +80.

Final Result.

The mean value of V from the tables is	299852
Correction for temperature	+12

Velocity of light in air	299864
Correction for vacuo	80

Velocity of light in vacuo	299944±51

The final value of the velocity of light from these experiments is then—299940 kilometers per second, or 186380 miles per second.

Objections Considered.

Measurement of the Deflection.

The chief objection, namely, that in the method of the revolving mirror the deflection is small, has already been sufficiently answered. The same objection, in another form, is that the image is more or less indistinct. This is answered by a glance at the tables. These show that in each individual observation the average error was only three ten-thousandths of the whole deflection.

Uncertainty of Laws of Reflection and Refraction in Media in Rapid Rotation.

What is probably hinted at under the above heading is that there may be a possibility that the rapid rotation of the mirror throws the reflected pencil in the direction of rotation. Granting that this is the case, an inspection of Fig. 14 shows that the deflection will not be affected.

In this figure let *m m* be the position of the mirror when the light first falls on it from the slit at *a*, and *m′ m′* the position when the light returns.

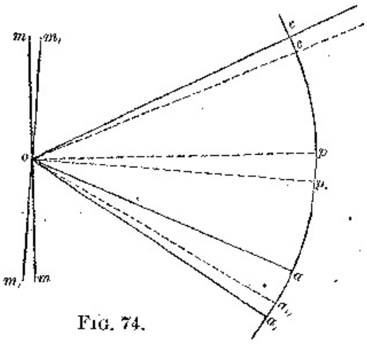

Fig. 74.

From the axis *o* draw *op op*, perpendicular to *m m* and to *m′ m′*, respectively. Then, supposing there is no such effect, the course of the axis of the pencil of light would be *a o c* mirror *c o a′*. That is, the angle of deflection would be *a o a′*, double the angle *p o p′*. If now the mirror be supposed to carry the pencil with it, let *o c′* be the direction of the pencil on leaving the mirror *m m*; i.e., the motion of the mirror has changed the direction of the reflected ray through the angle *c o c′*. The course would then be *a o c*, mirror *c′ o*. From *o* the reflection would take place in the direction *a″*, making the angles *c′ o p*, and *p′ o a″* equal. But the angle *c o c′* must be added to *p o a″*, in consequence of the motion of the mirror, or the angle of deviation will be *a o a″ + c o c′*; or *a o a″ + c o c′ = d*. (1)

By construction—

$$c\,o\,p' = p'\,o\,a' \quad (2)$$
$$c'\,o\,p' = p'\,o\,a'' \quad (3)$$

Subtracting (3) from (2) we have—

$$c\,o\,p' - c'\,o\,p' = p'\,o\,a' - p'\,o\,a'', \text{ or}$$
$$c\,o\,c' = a'\,o\,a''$$

Substituting *a′ o a″* for *c o c′* in (1) we have— *a o a″ + a′ o a″ = a o a′ = d*.

Or the deflection has remained unaltered.

Retardation Caused by Reflection.

Cornu, in answering the objection that there may be an unknown retardation by reflection from the distant mirror, says that if such existed the error it would introduce in his own work would be only 1/7000 that of Foucault, on account of the great distance used, and on account of there being in his own experiments but one reflection instead of twelve.

In my own experiments the same reasoning shows that if this possible error made a difference of 1 per cent. in Foucault's work (and his result is correct within that amount), then the error would be but .00003 part.

Distortion of the Revolving Mirror.

It, has been suggested that the distortion of the revolving mirror, either by twisting or by the effect of centrifugal force, might cause an error in the deflection.

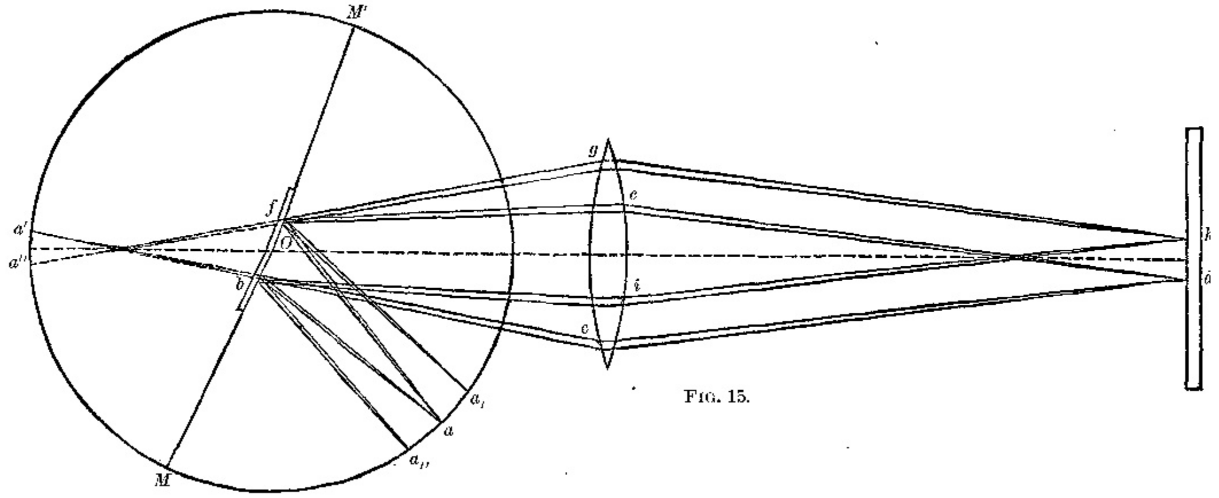

Fig. 15.

The only plane in which the deflection might be affected is the plane of rotation. Distortions in a vertical plane would have simply the effect of raising, lowering, or extending the slit.

Again, if the *mean* surface is plane there will be no effect on the deflection, but simply a blurring of the image.

Even if there be a distortion of any kind, there would be no effect on the deflection if the rays returned to the same portion whence they were reflected.

The only case which remains to be considered, then, is that given in Fig. 15, where the light from the slit *a*, falls upon a distorted mirror, and the return light upon a different portion of the same.

The one pencil takes the course *a b c d e f a′*, while the other follows the path *a f g h i b a′*.

In other words, besides the image coinciding with *a*, there would be two images, one on either side of *a*, and in case there were more than two portions having different inclinations there would be formed as many images to correspond. If the surfaces are not plane, the only effect is to produce a distortion of the image.

As no multiplication of images was observed, and no distortion of the one image, it follows that the distortion of the mirror was too small to be noticed, and that even if it were larger it could not affect the deflection.

The figure represents the distorted mirror at rest, but the reasoning is the same when it is in motion, save that all the images will be deflected in the direction of rotation.

Imperfection of the Lens.

It has also been suggested that, as the pencil goes through one-half of the lens and returns through the opposite half, if these two halves were not exactly similar, the return image would not coincide with the slit when the mirror was at rest. This would undoubtedly be true if we consider but one-half of the original pencil. It is evident, however, that the other half would pursue the contrary course, forming another image which falls on the other side of the slit, and that both these images would come into view, and the line midway between them would coincide with the true position. No such effect was observed, and would be very unlikely to occur. If the lens was imperfect, the faults would be all over the surface, and this would produce simply an indistinctness of the image.

Moreover, in the latter part of the observations the mirror was inverted, thus producing a positive rotation, whereas the rotation in the preceding sets was negative. This would correct the error mentioned if it existed, and shows also that no constant errors were introduced by having the rotation constantly in the same direction, the results in both cases being almost exactly the same.

Periodic Variations in Friction.

If the speed of rotation varied in the same manner in each revolution of the mirror, the chances would be that, at the particular time when the reflection took place, the speed would not be the same as the average speed found by the calculation. Such a periodic variation could only be caused by the influence of the frame or the pivots. For instance, the frame would be closer to the ring which holds the mirror twice in every revolution than at other times, and it would be more difficult for the mirror to turn here than at a position 90° from this. Or else there might be a certain position, due to want of trueness of shape of the sockets, which would cause a variation of friction at certain parts of the revolution.

To ascertain if there were any such variations, the position of the frame was changed in azimuth in several experiments. The results were unchanged showing that any such variation was too small to affect the result.

Change of Speed of Rotation.

In the last four sets of observations the speed was lowered from 256 turns to 192, 128, 96, and 64 turns per second. The results with these speeds were the same as with the greater speed within the limits of errors of experiment.

Bias.

Finally, to test the question if there were any bias in taking these observations, eight sets of observations were taken, in which the readings were made by another, the results being written down without divulging them. Five of these sets are given in the "specimen," pages 133-134.

It remains to notice the remarkable coincidence of the result of these experiments with that obtained by Cornu by the method of the "toothed wheel."

Cornu's result was 300400 kilometers, or as interpreted by Helmert 299990 kilometers. That of these experiments is 299940 kilometers.

Postscript.

The comparison of the micrometer with two scales made by Mr. Rogers, of the Harvard Observatory, has been completed. The scales were both on the same piece of silver, marked "Scales No. 25, on silver. Half inch at 58° F., too short .000009 inch. Centimeter at 67° F., too short .00008 cm."

It was found that the ratio .3937079 could be obtained almost exactly, if, instead of the centimeter being too short, it were too *long* by .00008 cm. at 67°.

On this supposition the following tables were obtained. They represent the value of one turn of the micrometer in millimeters.

Table 1 is the result from centimeter scale.

Table 2 is the result from half-inch scale.

Table 3 is the result from page 31.

It is seen from the correspondence in these results, that the previous work is correct.

	(1)	(2)	(3)
From 0 to 13	.99563	.99562	.99570
25	.99562	.99564	.99571
38	.99560	.99572	.99576
51	.99567	.99578	.99580
64	.99577	.99586	.99585
76	.99582	.99590	.99592
89	.99590	.99598	.99601
102	.99596	.99608	.99605
115	.99606	.99614	.99615
128	.99618	.99622	.99623
140	.99629	.99633	.99630