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## THE ENCYCLOPEDIA BRITANNICA

## A DICTIONARY OF ARTS, SCIENCES, LITERATURE AND GENERAL INFORMATION

## ELEVENTH EDITION

## VOLUME XIV SLICE II

Hydromechanics to Ichnography

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HYDROMECHANICS ( $\dot{\delta} \delta \rho о \mu \eta \chi \alpha \nu \iota \kappa \alpha ́)$, the science of the mechanics of water and fluids in general, including hydrostatics or the mathematical theory of fluids in equilibrium, and hydromechanics, the theory of fluids in motion. The practical application of hydromechanics forms the province of hydraulics (q.v.).

Historical.-The fundamental principles of hydrostatics were first given by Archimedes in his
 afterwards applied to experiments by Marino Ghetaldi (1566-1627) in his Promotus Archimedes (1603). Archimedes maintained that each particle of a fluid mass, when in equilibrium, is equally pressed in every direction; and he inquired into the conditions according to which a solid body floating in a fluid should assume and preserve a position of equilibrium.
In the Greek school at Alexandria, which flourished under the auspices of the Ptolemies, the first attempts were made at the construction of hydraulic machinery, and about 120 b.c. the fountain of compression, the siphon, and the forcing-pump were invented by Ctesibius and Hero. The siphon is a simple instrument; but the forcing-pump is a complicated invention, which could scarcely have been expected in the infancy of hydraulics. It was probably suggested to Ctesibius by the Egyptian Wheel or Noria, which was common at that time, and which was a kind of chain pump, consisting of a number of earthen pots carried round by a wheel. In some of these machines the pots have a valve in the bottom which enables them to descend without much resistance, and diminishes greatly the load upon the wheel; and, if we suppose that this valve was introduced so early as the time of Ctesibius, it is not difficult to perceive how such a machine might have led to the invention of the forcing-pump.

Notwithstanding these inventions of the Alexandrian school, its attention does not seem to have been directed to the motion of fluids; and the first attempt to investigate this subject was made by Sextus Julius Frontinus, inspector of the public fountains at Rome in the reigns of Nerva and Trajan. In his work De aquaeductibus urbis Romae commentarius, he considers the methods which were at that time employed for ascertaining the quantity of water discharged from ajutages, and the mode of distributing the waters of an aqueduct or a fountain. He remarked that the flow of water from an orifice depends not only on the magnitude of the orifice itself, but also on the height of the water in the reservoir; and that a pipe employed to carry off a portion of water from an aqueduct should, as circumstances required, have a position more or less inclined to the original direction of the current. But as he was unacquainted with the law of the velocities of running water as depending upon the depth of the orifice, the want of precision which appears in his results is not surprising.
Benedetto Castelli (1577-1644), and Evangelista Torricelli (1608-1647), two of the disciples of Galileo, applied the discoveries of their master to the science of hydrodynamics. In 1628 Castelli published a small work, Della misura dell' acque correnti, in which he satisfactorily explained several phenomena in the motion of fluids in rivers and canals; but he committed a great paralogism in supposing the velocity of the water proportional to the depth of the orifice below the surface of the vessel. Torricelli, observing that in a jet where the water rushed through a small ajutage it rose to nearly the same height with the reservoir from which it was supplied, imagined that it ought to move with the same velocity as if it had fallen through that height by the force of gravity, and hence he deduced the proposition that the velocities of liquids are as the square root of the head, apart from the resistance of the air and the friction of the orifice. This theorem was published in 1643, at the end of his treatise De motu gravium projectorum, and it was confirmed by the experiments of Raffaello Magiotti on the quantities of water discharged from different ajutages under different pressures (1648).

In the hands of Blaise Pascal (1623-1662) hydrostatics assumed the dignity of a science, and in a treatise on the equilibrium of liquids (Sur l'équilibre des liqueurs), found among his manuscripts after his death and published in 1663, the laws of the equilibrium of liquids were demonstrated in the most simple manner, and amply confirmed by experiments.

The theorem of Torricelli was employed by many succeeding writers, but particularly by Edmé Mariotte (1620-1684), whose Traité du mouvement des eaux, published after his death in the year 1686, is founded on a great variety of well-conducted experiments on the motion of fluids, performed at Versailles and Chantilly. In the discussion of some points he committed considerable mistakes. Others he treated very superficially, and in none of his experiments apparently did he attend to the diminution of efflux arising from the contraction of the liquid vein, when the orifice is merely a perforation in a thin plate; but he appears to have been the first who attempted to ascribe the discrepancy between theory and experiment to the retardation of the water's velocity through friction. His contemporary Domenico Guglielmini (1655-1710), who was inspector of the rivers and canals at Bologna, had ascribed this diminution of velocity in rivers to transverse motions arising from inequalities in their bottom. But as Mariotte observed similar obstructions even in glass pipes where no transverse currents could exist, the cause assigned by Guglielmini seemed destitute of foundation. The French philosopher, therefore, regarded these obstructions as the effects of friction. He supposed that the filaments of water which graze along the sides of the pipe lose a portion of their velocity; that the contiguous filaments, having on this account a greater velocity, rub upon the former,
and suffer a diminution of their celerity; and that the other filaments are affected with similar retardations proportional to their distance from the axis of the pipe. In this way the medium velocity of the current may be diminished, and consequently the quantity of water discharged in a given time must, from the effects of friction, be considerably less than that which is computed from theory.

The effects of friction and viscosity in diminishing the velocity of running water were noticed in the Principia of Sir Isaac Newton, who threw much light upon several branches of hydromechanics. At a time when the Cartesian system of vortices universally prevailed, he found it necessary to investigate that hypothesis, and in the course of his investigations he showed that the velocity of any stratum of the vortex is an arithmetical mean between the velocities of the strata which enclose it; and from this it evidently follows that the velocity of a filament of water moving in a pipe is an arithmetical mean between the velocities of the filaments which surround it. Taking advantage of these results, Henri Pitot (1695-1771) afterwards showed that the retardations arising from friction are inversely as the diameters of the pipes in which the fluid moves. The attention of Newton was also directed to the discharge of water from orifices in the bottom of vessels. He supposed a cylindrical vessel full of water to be perforated in its bottom with a small hole by which the water escaped, and the vessel to be supplied with water in such a manner that it always remained full at the same height. He then supposed this cylindrical column of water to be divided into two parts,-the first, which he called the "cataract," being an hyperboloid generated by the revolution of an hyperbola of the fifth degree around the axis of the cylinder which should pass through the orifice, and the second the remainder of the water in the cylindrical vessel. He considered the horizontal strata of this hyperboloid as always in motion, while the remainder of the water was in a state of rest, and imagined that there was a kind of cataract in the middle of the fluid. When the results of this theory were compared with the quantity of water actually discharged, Newton concluded that the velocity with which the water issued from the orifice was equal to that which a falling body would receive by descending through half the height of water in the reservoir. This conclusion, however, is absolutely irreconcilable with the known fact that jets of water rise nearly to the same height as their reservoirs, and Newton seems to have been aware of this objection. Accordingly, in the second edition of his Principia, which appeared in 1713, he reconsidered his theory. He had discovered a contraction in the vein of fluid (vena contracta) which issued from the orifice, and found that, at the distance of about a diameter of the aperture, the section of the vein was contracted in the subduplicate ratio of two to one. He regarded, therefore, the section of the contracted vein as the true orifice from which the discharge of water ought to be deduced, and the velocity of the effluent water as due to the whole height of water in the reservoir; and by this means his theory became more conformable to the results of experience, though still open to serious objections. Newton was also the first to investigate the difficult subject of the motion of waves (q.v.).

In 1738 Daniel Bernoulli (1700-1782) published his Hydrodynamica seu de viribus et motibus fluidorum commentarii. His theory of the motion of fluids, the germ of which was first published in his memoir entitled Theoria nova de motu aquarum per canales quocunque fluentes, communicated to the Academy of St Petersburg as early as 1726, was founded on two suppositions, which appeared to him conformable to experience. He supposed that the surface of the fluid, contained in a vessel which is emptying itself by an orifice, remains always horizontal; and, if the fluid mass is conceived to be divided into an infinite number of horizontal strata of the same bulk, that these strata remain contiguous to each other, and that all their points descend vertically, with velocities inversely proportional to their breadth, or to the horizontal sections of the reservoir. In order to determine the motion of each stratum, he employed the principle of the conservatio virium vivarum, and obtained very elegant solutions. But in the absence of a general demonstration of that principle, his results did not command the confidence which they would otherwise have deserved, and it became desirable to have a theory more certain, and depending solely on the fundamental laws of mechanics. Colin Maclaurin (1698-1746) and John Bernoulli (1667-1748), who were of this opinion, resolved the problem by more direct methods, the one in his Fluxions, published in 1742, and the other in his Hydraulica nunc primum detecta, et demonstrata directe ex fundamentis pure mechanicis, which forms the fourth volume of his works. The method employed by Maclaurin has been thought not sufficiently rigorous; and that of John Bernoulli is, in the opinion of Lagrange, defective in clearness and precision. The theory of Daniel Bernoulli was opposed also by Jean le Rond d'Alembert. When generalizing the theory of pendulums of Jacob Bernoulli (1654-1705) he discovered a principle of dynamics so simple and general that it reduced the laws of the motions of bodies to that of their equilibrium. He applied this principle to the motion of fluids, and gave a specimen of its application at the end of his Dynamics in 1743. It was more fully developed in his Traité des fluides, published in 1744, in which he gave simple and elegant solutions of problems relating to the equilibrium and motion of fluids. He made use of the same suppositions as Daniel Bernoulli, though his calculus was established in a very different manner. He considered, at every instant, the actual motion of a stratum as composed of a motion which it had in the preceding instant and of a motion which it had lost; and the laws of equilibrium between the motions lost furnished him with equations representing the motion of the fluid. It remained a desideratum to express by equations the motion of a particle of the fluid
in any assigned direction. These equations were found by d'Alembert from two principles-that a rectangular canal, taken in a mass of fluid in equilibrium, is itself in equilibrium, and that a portion of the fluid, in passing from one place to another, preserves the same volume when the fluid is incompressible, or dilates itself according to a given law when the fluid is elastic. His ingenious method, published in 1752, in his Essai sur la résistance des fluides, was brought to perfection in his Opuscules mathématiques, and was adopted by Leonhard Euler.

The resolution of the questions concerning the motion of fluids was effected by means of Euler's partial differential coefficients. This calculus was first applied to the motion of water by d'Alembert, and enabled both him and Euler to represent the theory of fluids in formulae restricted by no particular hypothesis.

One of the most successful labourers in the science of hydrodynamics at this period was Pierre Louis Georges Dubuat (1734-1809). Following in the steps of the Abbé Charles Bossut (Nouvelles Experiences sur la résistance des fluides, 1777), he published, in 1786, a revised edition of his Principes d'hydraulique, which contains a satisfactory theory of the motion of fluids, founded solely upon experiments. Dubuat considered that if water were a perfect fluid, and the channels in which it flowed infinitely smooth, its motion would be continually accelerated, like that of bodies descending in an inclined plane. But as the motion of rivers is not continually accelerated, and soon arrives at a state of uniformity, it is evident that the viscosity of the water, and the friction of the channel in which it descends, must equal the accelerating force. Dubuat, therefore, assumed it as a proposition of fundamental importance that, when water flows in any channel or bed, the accelerating force which obliges it to move is equal to the sum of all the resistances which it meets with, whether they arise from its own viscosity or from the friction of its bed. This principle was employed by him in the first edition of his work, which appeared in 1779. The theory contained in that edition was founded on the experiments of others, but he soon saw that a theory so new, and leading to results so different from the ordinary theory, should be founded on new experiments more direct than the former, and he was employed in the performance of these from 1780 to 1783 . The experiments of Bossut were made only on pipes of a moderate declivity, but Dubuat used declivities of every kind, and made his experiments upon channels of various sizes.

The theory of running water was greatly advanced by the researches of Gaspard Riche de Prony (1755-1839). From a collection of the best experiments by previous workers he selected eighty-two (fifty-one on the velocity of water in conduit pipes, and thirty-one on its velocity in open canals); and, discussing these on physical and mechanical principles, he succeeded in drawing up general formulae, which afforded a simple expression for the velocity of running water.
J. A. Eytelwein (1764-1848) of Berlin, who published in 1801 a valuable compendium of hydraulics entitled Handbuch der Mechanik und der Hydraulik, investigated the subject of the discharge of water by compound pipes, the motions of jets and their impulses against plane and oblique surfaces; and he showed theoretically that a water-wheel will have its maximum effect when its circumference moves with half the velocity of the stream.
J. N. P. Hachette (1769-1834) in 1816-1817 published memoirs containing the results of experiments on the spouting of fluids and the discharge of vessels. His object was to measure the contracted part of a fluid vein, to examine the phenomena attendant on additional tubes, and to investigate the form of the fluid vein and the results obtained when different forms of orifices are employed. Extensive experiments on the discharge of water from orifices (Expériences hydrauliques, Paris, 1832) were conducted under the direction of the French government by J. V. Poncelet (1788-1867) and J. A. Lesbros (1790-1860). P. P. Boileau (18111891) discussed their results and added experiments of his own (Traité de la mésure des eaux courantes, Paris, 1854). K. R. Bornemann re-examined all these results with great care, and gave formulae expressing the variation of the coefficients of discharge in different conditions (Civil Ingénieur, 1880). Julius Weisbach (1806-1871) also made many experimental investigations on the discharge of fluids. The experiments of J. B. Francis (Lowell Hydraulic Experiments, Boston, Mass., 1855) led him to propose variations in the accepted formulae for the discharge over weirs, and a generation later a very complete investigation of this subject was carried out by H. Bazin. An elaborate inquiry on the flow of water in pipes and channels was conducted by H. G. P. Darcy (1803-1858) and continued by H. Bazin, at the expense of the French government (Recherches hydrauliques, Paris, 1866). German engineers have also devoted special attention to the measurement of the flow in rivers; the Beiträge zur Hydrographie des Königreiches Böhmen (Prague, 1872-1875) of A. R. Harlacher (1842-1890) contained valuable measurements of this kind, together with a comparison of the experimental results with the formulae of flow that had been proposed up to the date of its publication, and important data were yielded by the gaugings of the Mississippi made for the United States government by A. A. Humphreys and H. L. Abbot, by Robert Gordon's gaugings of the Irrawaddy, and by Allen J. C. Cunningham's experiments on the Ganges canal. The friction of water, investigated for slow speeds by Coulomb, was measured for higher speeds by William Froude (1810-1879), whose work is of great value in the theory of ship resistance (Brit. Assoc. Report., 1869), and stream line motion was studied by Professor Osborne Reynolds and by

## Hydrostatics

Hydrostatics is a science which grew originally out of a number of isolated practical problems; but it satisfies the requirement of perfect accuracy in its application to phenomena, the largest and smallest, of the behaviour of a fluid. At the same time, it delights the pure theorist by the simplicity of the logic with which the fundamental theorems may be established, and by the elegance of its mathematical operations, insomuch that hydrostatics may be considered as the Euclidean pure geometry of mechanical science.

1. The Different States of a Substance or Matter.-All substance in nature falls into one of the two classes, solid and fluid; a solid substance, the land, for instance, as contrasted with a fluid, like water, being a substance which does not flow of itself.

A fluid, as the name implies, is a substance which flows, or is capable of flowing; water and air are the two fluids distributed most universally over the surface of the earth.

Fluids again are divided into two classes, termed a liquid and a gas, of which water and air are the chief examples.

A liquid is a fluid which is incompressible or practically so, i.e. it does not change in volume sensibly with change of pressure.

A gas is a compressible fluid, and the change in volume is considerable with moderate variation of pressure.

Liquids, again, can be poured from one open vessel into another, and can be kept in an uncovered vessel, but a gas tends to diffuse itself indefinitely and must be preserved in a closed reservoir.

The distinguishing characteristics of the three kinds of substance or states of matter, the solid, liquid and gas, are summarized thus in O. Lodge's Mechanics:-

A solid has both size and shape.
A liquid has size but not shape.
A gas has neither size nor shape.
2. The Change of State of Matter.-By a change of temperature and pressure combined, a substance can in general be made to pass from one state into another; thus by gradually increasing the temperature a solid piece of ice can be melted into the liquid state of water, and the water again can be boiled off into the gaseous state as steam. Again, by raising the temperature, a metal in the solid state can be melted and liquefied, and poured into a mould to assume any form desired, which is retained when the metal cools and solidifies again; the gaseous state of a metal is revealed by the spectroscope. Conversely, a combination of increased pressure and lowering of temperature will, if carried far enough, reduce a gas to a liquid, and afterwards to the solid state; and nearly every gaseous substance has now undergone this operation.

A certain critical temperature is observed in a gas, above which the liquefaction is impossible; so that the gaseous state has two subdivisions into (i.) a true gas, which cannot be liquefied, because its temperature is above the critical temperature, (ii.) a vapour, where the temperature is below the critical, and which can ultimately be liquefied by further lowering of temperature or increase of pressure.
3. Plasticity and Viscosity.-Every solid substance is found to be plastic more or less, as exemplified by punching, shearing and cutting; but the plastic solid is distinguished from the viscous fluid in that a plastic solid requires a certain magnitude of stress to be exceeded to make it flow, whereas the viscous liquid will yield to the slightest stress, but requires a certain length of time for the effect to be appreciable.

According to Maxwell (Theory of Heat) "When a continuous alteration of form is produced only by a stress exceeding a certain value, the substance is called a solid, however soft and plastic it may be. But when the smallest stress, if only continued long enough, will cause a perceptible and increasing change of form, the substance must be regarded as a viscous fluid, however hard it may be." Maxwell illustrates the difference between a soft solid and a hard liquid by a jelly and a block of pitch; also by the experiment of supporting a candle and a stick of sealing-wax; after a considerable time the sealing-wax will be found bent and so is a fluid, but the candle remains straight as a solid.
4. Definition of a Fluid.-A fluid is a substance which yields continually to the slightest tangential stress in its interior; that is, it can be divided very easily along any plane (given plenty of time if the fluid is viscous). It follows that when the fluid has come to rest, the tangential stress in any plane in its interior must vanish, and the stress must be entirely normal to the plane. This mechanical axiom of the normality of fluid pressure is the foundation of the mathematical theory of hydrostatics.

The theorems of hydrostatics are thus true for all stationary fluids, however viscous they may be; it is only when we come to hydrodynamics, the science of the motion of a fluid, that viscosity will make itself felt and modify the theory; unless we begin by postulating the perfect fluid, devoid of viscosity, so that the principle of the normality of fluid pressure is taken to hold when the fluid is in movement.
5. The Measurement of Fluid Pressure.-The pressure at any point of a plane in the interior of a fluid is the intensity of the normal thrust estimated per unit area of the plane.

Thus, if a thrust of P tb is distributed uniformly over a plane area of A sq. ft ., as on the horizontal bottom of the sea or any reservoir, the pressure at any point of the plane is P/A to per sq. ft., or $\mathrm{P} / 144 \mathrm{~A} \mathrm{t}_{\mathrm{b}}$ per sq. in. ( $\mathrm{\pi} / \mathrm{ft} .{ }^{2}$ and $\mathrm{t} / \mathrm{in}^{2}{ }^{2}$, in the Hospitalier notation, to be employed in the sequel). If the distribution of the thrust is not uniform, as, for instance, on a vertical or inclined face or wall of a reservoir, then P/A represents the average pressure over the area; and the actual pressure at any point is the average pressure over a small area enclosing the point. Thus, if a thrust $\Delta \mathrm{P} \mathrm{tb}$ acts on a small plane area $\Delta \mathrm{Aft}{ }^{2}$ enclosing a point $B$, the pressure $p$ at $B$ is the limit of $\Delta \mathrm{P} / \Delta \mathrm{A}$; and

$$
\begin{equation*}
\mathrm{p}=\mathrm{lt}(\Delta \mathrm{P} / \Delta \mathrm{A})=\mathrm{dP} / \mathrm{dA} \tag{1}
\end{equation*}
$$

in the notation of the differential calculus.
6. The Equality of Fluid Pressure in all Directions.-This fundamental principle of hydrostatics follows at once from the principle of the normality of fluid pressure implied in the definition of a fluid in § 4. Take any two arbitrary directions in the plane of the paper, and draw a small isosceles triangle abc, whose sides are perpendicular to the two directions, and consider the equilibrium of a small triangular prism of fluid, of which the triangle is the cross section. Let $P$, Q denote the normal thrust across the sides bc , ca, and R the normal thrust across the base ab . Then, since these three forces maintain equilibrium, and $R$ makes equal angles with $P$ and $Q$, therefore P and Q must be equal. But the faces bc, ca, over which P and Q act, are also equal, so that the pressure on each face is equal. A scalene triangle abc might also be employed, or a tetrahedron.

It follows that the pressure of a fluid requires to be calculated in one direction only, chosen as the simplest direction for convenience.
7. The Transmissibility of Fluid Pressure.-Any additional pressure applied to the fluid will be transmitted equally to every point in the case of a liquid; this principle of the transmissibility of pressure was enunciated by Pascal, 1653, and applied by him to the invention of the hydraulic press.

This machine consists essentially of two communicating cylinders (fig. 1a), filled with liquid and closed by pistons. If a thrust P tb is applied to one piston of area $\mathrm{A} \mathrm{ft}^{2}{ }^{2}$, it will be balanced by a thrust W ib applied to the other piston of area B ft. ${ }^{2}$, where

$$
\begin{equation*}
\mathrm{p}=\mathrm{P} / \mathrm{A}=\mathrm{W} / \mathrm{B}, \tag{1}
\end{equation*}
$$

the pressure p of the liquid being supposed uniform; and, by making the ratio B/A sufficiently large, the mechanical advantage can be increased to any desired amount, and in the simplest manner possible, without the intervention of levers and machinery.
Fig. $1 b$ shows also a modern form of the hydraulic press, applied to the operation of covering an electric cable with a lead coating.
8. Theorem.-In a fluid at rest under gravity the pressure is the same at any two points in the same horizontal plane; in other words, a surface of equal pressure is a horizontal plane.

This is proved by taking any two points A and B at the same level, and considering the equilibrium of a thin prism of liquid $A B$, bounded by planes at $A$ and $B$ perpendicular to $A B$. $A$ s gravity and the fluid pressure on the sides of the prism act at right angles to AB , the equilibrium requires the equality of thrust on the ends $A$ and $B$; and as the areas are equal, the pressure must be equal at A and B; and so the pressure is the same at all points in the same horizontal plane. If the fluid is a liquid, it can have a free surface without diffusing itself, as a gas would; and this free surface, being a surface of zero pressure, or more generally of uniform atmospheric pressure, will also be a surface of equal pressure, and therefore a horizontal plane.

Hence the theorem.-The free surface of a liquid at rest under gravity is a horizontal plane. This is the characteristic distinguishing between a solid and a liquid; as, for instance, between land and water. The land has hills and valleys, but the surface of water at rest is a horizontal plane; and if disturbed the surface moves in waves.
9. Theorem.-In a homogeneous liquid at rest under gravity the pressure increases uniformly with the depth.

This is proved by taking the two points A and B in the same vertical line, and considering the equilibrium of the prism by resolving vertically. In this case the thrust at the lower end B must exceed the thrust at A , the upper end, by the weight of the prism of liquid; so that, denoting the cross section of the prism by $\alpha \mathrm{ft}^{2}$, the pressure at A and By by $\mathrm{p}_{0}$ and $\mathrm{p} \mathrm{t} / \mathrm{ft} .^{2}$, and by w the density of the liquid estimated in $\mathrm{t} / \mathrm{ft} .^{3}$,

$$
\begin{gather*}
p \alpha-p_{0} \alpha=w \alpha \cdot A B,  \tag{1}\\
p=w \cdot A B+p_{0} . \tag{2}
\end{gather*}
$$



Fig. $1 b$.

Thus in water, where $\mathrm{w}=62.4 \mathrm{tb} / \mathrm{ft}{ }^{3}$, the pressure increases 62.4 $\mathrm{tb} / \mathrm{ft} .^{2}$, or $62.4 \div 144=0.433 \mathrm{\pi b} / \mathrm{in} .^{2}$ for every additional foot of depth.
10. Theorem.-If two liquids of different density are resting in vessels in communication, the height of the free surface of such liquid above the surface of separation is inversely as the density.

For if the liquid of density $\sigma$ rises to the height $h$ and of density $\rho$ to the height $k$, and $p_{0}$ denotes the atmospheric pressure, the pressure in the liquid at the level of the surface of separation will be $\sigma \mathrm{h}+\mathrm{p}_{0}$ and $\rho \mathrm{k}+\mathrm{p}_{0}$, and these being equal we have

$$
\begin{equation*}
\sigma \mathrm{h}=\rho \mathrm{k} . \tag{1}
\end{equation*}
$$

The principle is illustrated in the article BAROMETER, where a column of mercury of density $\sigma$ and height $h$, rising in the tube to the Torricellian vacuum, is balanced by a column of air of density $\rho$, which may be supposed to rise as a homogeneous fluid to a height $k$, called the height of the homogeneous atmosphere. Thus water being about 800 times denser than air and mercury 13.6 times denser than water,

$$
\begin{equation*}
\mathrm{k} / \mathrm{h}=\sigma / \rho=800 \times 13.6=10,880 ; \tag{2}
\end{equation*}
$$

and with an average barometer height of 30 in . this makes $\mathrm{k} 27,200 \mathrm{ft}$., about 8300 metres.
11. The Head of Water or a Liquid.-The pressure $\sigma \mathrm{h}$ at a depth h ft . in liquid of density $\sigma$ is called the pressure due to a head of h ft . of the liquid. The atmospheric pressure is thus due to an average head of 30 in . of mercury, or $30 \times 13.6 \div 12=34 \mathrm{ft}$. of water, or $27,200 \mathrm{ft}$. of air. The pressure of the air is a convenient unit to employ in practical work, where it is called an "atmosphere"; it is made the equivalent of a pressure of one $\mathrm{kg} / \mathrm{cm}^{2}$; and one ton $/ \mathrm{inch}^{2}$, employed as the unit with high pressure as in artillery, may be taken as 150 atmospheres.
12. Theorem.-A body immersed in a fluid is buoyed up by a force equal to the weight of the liquid displaced, acting vertically upward through the centre of gravity of the displaced liquid.

For if the body is removed, and replaced by the fluid as at first, this fluid is in equilibrium under its own weight and the thrust of the surrounding fluid, which must be equal and opposite, and the surrounding fluid acts in the same manner when the body replaces the displaced fluid again; so that the resultant thrust of the fluid acts vertically upward through the centre of gravity of the fluid displaced, and is equal to the weight.

When the body is floating freely like a ship, the equilibrium of this liquid thrust with the weight of the ship requires that the weight of water displaced is equal to the weight of the ship and the two centres of gravity are in the same vertical line. So also a balloon begins to rise when the weight of air displaced is greater than the weight of the balloon, and it is in equilibrium when the weights are equal. This theorem is called generally the principle of Archimedes.

It is used to determine the density of a body experimentally; for if W is the weight of a body weighed in a balance in air (strictly in vacuo), and if $\mathrm{W}^{\prime}$ is the weight required to balance when the body is suspended in water, then the upward thrust of the liquid or weight of liquid displaced is $\mathrm{W}-\mathrm{W}^{\prime}$, so that the specific gravity (S.G.), defined as the ratio of the weight of a body to the weight of an equal volume of water, is $\mathrm{W} /\left(\mathrm{W}-\mathrm{W}^{\prime}\right)$.

As stated first by Archimedes, the principle asserts the obvious fact that a body displaces its
own volume of water; and he utilized it in the problem of the determination of the adulteration of the crown of Hiero. He weighed out a lump of gold and of silver of the same weight as the crown; and, immersing the three in succession in water, he found they spilt over measures of water in the ratio $1 / 14: 4 / 77: 2 / 21$ or $33: 24: 44$; thence it follows that the gold : silver alloy of the crown was as 11:9 by weight.
13. Theorem.-The resultant vertical thrust on any portion of a curved surface exposed to the pressure of a fluid at rest under gravity is the weight of fluid cut out by vertical lines drawn round the boundary of the curved surface.

Theorem.-The resultant horizontal thrust in any direction is obtained by drawing parallel horizontal lines round the boundary, and intersecting a plane perpendicular to their direction in a plane curve; and then investigating the thrust on this plane area, which will be the same as on the curved surface.

The proof of these theorems proceeds as before, employing the normality principle; they are required, for instance, in the determination of the liquid thrust on any portion of the bottom of a ship.

In casting a thin hollow object like a bell, it will be seen that the resultant upward thrust on the mould may be many times greater than the weight of metal; many a curious experiment has been devised to illustrate this property and classed as a hydrostatic paradox (Boyle, Hydrostatical Paradoxes, 1666).

Consider, for instance, the operation of casting a hemispherical bell, in fig. 2. As the molten metal is run in, the upward thrust on the outside mould, when the level has reached PP', is the weight of metal in the volume generated by the revolution of APQ; and this, by a theorem of Archimedes, has the same volume as the cone ORR', or $1 / 3$ пу $^{3}$, where y is the depth of metal, the horizontal sections being equal so long as y is less than the radius of the outside hemisphere. Afterwards, when the metal has risen above B , to the level $\mathrm{KK}^{\prime}$, the additional thrust is the weight of the


Fig. 2. cylinder of diameter KK' and height BH. The upward thrust is the same, however thin the metal may be in the interspace between the outer mould and the core inside; and this was formerly considered paradoxical.

## Analytical Equations of Equilibrium of a Fluid at rest under any System of Force.

14. Referred to three fixed coordinate axes, a fluid, in which the pressure is $p$, the density $\rho$, and $\mathrm{X}, \mathrm{Y}, \mathrm{Z}$ the components of impressed force per unit mass, requires for the equilibrium of the part filling a fixed surface $S$, on resolving parallel to $O x$,

$$
\begin{equation*}
\iint \operatorname{lpdS}=\iiint \rho \mathrm{X} d x \mathrm{dy} \mathrm{dz} \tag{1}
\end{equation*}
$$

where $\mathrm{l}, \mathrm{m}, \mathrm{n}$ denote the direction cosines of the normal drawn outward of the surface S .
But by Green's transformation

$$
\begin{equation*}
\iint \operatorname{lp} \mathrm{dS}=\iiint \frac{\mathrm{dp}}{\mathrm{dx}} \mathrm{dx} d y \mathrm{dz} \tag{2}
\end{equation*}
$$

thus leading to the differential relation at every point

$$
\begin{equation*}
\frac{d p}{d x}=\rho X, \quad \frac{d p}{d y}=\rho Y, \quad \frac{d p}{d z}=\rho Z . \tag{3}
\end{equation*}
$$

The three equations of equilibrium obtained by taking moments round the axes are then found to be satisfied identically.

Hence the space variation of the pressure in any direction, or the pressure-gradient, is the resolved force per unit volume in that direction. The resultant force is therefore in the direction of the steepest pressure-gradient, and this is normal to the surface of equal pressure; for equilibrium to exist in a fluid the lines of force must therefore be capable of being cut orthogonally by a system of surfaces, which will be surfaces of equal pressure.

Ignoring temperature effect, and taking the density as a function of the pressure, surfaces of equal pressure are also of equal density, and the fluid is stratified by surfaces orthogonal to the lines of force;

$$
\begin{equation*}
\bar{\rho} \overline{\mathrm{dx}}, \quad \bar{\rho} \overline{\mathrm{dy}}, \quad \bar{\rho} \overline{\mathrm{dz}}, \text { or } \mathrm{X}, \mathrm{Y}, \mathrm{Z} \tag{4}
\end{equation*}
$$

are the partial differential coefficients of some function $P,=\int d p / \rho$, of $x, y, z$; so that $X, Y, Z$ must be the partial differential coefficients of a potential -V , such that the force in any direction is the downward gradient of V ; and then

$$
\begin{equation*}
\frac{d P}{d x}+\frac{d V}{d x}=0, \text { or } P+V=\text { constant } \tag{5}
\end{equation*}
$$

in which P may be called the hydrostatic head and V the head of potential.
With variation of temperature, the surfaces of equal pressure and density need not coincide; but, taking the pressure, density and temperature as connected by some relation, such as the gas-equation, the surfaces of equal density and temperature must intersect in lines lying on a surface of equal pressure.
15. As an example of the general equations, take the simplest case of a uniform field of gravity, with Oz directed vertically downward; employing the gravitation unit of force,

$$
\begin{align*}
\frac{1}{\rho} \frac{d p}{d x} & =0, \quad \frac{1}{\rho} \quad \frac{d p}{d y}=0, \quad \frac{1}{\rho} \quad \frac{d p}{d z}=1  \tag{1}\\
P & =\int d p / \rho=z+\text { a constant. } \tag{2}
\end{align*}
$$

When the density $\rho$ is uniform, this becomes, as before in (2) § 9

$$
\begin{equation*}
\mathrm{p}=\rho \mathrm{z}+\mathrm{p}_{0} . \tag{3}
\end{equation*}
$$

Suppose the density $\rho$ varies as some nth power of the depth below O , then

$$
\begin{gather*}
d p / d z=\rho=\mu z^{n} \\
p=\mu \frac{z^{n+1}}{n+1}=\frac{\rho z}{n+1}=\frac{\rho}{n+1}\left(\frac{\rho}{\mu}\right)^{1 / n}, \tag{4}
\end{gather*}
$$

supposing p and $\rho$ to vanish together.
These equations can be made to represent the state of convective equilibrium of the atmosphere, depending on the gas-equation

$$
\begin{equation*}
\mathrm{p}=\rho \mathrm{k}=\mathrm{R} \rho \theta, \tag{6}
\end{equation*}
$$

where $\theta$ denotes the absolute temperature; and then

$$
\begin{equation*}
\mathrm{R} \frac{\mathrm{~d} \theta}{\mathrm{dz}}=\frac{\mathrm{d}}{\mathrm{dz}}\left(\frac{\mathrm{p}}{\rho}\right)=\frac{1}{\mathrm{n}+1}, \tag{7}
\end{equation*}
$$

so that the temperature-gradient $\mathrm{d} \theta / \mathrm{dz}$ is constant, as in convective equilibrium in (11).
From the gas-equation in general, in the atmosphere

$$
\begin{equation*}
\frac{1}{\rho} \frac{d p}{d z}=\frac{1}{p} \frac{d p}{d z}-\frac{1}{\theta} \frac{d \theta}{d z}=\frac{\rho}{p}-\frac{1}{\theta} \frac{d \theta}{d z}=\frac{1}{k}-\frac{1}{\theta} \frac{d \theta}{d z}, \tag{8}
\end{equation*}
$$

which is positive, and the density $\rho$ diminishes with the ascent, provided the temperaturegradient $\mathrm{d} \theta / \mathrm{dz}$ does not exceed $\theta / \mathrm{k}$.

With uniform temperature, taking k constant in the gas-equation,

$$
\begin{equation*}
\mathrm{dp} / \mathrm{dz}=\rho=\mathrm{p} / \mathrm{k}, \quad \mathrm{p}=\mathrm{p}_{0} \mathrm{e}^{\mathrm{z} / \mathrm{k}} \tag{9}
\end{equation*}
$$

so that in ascending in the atmosphere of thermal equilibrium the pressure and density diminish at compound discount, and for pressures $p_{1}$ and $p_{2}$ at heights $z_{1}$ and $z_{2}$

$$
\begin{equation*}
\left(\mathrm{z}_{1}-\mathrm{z}_{2}\right) / \mathrm{k}=\log _{\mathrm{e}}\left(\mathrm{p}_{2} / \mathrm{p}_{1}\right)=2.3 \log _{10}\left(\mathrm{p}_{2} / \mathrm{p}_{1}\right) . \tag{10}
\end{equation*}
$$

In the convective equilibrium of the atmosphere, the air is supposed to change in density and pressure without exchange of heat by conduction; and then

$$
\begin{gather*}
\rho / \rho_{0}=\left(\theta / \theta_{0}\right)^{\mathrm{n}}, \mathrm{p} / \mathrm{p}_{0}=\left(\theta / \theta_{0}\right)^{\mathrm{n}+1},  \tag{11}\\
\frac{\mathrm{dz}}{\mathrm{~d} \theta}=\frac{1}{\rho} \frac{\mathrm{dp}}{\mathrm{~d} \theta}=(\mathrm{n}+1) \frac{\mathrm{p}}{\rho \theta}=(\mathrm{n}+1) \mathrm{R}, \gamma=1+\frac{1}{\mathrm{n}},
\end{gather*}
$$

where $\gamma$ is the ratio of the specific heat at constant pressure and constant volume.
In the more general case of the convective equilibrium of a spherical atmosphere surrounding
the earth, of radius $a$,

$$
\begin{equation*}
\frac{d p}{\rho}=(n+1) \frac{p_{0}}{\rho_{0}} \frac{d \theta}{\theta_{0}}=-\frac{a^{2}}{r^{2}} d r, \tag{12}
\end{equation*}
$$

gravity varying inversely as the square of the distance $r$ from the centre; so that, $k=p_{0} / \rho_{0}$, denoting the height of the homogeneous atmosphere at the surface, $\theta$ is given by

$$
\begin{equation*}
(n+1) k\left(1-\theta / \theta_{0}\right)=a(1-a / r) \tag{13}
\end{equation*}
$$

or if c denotes the distance where $\theta=0$,

$$
\begin{equation*}
\frac{\theta}{\theta_{0}}=\frac{a}{r} \cdot \frac{c-r}{c-a} . \tag{14}
\end{equation*}
$$

When the compressibility of water is taken into account in a deep ocean, an experimental law must be employed, such as

$$
\begin{equation*}
\mathrm{p}-\mathrm{p}_{0}=\mathrm{k}\left(\rho-\rho_{0}\right), \text { or } \rho / \rho_{0}=1+\left(\mathrm{p}-\mathrm{p}_{0}\right) / \lambda, \lambda=\mathrm{k} \rho_{0} \tag{15}
\end{equation*}
$$

so that $\lambda$ is the pressure due to a head k of the liquid at density $\rho_{0}$ under atmospheric pressure $\mathrm{p}_{0}$; and it is the gauge pressure required on this law to double the density. Then

$$
\begin{equation*}
\mathrm{dp} / \mathrm{dz}=\mathrm{kd} \rho / \mathrm{dz}=\rho, \quad \rho=\rho_{0} \mathrm{e}^{\mathrm{z} / \mathrm{k}}, \quad \mathrm{p}-\mathrm{p}_{0}=\mathrm{k} \rho_{0}\left(\mathrm{e}^{\mathrm{z} / \mathrm{k}}-1\right) ; \tag{16}
\end{equation*}
$$

and if the liquid was incompressible, the depth at pressure $p$ would be $\left(p-p_{0}\right) / p_{0}$, so that the lowering of the surface due to compression is

$$
\begin{equation*}
\mathrm{ke}^{\mathrm{z} / \mathrm{k}}-\mathrm{k}-\mathrm{z}=1 / 2 \mathrm{z}^{2} / \mathrm{k} \text {, when } \mathrm{k} \text { is large. } \tag{17}
\end{equation*}
$$

For sea water, $\lambda$ is about 25,000 atmospheres, and $k$ is then 25,000 times the height of the water barometer, about 250,000 metres, so that in an ocean 10 kilometres deep the level is lowered about 200 metres by the compressibility of the water; and the density at the bottom is increased 4\%.

On another physical assumption of constant cubical elasticity $\lambda$,

$$
\begin{gather*}
d p=\lambda d \rho / \rho, \quad\left(p-p_{0}\right) / \lambda=\log \left(\rho / \rho_{0}\right)  \tag{18}\\
\frac{d p}{z d}=\frac{\lambda}{\rho} \frac{d \rho}{d z}=\rho, \quad \lambda\left(\frac{1}{\rho_{0}}-\frac{1}{\rho}\right)=z, \quad 1-\frac{\rho_{0}}{\rho}=\frac{z}{k}, \quad \lambda=k \rho_{0}, \tag{19}
\end{gather*}
$$

and the lowering of the surface is

$$
\begin{equation*}
\frac{\mathrm{p}-\mathrm{p}_{0}}{\rho_{0}}-\mathrm{z}=\mathrm{k} \log \frac{\rho}{\rho_{0}}-\mathrm{z}=-\mathrm{k} \log \left(1-\frac{\mathrm{z}}{\mathrm{k}}\right)-\mathrm{z} \approx \frac{\mathrm{z}^{2}}{2 \mathrm{k}} \tag{20}
\end{equation*}
$$

as before in (17).
16. Centre of Pressure.-A plane area exposed to fluid pressure on one side experiences a single resultant thrust, the integrated pressure over the area, acting through a definite point called the centre of pressure (C.P.) of the area.

Thus if the plane is normal to Oz , the resultant thrust

$$
\begin{equation*}
R=\iint p d x d y \tag{1}
\end{equation*}
$$

and the coordinates $\bar{x}, \bar{y}$ of the C.P. are given by

$$
\begin{equation*}
\overline{\mathrm{x}} \mathrm{R}=\iint \mathrm{xp} \mathrm{dx} d y, \quad \overline{\mathrm{y}} \mathrm{R}=\iint \mathrm{yp} \mathrm{dx} d y \tag{2}
\end{equation*}
$$

The C•P. is thus the C•G. of a plane lamina bounded by the area, in which the surface density is $p$.

If $p$ is uniform, the $C \cdot P$. and $C \cdot G$. of the area coincide.
For a homogeneous liquid at rest under gravity, p is proportional to the depth below the surface, i.e. to the perpendicular distance from the line of intersection of the plane of the area with the free surface of the liquid.

If the equation of this line, referred to new coordinate axes in the plane area, is written

$$
\begin{equation*}
\mathrm{x} \cos \alpha+\mathrm{y} \sin \alpha-\mathrm{h}=0 \tag{3}
\end{equation*}
$$

$$
R=\iint \rho(h-x \cos \alpha-y \sin \alpha) d x d y
$$

$$
\begin{align*}
& \overline{\mathrm{x}} \mathrm{R}=\iint \rho \mathrm{x}(\mathrm{~h}-\mathrm{x} \cos \alpha-\mathrm{y} \sin \alpha) \mathrm{dx} d \mathrm{y}  \tag{5}\\
& \overline{\mathrm{y} R}=\iint \rho \mathrm{y}(\mathrm{~h}-\mathrm{x} \cos \alpha-\mathrm{y} \sin \alpha) \mathrm{dx} d y
\end{align*}
$$

Placing the new origin at the C.G. of the area A,

$$
\begin{gather*}
\iint x d x d y=0, \iint y d x d y=0  \tag{6}\\
R=\rho h A \tag{7}
\end{gather*}
$$

$$
\begin{align*}
& \overline{\mathrm{xh}} \mathrm{~A}=-\cos \alpha \iint \mathrm{x}^{2} \mathrm{dA}-\sin \alpha \iint \mathrm{xy} \mathrm{dA}  \tag{8}\\
& \overline{\mathrm{yh}} \mathrm{~A}=-\cos \alpha \iint \mathrm{xy} \mathrm{dA}-\sin \alpha \iint \mathrm{y}^{2} \mathrm{dA} . \tag{9}
\end{align*}
$$

Turning the axes to make them coincide with the principal axes of the area A , thus making $\iint$ $x y d A=0$,

$$
\begin{equation*}
\overline{\mathrm{x}} \mathrm{~h}=-\mathrm{a}^{2} \cos \alpha, \overline{\mathrm{y}} \mathrm{~h}=-\mathrm{b}^{2} \sin \alpha, \tag{10}
\end{equation*}
$$

where

$$
\begin{equation*}
\iint \mathrm{x}^{2} \mathrm{dA}=\mathrm{Aa}^{2}, \quad \iint \mathrm{y}^{2} \mathrm{dA}=\mathrm{Ab}^{2} \tag{11}
\end{equation*}
$$

a and b denoting the semi-axes of the momental ellipse of the area.
This shows that the C.P. is the antipole of the line of intersection of its plane with the free surface with respect to the momental ellipse at the C.G. of the area.
Thus the C.P. of a rectangle or parallelogram with a side in the surface is at $2 / 3$ of the depth of the lower side; of a triangle with a vertex in the surface and base horizontal is $3 / 4$ of the depth of the base; but if the base is in the surface, the C•P. is at half the depth of the vertex; as on the faces of a tetrahedron, with one edge in the surface.

The core of an area is the name given to the limited area round its C.G. within which the C•P. must lie when the area is immersed completely; the boundary of the core is therefore the locus of the antipodes with respect to the momental ellipse of water lines which touch the boundary of the area. Thus the core of a circle or an ellipse is a concentric circle or ellipse of one quarter the size.

The C.P. of water lines passing through a fixed point lies on a straight line, the antipolar of the point; and thus the core of a triangle is a similar triangle of one quarter the size, and the core of a parallelogram is another parallelogram, the diagonals of which are the middle third of the median lines.

In the design of a structure such as a tall reservoir dam it is important that the line of thrust in the material should pass inside the core of a section, so that the material should not be in a state of tension anywhere and so liable to open and admit the water.
17. Equilibrium and Stability of a Ship or Floating Body. The Metacentre.-The principle of Archimedes in § 12 leads immediately to the conditions of equilibrium of a body supported freely in fluid, like a fish in water or a balloon in the air, or like a ship (fig. 3) floating partly immersed in water and the rest in air. The body is in equilibrium under two forces:-(i.) its weight W acting vertically downward through G, the C.G. of the body, and (ii.) the buoyancy of the fluid, equal to the weight of the displaced fluid, and acting vertically upward through B, the C.G. of the displaced fluid; for equilibrium these two forces must be


Fig. 3. equal and opposite in the same line.

The conditions of equilibrium of a body, floating like a ship on the surface of a liquid, are therefore:-
(i.) the weight of the body must be less than the weight of the total volume of liquid it can displace; or else the body will sink to the bottom of the liquid; the difference of the weights is
called the "reserve of buoyancy."
(ii.) the weight of liquid which the body displaces in the position of equilibrium is equal to the weight W of the body; and
(iii.) the C.G., B, of the liquid displaced and $G$ of the body, must lie in the same vertical line GB.
18. In addition to satisfying these conditions of equilibrium, a ship must fulfil the further condition of stability, so as to keep upright; if displaced slightly from this position, the forces called into play must be such as to restore the ship to the upright again. The stability of a ship is investigated practically by inclining it; a weight is moved across the deck and the angle is observed of the heel produced.

Suppose P tons is moved c ft. across the deck of a ship of W tons displacement; the C.G. will move from $G$ to $G_{1}$ the reduced distance $G_{1} G_{2}=c(P / W)$; and if $B$, called the centre of buoyancy, moves to $B_{1}$, along the curve of buoyancy $B_{1}$, the normal of this curve at $B_{1}$ will be the new vertical $B_{1} G_{1}$, meeting the old vertical in a point $M$, the centre of curvature of $\mathrm{BB}_{1}$, called the metacentre.

If the ship heels through an angle $\theta$ or a slope of 1 in m ,

$$
\begin{equation*}
\mathrm{GM}=\mathrm{GG}_{1} \cot \theta=\mathrm{mc}(\mathrm{P} / \mathrm{W}), \tag{1}
\end{equation*}
$$

and GM is called the metacentric height; and the ship must be ballasted, so that G lies below M . If $G$ was above $M$, the tangent drawn from $G$ to the evolute of $B$, and normal to the curve of buoyancy, would give the vertical in a new position of equilibrium. Thus in H.M.S. "Achilles" of 9000 tons displacement it was found that moving 20 tons across the deck, a distance of 42 ft ., caused the bob of a pendulum 20 ft . long to move through 10 in ., so that

$$
\begin{equation*}
\mathrm{GM}=\frac{240}{10} \times 42 \times \frac{20}{9000} 2.24 \mathrm{ft} . \tag{2}
\end{equation*}
$$

also

$$
\begin{equation*}
\cot \theta=24, \theta=2^{\circ} 24^{\prime} . \tag{3}
\end{equation*}
$$

In a diagram it is conducive to clearness to draw the ship in one position, and to incline the water-line; and the page can be turned if it is desired to bring the new water-line horizontal.

Suppose the ship turns about an axis through $F$ in the water-line area, perpendicular to the plane of the paper; denoting by $y$ the distance of an element dA if the water-line area from the axis of rotation, the change of displacement is $\Sigma y d A \tan \theta$, so that there is no change of displacement if $\Sigma \mathrm{ydA}=0$, that is, if the axis passes through the C.G. of the water-line area, which we denote by F and call the centre of flotation.

The righting couple of the wedges of immersion and emersion will be

$$
\begin{equation*}
\Sigma \mathrm{wy} \mathrm{dA} \tan \theta \cdot \mathrm{y}=\mathrm{w} \tan \theta \Sigma \mathrm{y}^{2} \mathrm{dA}=\mathrm{w} \tan \theta \cdot \mathrm{Ak}^{2} \mathrm{ft} . \text { tons, } \tag{4}
\end{equation*}
$$

w denoting the density of water in tons/ $\mathrm{ft} .^{3}$, and $\mathrm{W}=\mathrm{wV}$, for a displacement of $\mathrm{Vft} .^{3}$
This couple, combined with the original buoyancy $W$ through $B$, is equivalent to the new buoyancy through B, so that

$$
\begin{equation*}
\mathrm{W} \cdot \mathrm{BB}_{1}=\mathrm{wAk}^{2} \tan \theta, \tag{5}
\end{equation*}
$$

$$
\begin{equation*}
\mathrm{BM}=\mathrm{BB}_{1} \cot \theta=\mathrm{Ak}^{2} / \mathrm{V}, \tag{6}
\end{equation*}
$$

giving the radius of curvature BM of the curve of buoyancy B , in terms of the displacement V , and $A k^{2}$ the moment of inertia of the water-line area about an axis through $F$, perpendicular to the plane of displacement.

An inclining couple due to moving a weight about in a ship will heel the ship about an axis perpendicular to the plane of the couple, only when this axis is a principal axis at $F$ of the momental ellipse of the water-line area A. For if the ship turns through a small angle $\theta$ about the line $\mathrm{FF}^{\prime}$, then $\mathrm{b}_{1}, \mathrm{~b}_{2}$, the C•G. of the wedge of immersion and emersion, will be the $\mathrm{C} \cdot \mathrm{P}$. with respect to $\mathrm{FF}^{\prime}$ of the two parts of the water-line area, so that $\mathrm{b}_{1} \mathrm{~b}_{2}$ will be conjugate to $\mathrm{FF}^{\prime}$ with respect to the momental ellipse at $F$.

The naval architect distinguishes between the stability of form, represented by the righting couple W.BM, and the stability of ballasting, represented by W.BG. Ballasted with G at B, the righting couple when the ship is heeled through $\theta$ is given by W.BM. tan $\theta$; but if weights inside the ship are raised to bring $G$ above $B$, the righting couple is diminished by $\mathrm{W} \cdot \mathrm{BG} . \tan \theta$, so that
the resultant righting couple is $\mathrm{W} \cdot \mathrm{GM} \cdot \tan \theta$. Provided the ship is designed to float upright at the smallest draft with no load on board, the stability at any other draft of water can be arranged by the stowage of the weight, high or low.
19. Proceeding as in $\S 16$ for the determination of the C.P. of an area, the same argument will show that an inclining couple due to the movement of a weight $P$ through a distance $c$ will cause the ship to heel through an angle $\theta$ about an axis $\mathrm{FF}^{\prime}$ through F , which is conjugate to the direction of the movement of P with respect to an ellipse, not the momental ellipse of the waterline area A, but a confocal to it, of squared semi-axes

$$
\begin{equation*}
\mathrm{a}^{2}-\mathrm{hV} / \mathrm{A}, \mathrm{~b}^{2}-\mathrm{hV} / \mathrm{A}, \tag{1}
\end{equation*}
$$

h denoting the vertical height BG between C.G. and centre of buoyancy. The varying direction of the inclining couple Pc may be realized by swinging the weight P from a crane on the ship, in a circle of radius $c$. But if the weight $P$ was lowered on the ship from a crane on shore, the vessel would sink bodily a distance $\mathrm{P} / \mathrm{wA}$ if P was deposited over F ; but deposited anywhere else, say over Q on the water-line area, the ship would turn about a line the antipolar of Q with respect to the confocal ellipse, parallel to $\mathrm{FF}^{\prime}$, at a distance FK from F

$$
\begin{equation*}
\mathrm{FK}=\left(\mathrm{k}^{2}-\mathrm{hV} / \mathrm{A}\right) / \mathrm{FQ} \sin \mathrm{QFF}^{\prime} \tag{2}
\end{equation*}
$$

through an angle $\theta$ or a slope of one in $m$, given by

$$
\begin{equation*}
\sin \theta=\frac{1}{m}=\frac{P}{w A \cdot F K}=\frac{P}{W} \cdot \frac{V}{A k^{2}-h V} F Q \sin Q F F^{\prime} \tag{3}
\end{equation*}
$$

where k denotes the radius of gyration about $\mathrm{FF}^{\prime}$ of the water-line area. Burning the coal on a voyage has the reverse effect on a steamer.

## Hydrodynamics

20. In considering the motion of a fluid we shall suppose it non-viscous, so that whatever the state of motion the stress across any section is normal, and the principle of the normality and thence of the equality of fluid pressure can be employed, as in hydrostatics. The practical problems of fluid motion, which are amenable to mathematical analysis when viscosity is taken into account, are excluded from treatment here, as constituting a separate branch called "hydraulics" (q.v.). Two methods are employed in hydrodynamics, called the Eulerian and Lagrangian, although both are due originally to Leonhard Euler. In the Eulerian method the attention is fixed on a particular point of space, and the change is observed there of pressure, density and velocity, which takes place during the motion; but in the Lagrangian method we follow up a particle of fluid and observe how it changes. The first may be called the statistical method, and the second the historical, according to J. C. Maxwell. The Lagrangian method being employed rarely, we shall confine ourselves to the Eulerian treatment.

## The Eulerian Form of the Equations of Motion.

21. The first equation to be established is the equation of continuity, which expresses the fact that the increase of matter within a fixed surface is due to the flow of fluid across the surface into its interior.

In a straight uniform current of fluid of density $\rho$, flowing with velocity $q$, the flow in units of mass per second across a plane area A, placed in the current with the normal of the plane making an angle $\theta$ with the velocity, is $\rho A q \cos \theta$, the product of the density $\rho$, the area $A$, and $q$ $\cos \theta$ the component velocity normal to the plane.

Generally if S denotes any closed surface, fixed in the fluid, M the mass of the fluid inside it at any time t , and $\theta$ the angle which the outward-drawn normal makes with the velocity q at that point,

$$
\begin{align*}
\mathrm{dM} / \mathrm{dt} & =\text { rate of increase of fluid inside the surface, } \\
& =\text { flux across the surface into the interior } \\
& =-\iint \rho \mathrm{q} \cos \theta \mathrm{dS}, \tag{1}
\end{align*}
$$

the integral equation of continuity.
In the Eulerian notation $\mathrm{u}, \mathrm{v}, \mathrm{w}$ denote the components of the velocity q parallel to the coordinate axes at any point ( $\mathrm{x}, \mathrm{y}, \mathrm{z}$ ) at the time t ; $\mathrm{u}, \mathrm{v}, \mathrm{w}$ are functions of $\mathrm{x}, \mathrm{y}, \mathrm{z}, \mathrm{t}$, the independent variables; and $d$ is used here to denote partial differentiation with respect to any one of these four independent variables, all capable of varying one at a time.
transformation is required again, namely,

$$
\begin{equation*}
\iiint\left(\frac{d \xi}{d x}+\frac{d \eta}{d y}+\frac{d \zeta}{d z}\right) d x d y d z=\iint(l \xi+m \eta+n \zeta) d S \tag{2}
\end{equation*}
$$

or individually

$$
\begin{equation*}
\iiint \frac{d \xi}{d x} d x d y d z=\iint l \xi d S, \ldots \tag{3}
\end{equation*}
$$

where the integrations extend throughout the volume and over the surface of a closed space $S$; $\mathrm{l}, \mathrm{m}, \mathrm{n}$ denoting the direction cosines of the outward-drawn normal at the surface element dS, and $\xi, \eta, \zeta$ any continuous functions of $x, y, z$.

The integral equation of continuity (1) may now be written

$$
\begin{equation*}
\iiint \frac{d \rho}{d t} d x d y d z=\iint(l \rho u+m \rho v+n \rho w) d S=0 \tag{4}
\end{equation*}
$$

which becomes by Green's transformation

$$
\begin{equation*}
\iiint\left(\frac{d \rho}{d t}+\frac{d(\rho u)}{d x}+\frac{d(\rho v)}{d y}+\frac{d(\rho w)}{d z}\right) d x d y d z=0 \tag{5}
\end{equation*}
$$

leading to the differential equation of continuity when the integration is removed.
22. The equations of motion can be established in a similar way by considering the rate of increase of momentum in a fixed direction of the fluid inside the surface, and equating it to the momentum generated by the force acting throughout the space $S$, and by the pressure acting over the surface S .

Taking the fixed direction parallel to the axis of $x$, the time-rate of increase of momentum, due to the fluid which crosses the surface, is

$$
\begin{equation*}
-\iint \rho u q \cos \theta d S=-\iint\left(l \rho u^{2}+m \rho u v+n \rho u w\right) d S \tag{1}
\end{equation*}
$$

which by Green's transformation is

$$
\begin{equation*}
-\iiint\left(\frac{d\left(\rho u^{2}\right)}{d x}+\frac{d(\rho u v)}{d y}+\frac{d(\rho u w)}{d z}\right) d x d y d z \tag{2}
\end{equation*}
$$

The rate of generation of momentum in the interior of $S$ by the component of force, $X$ per unit mass, is

$$
\begin{equation*}
\iiint \rho X d x d y d z, \tag{3}
\end{equation*}
$$

and by the pressure at the surface S is

$$
\begin{equation*}
-\iint \operatorname{lp} \mathrm{dS}=-\iiint \frac{\mathrm{dp}}{\mathrm{dx}} \mathrm{dx} \mathrm{dy} \mathrm{dz} \tag{4}
\end{equation*}
$$

by Green's transformation.
The time rate of increase of momentum of the fluid inside $S$ is

$$
\begin{equation*}
\iiint \frac{\mathrm{d}(\rho \mathrm{u})}{\mathrm{dt}} \mathrm{dx} d y \mathrm{dz} \tag{5}
\end{equation*}
$$

and (5) is the sum of (1), (2), (3), (4), so that

$$
\begin{equation*}
\iiint\left(\frac{d \rho u}{d t}+\frac{d \rho u^{2}}{d x}+\frac{d \rho u v}{d y}+\frac{d \rho u w}{d z}-\rho X+\frac{d p}{d x}\right) d x d y d z=0 \tag{6}
\end{equation*}
$$

leading to the differential equation of motion

$$
\begin{equation*}
\frac{d \rho u}{d t}+\frac{d \rho u^{2}}{d x}+\frac{d \rho u v}{d y}+\frac{d \rho u w}{d z}=\rho X-\frac{d p}{d x} \tag{7}
\end{equation*}
$$

with two similar equations.
The absolute unit of force is employed here, and not the gravitation unit of hydrostatics; in a numerical application it is assumed that C.G.S. units are intended.

These equations may be simplified slightly, using the equation of continuity (5) § 21 ; for

$$
\begin{aligned}
& \frac{d \rho u}{d t}+\frac{d \rho u^{2}}{d x}+\frac{d \rho u v}{d y}+\frac{d \rho u w}{d z} \\
= & \rho\left(\frac{d u}{d t}+u \frac{d u}{d x}+v \frac{d u}{d y}+w \frac{d u}{d z}\right)
\end{aligned}
$$

$$
\begin{equation*}
+u\left(\frac{d \rho}{d t}+\frac{d \rho u}{d x}+\frac{d \rho v}{d y}+\frac{d \rho w}{d z}\right) \tag{8}
\end{equation*}
$$

reducing to the first line, the second line vanishing in consequence of the equation of continuity; and so the equation of motion may be written in the more usual form

$$
\begin{equation*}
\frac{d u}{d t}+u \frac{d u}{d x}+v \frac{d u}{d y}+w \frac{d u}{d z}=x-\frac{1}{\rho} \frac{d p}{d x} \tag{9}
\end{equation*}
$$

with the two others

$$
\begin{align*}
& \frac{d v}{d t}+u \frac{d v}{d x}+v \frac{d v}{d y}+w \frac{d v}{d z}=Y-\frac{1}{\rho} \frac{d p}{d y}  \tag{10}\\
& \frac{d w}{d t}+u \frac{d w}{d x}+v \frac{d w}{d y}+w \frac{d w}{d z}=Z-\frac{1}{\rho} \frac{d p}{d z} \tag{11}
\end{align*}
$$

23. As a rule these equations are established immediately by determining the component acceleration of the fluid particle which is passing through ( $x, y, z$ ) at the instant $t$ of time considered, and saying that the reversed acceleration or kinetic reaction, combined with the impressed force per unit of mass and pressure-gradient, will according to d'Alembert's principle form a system in equilibrium.

To determine the component acceleration of a particle, suppose F to denote any function of x , $\mathrm{y}, \mathrm{z}, \mathrm{t}$, and investigate the time rate of F for a moving particle; denoting the change by $\mathrm{DF} / \mathrm{dt}$,

$$
\begin{gather*}
\frac{D F}{d t}=l t \cdot \frac{F(x+u \delta t, y+v \delta t, z+w \delta t, t+\delta t)-F(x, y, z, t)}{\delta t} \\
=\frac{d F}{d t}+u \frac{d F}{d x}+v \frac{d F}{d y}+w \frac{d F}{d z} \tag{1}
\end{gather*}
$$

and $\mathrm{D} / \mathrm{dt}$ is called particle differentiation, because it follows the rate of change of a particle as it leaves the point $\mathrm{x}, \mathrm{y}, \mathrm{z}$; but

$$
\begin{equation*}
\mathrm{dF} / \mathrm{dt}, \mathrm{dF} / \mathrm{dx}, \mathrm{dF} / \mathrm{dy}, \mathrm{dF} / \mathrm{dz} \tag{2}
\end{equation*}
$$

represent the rate of change of F at the time t , at the point, $\mathrm{x}, \mathrm{y}, \mathrm{z}$, fixed in space.
The components of acceleration of a particle of fluid are consequently

$$
\begin{align*}
& \frac{\mathrm{Du}}{\mathrm{dt}}=\frac{\mathrm{du}}{\mathrm{dt}}+\mathrm{u} \frac{\mathrm{du}}{\mathrm{dx}}+\mathrm{v} \frac{\mathrm{du}}{\mathrm{dy}}+\mathrm{w} \frac{\mathrm{du}}{\mathrm{dz}}  \tag{3}\\
& \frac{\mathrm{Dv}}{\mathrm{dt}}=\frac{\mathrm{dv}}{\mathrm{dt}}+\mathrm{u} \frac{d v}{d x}+\mathrm{v} \frac{d v}{d y}+\mathrm{w} \frac{d v}{d z}  \tag{4}\\
& \frac{\mathrm{Dw}}{d t}=\frac{d w}{d t}+u \frac{d w}{d x}+v \frac{d w}{d y}+w \frac{d w}{d z} \tag{5}
\end{align*}
$$

leading to the equations of motion above.
If $\mathrm{F}(\mathrm{x}, \mathrm{y}, \mathrm{z}, \mathrm{t})=0$ represents the equation of a surface containing always the same particles of fluid,

$$
\begin{equation*}
\frac{\mathrm{DF}}{\mathrm{dt}}=0, \text { or } \frac{\mathrm{dF}}{\mathrm{dt}}+\mathrm{u} \frac{\mathrm{dF}}{\mathrm{dx}}+\mathrm{v} \frac{\mathrm{dF}}{\mathrm{dy}}+\mathrm{w} \frac{\mathrm{dF}}{\mathrm{dz}}=0, \tag{6}
\end{equation*}
$$

which is called the differential equation of the bounding surface. A bounding surface is such that there is no flow of fluid across it, as expressed by equation (6). The surface always contains the same fluid inside it, and condition (6) is satisfied over the complete surface, as well as any part of it.
But turbulence in the motion will vitiate the principle that a bounding surface will always consist of the same fluid particles, as we see on the surface of turbulent water.
24. To integrate the equations of motion, suppose the impressed force is due to a potential V , such that the force in any direction is the rate of diminution of V , or its downward gradient; and then

$$
\begin{equation*}
X=-d V / d x, Y=-d V / d y, Z=-d V / d z ; \tag{1}
\end{equation*}
$$

and putting

$$
\begin{gather*}
\frac{d w}{d y}-\frac{d v}{d z}=2 \xi, \frac{d u}{d z}-\frac{d w}{d x}=2 \eta, \frac{d v}{d x}-\frac{d u}{d y}=2 \zeta  \tag{2}\\
\frac{d \xi}{d x}+\frac{d \eta}{d y}+\frac{d \zeta}{d z}=0 \tag{3}
\end{gather*}
$$

the equations of motion may be written

$$
\begin{align*}
& \frac{d u}{d t}-2 v \zeta+2 w \eta+\frac{d H}{d x}=0  \tag{4}\\
& \frac{d v}{d t}-2 w \xi+2 u \zeta+\frac{d H}{d y}=0  \tag{5}\\
& \frac{d w}{d t}-2 u \eta+2 w \xi+\frac{d H}{d z}=0 \tag{6}
\end{align*}
$$

where

$$
\begin{gather*}
\mathrm{H}=\int \mathrm{dp} / \rho+\mathrm{V}+\frac{1}{2} \mathrm{q}^{2},  \tag{7}\\
\mathrm{q}^{2}=\mathrm{u}^{2}+\mathrm{v}^{2}+\mathrm{w}^{2}, \tag{8}
\end{gather*}
$$

and the three terms in H may be called the pressure head, potential head, and head of velocity, when the gravitation unit is employed and $1 / 2 q^{2}$ is replaced by $1 / 2 q^{2} / g$.

Eliminating H between (5) and (6)

$$
\begin{equation*}
\frac{D \xi}{d t}-\xi \frac{d u}{d x}-\eta \frac{d w}{d x}-\zeta \frac{d v}{d x}+\xi\left(\frac{d u}{d x}+\frac{d v}{d y}+\frac{d w}{d z}\right)=0 \tag{9}
\end{equation*}
$$

and combining this with the equation of continuity

$$
\begin{equation*}
\frac{1}{\rho} \frac{\mathrm{D} \rho}{\mathrm{dt}}+\frac{\mathrm{du}}{\mathrm{dx}}+\frac{\mathrm{dv}}{\mathrm{dy}}+\frac{\mathrm{dw}}{\mathrm{dz}}=0 \tag{10}
\end{equation*}
$$

we have

$$
\begin{equation*}
\frac{D}{d t}\left(\frac{\xi}{\rho}\right)-\frac{\xi}{\rho} \frac{d u}{d x}-\frac{\eta}{\rho} \frac{d v}{d x}-\frac{\zeta}{\rho} \frac{d w}{d x}=0 \tag{11}
\end{equation*}
$$

with two similar equations.

## Putting

$$
\begin{equation*}
\omega^{2}=\xi^{2}+\eta^{2}+\zeta^{2} \tag{12}
\end{equation*}
$$

a vortex line is defined to be such that the tangent is in the direction of $\omega$, the resultant of $\xi, \eta$, $\zeta$, called the components of molecular rotation. A small sphere of the fluid, if frozen suddenly, would retain this angular velocity.

If $\omega$ vanishes throughout the fluid at any instant, equation (11) shows that it will always be zero, and the fluid motion is then called irrotational; and a function $\varphi$ exists, called the velocity function, such that

$$
\begin{equation*}
u d x+v d y+w d z=-d \varphi \tag{13}
\end{equation*}
$$

and then the velocity in any direction is the space-decrease or downward gradient of $\varphi$.
25. But in the most general case it is possible to have three functions $\varphi, \psi, m$ of $x, y, z$, such that

$$
\begin{equation*}
u d x+v d y+w d z=-d \varphi-m d \psi \tag{1}
\end{equation*}
$$

as A. Clebsch has shown, from purely analytical considerations (Crelle, lvi.); and then

$$
\begin{equation*}
\xi=1 / 2 \frac{d(\psi, m)}{d(y, z)}, \quad \eta=1 / 2 \frac{d(\psi, m)}{d(z, x)}, \quad \zeta=1 / 2 \frac{d(\psi, m)}{d(x, y)}, \tag{2}
\end{equation*}
$$

and

$$
\begin{equation*}
\xi \frac{d \psi}{d x}+\eta \frac{d \psi}{d y}+\zeta \frac{d \psi}{d z}=0, \quad \xi \frac{d m}{d x}+\eta \frac{d m}{d y}+\zeta \frac{d m}{d z}=0 \tag{3}
\end{equation*}
$$

so that, at any instant, the surfaces over which $\psi$ and $m$ are constant intersect in the vortex lines.

Putting

$$
\begin{equation*}
\mathrm{H}-\frac{\mathrm{d} \varphi}{\mathrm{dt}}-\mathrm{m} \frac{\mathrm{~d} \psi}{\mathrm{dt}}=\mathrm{K} \tag{4}
\end{equation*}
$$

the equations of motion (4), (5), (6) § 24 can be written

$$
\begin{equation*}
\overline{d x}-2 u \zeta+2 w \eta-\overline{d(x, t)}=0, \ldots, \ldots ; \tag{5}
\end{equation*}
$$

and therefore

$$
\begin{equation*}
\xi \frac{d K}{d x}+\eta \frac{d K}{d y}+\zeta \frac{d K}{d z}=0 . \tag{6}
\end{equation*}
$$

Equation (5) becomes, by a rearrangement,

$$
\begin{gather*}
\frac{d K}{d x}-\frac{d \psi}{d x}\left(\frac{d m}{d t}+u \frac{d m}{d x}+v \frac{d m}{d y}+w \frac{d m}{d z}\right) \\
+\frac{d m}{d x}\left(\frac{d \psi}{d t}+u \frac{d \psi}{d x}+v \frac{d \psi}{d y}+w \frac{d \psi}{d z}\right)=0, \ldots, \ldots,  \tag{7}\\
\frac{d K}{d x}-\frac{d \psi}{d x} \frac{D m}{d t}+\frac{d m}{d x} \frac{D \psi}{d t}=0, \ldots, \ldots, \tag{8}
\end{gather*}
$$

and as we prove subsequently (§ 37) that the vortex lines are composed of the same fluid particles throughout the motion, the surface $m$ and $\psi$ satisfies the condition of (6) § 23; so that K is uniform throughout the fluid at any instant, and changes with the time only, and so may be replaced by $\mathrm{F}(\mathrm{t})$.
26. When the motion is steady, that is, when the velocity at any point of space does not change with the time,

$$
\begin{gather*}
\frac{d K}{d x}-2 v \zeta+2 w \eta=0, \ldots, \ldots  \tag{1}\\
\xi \frac{d K}{d x}+\eta \frac{d K}{d y}+\zeta \frac{d K}{d z}=0, \quad u \frac{d K}{d x}+v \frac{d K}{d y}+w \frac{d K}{d z}=0,
\end{gather*}
$$

and

$$
\begin{equation*}
\mathrm{K}=\int \mathrm{dp} / \rho+\mathrm{V}+1 / 2 \mathrm{q}^{2}=\mathrm{H} \tag{3}
\end{equation*}
$$

is constant along a vortex line, and a stream line, the path of a fluid particle, so that the fluid is traversed by a series of $H$ surfaces, each covered by a network of stream lines and vortex lines; and if the motion is irrotational H is a constant throughout the fluid.

Taking the axis of x for an instant in the normal through a point on the surface $\mathrm{H}=$ constant, this makes $u=0, \xi=0$; and in steady motion the equations reduce to

$$
\begin{equation*}
\mathrm{dH} / \mathrm{d} \nu=2 \mathrm{v} \zeta-2 \mathrm{w} \eta=2 \mathrm{q} \omega \sin \theta, \tag{4}
\end{equation*}
$$

where $\theta$ is the angle between the stream line and vortex line; and this holds for their projection on any plane to which $\mathrm{d} \nu$ is drawn perpendicular.

In plane motion (4) reduces to

$$
\begin{equation*}
\frac{\mathrm{dH}}{\mathrm{~d} \nu}=2 \mathrm{q} \zeta=\mathrm{q}\left(\frac{\mathrm{dQ}}{\mathrm{dv}}+\frac{\mathrm{q}}{\mathrm{r}}\right) \tag{5}
\end{equation*}
$$

if $r$ denotes the radius of curvature of the stream line, so that

$$
\begin{equation*}
\frac{1}{\rho} \frac{d p}{d \nu}+\frac{d V}{d \nu}=\frac{d H}{d \nu}-\frac{\mathrm{d}^{1} / 2 q^{2}}{d \nu}=\frac{q^{2}}{r} \tag{6}
\end{equation*}
$$

the normal acceleration.
The osculating plane of a stream line in steady motion contains the resultant acceleration, the direction ratios of which are

$$
\begin{equation*}
u \frac{d u}{d x}+v \frac{d u}{d y}+w \frac{d u}{d z}=\frac{d^{1} / 2 q^{2}}{d x}-2 v \zeta+2 w \eta=\frac{d^{1} / 2 q^{2}}{d x}-\frac{d H}{d x}, \ldots, \tag{7}
\end{equation*}
$$

and when $q$ is stationary, the acceleration is normal to the surface $H=$ constant, and the stream line is a geodesic.

Calling the sum of the pressure and potential head the statical head, surfaces of constant statical and dynamical head intersect in lines on $H$, and the three surfaces touch where the velocity is stationary.

Equation (3) is called Bernoulli's equation, and may be interpreted as the balance-sheet of the energy which enters and leaves a given tube of flow.

If homogeneous liquid is drawn off from a vessel so large that the motion at the free surface at a distance may be neglected, then Bernoulli's equation may be written

$$
\mathrm{H}=\mathrm{p} / \rho+\mathrm{z}+\mathrm{q}^{2} / 2 \mathrm{~g}=\mathrm{P} / \rho+\mathrm{h},
$$

where $P$ denotes the atmospheric pressure and $h$ the height of the free surface, a fundamental equation in hydraulics; a return has been made here to the gravitation unit of hydrostatics, and Oz is taken vertically upward.

In particular, for a jet issuing into the atmosphere, where $p=P$,

$$
\begin{equation*}
\mathrm{q}^{2} / 2 \mathrm{~g}=\mathrm{h}-\mathrm{z}, \tag{9}
\end{equation*}
$$

or the velocity of the jet is due to the head $\mathrm{k}-\mathrm{z}$ of the still free surface above the orifice; this is Torricelli's theorem (1643), the foundation of the science of hydrodynamics.
27. Uniplanar Motion.-In the uniplanar motion of a homogeneous liquid the equation of continuity reduces to

$$
\begin{equation*}
\frac{d u}{d x}+\frac{d v}{d y}=0 \tag{1}
\end{equation*}
$$

so that we can put

$$
\begin{equation*}
u=-d \psi / d y, \quad v=d \psi / d x \tag{2}
\end{equation*}
$$

where $\psi$ is a function of $\mathrm{x}, \mathrm{y}$, called the stream- or current-function; interpreted physically, $\psi-$ $\psi_{0}$, the difference of the value of $\psi$ at a fixed point A and a variable point P is the flow, in $\mathrm{ft}^{3}$ / second, across any curved line AP from A to P , this being the same for all lines in accordance with the continuity.
Thus if $\mathrm{d} \psi$ is the increase of $\psi$ due to a displacement from P to $\mathrm{P}^{\prime}$, and k is the component of velocity normal to $\mathrm{PP}^{\prime}$, the flow across $\mathrm{PP}^{\prime}$ is $\mathrm{d} \psi=\mathrm{k} \cdot \mathrm{PP}^{\prime}$; and taking $\mathrm{PP}^{\prime}$ parallel to $\mathrm{Ox}, \mathrm{d} \psi=\mathrm{v}$ $d x$; and similarly $d \psi=-u$ dy with $P^{\prime}$ parallel to Oy; and generally $d \psi / d s$ is the velocity across ds, in a direction turned through a right angle forward, against the clock.

In the equations of uniplanar motion

$$
\begin{equation*}
2 \zeta=\frac{d v}{d x}-\frac{d u}{d y}=\frac{d^{2} \psi}{d x^{2}}+\frac{d^{2} \psi}{d y^{2}}=-\nabla^{2} \psi, \text { suppose } \tag{3}
\end{equation*}
$$

so that in steady motion

$$
\begin{equation*}
\frac{\mathrm{dH}}{\mathrm{dx}}+\nabla^{2} \psi \frac{\mathrm{~d} \psi}{\mathrm{dx}}=0, \frac{\mathrm{dH}}{\mathrm{dy}}+\nabla^{2} \psi \frac{\mathrm{~d} \psi}{\mathrm{dy}}=0, \frac{\mathrm{dH}}{\mathrm{~d} \psi}+\nabla^{2} \psi=0 \tag{4}
\end{equation*}
$$

and $\nabla^{2} \psi$ must be a function of $\psi$.
If the motion ia irrotational,

$$
\begin{equation*}
u=-\frac{d \varphi}{d x}=-\frac{d \psi}{d y}, v=-\frac{d \varphi}{d y}=\frac{d \psi}{d x} \tag{5}
\end{equation*}
$$

so that $\psi$ and $\varphi$ are conjugate functions of x and y ,

$$
\begin{equation*}
\varphi+\psi i=f(x+y i), \nabla^{2} \psi=0, \nabla^{2} \varphi=0 ; \tag{6}
\end{equation*}
$$

or putting

$$
\varphi+\psi i=w, x+y i=z, w=f(z)
$$

The curves $\varphi=$ constant and $\psi=$ constant form an orthogonal system; and the interchange of $\varphi$ and $\psi$ will give a new state of uniplanar motion, in which the velocity at every point is turned through a right angle without alteration of magnitude.

For instance, in a uniplanar flow, radially inward towards $O$, the flow across any circle of radius $r$ being the same and denoted by $2 \pi m$, the velocity must be $\mathrm{m} / \mathrm{r}$, and

$$
\begin{equation*}
\varphi=\mathrm{m} \log \mathrm{r}, \psi=\mathrm{m} \theta, \varphi+\psi \mathrm{i}=\mathrm{m} \log \mathrm{re}^{\mathrm{i} \theta}, \mathrm{w}=\mathrm{m} \log \mathrm{z} \tag{7}
\end{equation*}
$$

Interchanging these values

$$
\begin{equation*}
\psi=\mathrm{m} \log \mathrm{r}, \quad \varphi=\mathrm{m} \theta, \quad \psi+\varphi \mathrm{i}=\mathrm{m} \log \mathrm{re} \mathrm{e}^{\mathrm{i} \theta} \tag{8}
\end{equation*}
$$

gives a state of vortex motion, circulating round Oz , called a straight or columnar vortex.
A single vortex will remain at rest, and cause a velocity at any point inversely as the distance from the axis and perpendicular to its direction; analogous to the magnetic field of a straight electric current.

If other vortices are present, any one may be supposed to move with the velocity due to the
others, the resultant stream-function being

$$
\begin{equation*}
\psi=\Sigma \mathrm{m} \log \mathrm{r}=\log \Pi \mathrm{r}^{\mathrm{m}} ; \tag{9}
\end{equation*}
$$

the path of a vortex is obtained by equating the value of $\psi$ at the vortex to a constant, omitting the $\mathrm{r}^{\mathrm{m}}$ of the vortex itself.

When the liquid is bounded by a cylindrical surface, the motion of a vortex inside may be determined as due to a series of vortex-images, so arranged as to make the flow zero across the boundary.

For a plane boundary the image is the optical reflection of the vortex. For example, a pair of equal opposite vortices, moving on a line parallel to a plane boundary, will have a corresponding pair of images, forming a rectangle of vortices, and the path of a vortex will be the Cotes' spiral

$$
\begin{equation*}
\mathrm{r} \sin 2 \theta=2 \mathrm{a}, \text { or } \mathrm{x}^{-2}+\mathrm{y}^{-2}=\mathrm{a}^{-2} \tag{10}
\end{equation*}
$$

this is therefore the path of a single vortex in a right-angled corner; and generally, if the angle of the corner is $\pi / n$, the path is the Cotes' spiral

$$
\begin{equation*}
\mathrm{r} \sin \mathrm{n} \theta=\mathrm{na} . \tag{11}
\end{equation*}
$$

A single vortex in a circular cylinder of radius a at a distance c from the centre will move with the velocity due to an equal opposite image at a distance $\mathrm{a}^{2} / \mathrm{c}$, and so describe a circle with velocity

$$
\begin{equation*}
\mathrm{mc} /\left(\mathrm{a}^{2}-\mathrm{c}^{2}\right) \text { in the periodic time } 2 \pi\left(\mathrm{a}^{2}-\mathrm{c}^{2}\right) / \mathrm{m} . \tag{12}
\end{equation*}
$$

Conjugate functions can be employed also for the motion of liquid in a thin sheet between two concentric spherical surfaces; the components of velocity along the meridian and parallel in colatitude $\theta$ and longitude $\lambda$ can be written

$$
\begin{equation*}
\frac{d \varphi}{d \theta}=\frac{1}{\sin \theta} \frac{d \psi}{d \lambda}, \frac{1}{\sin \theta} \frac{d \psi}{d \lambda}=-\frac{d \psi}{d \theta} \tag{13}
\end{equation*}
$$

and then

$$
\begin{equation*}
\varphi+\psi \mathrm{i}=\mathrm{F}\left(\tan \underline{1} / 2 \theta \cdot \mathrm{e}^{\lambda \mathrm{i}}\right) \tag{14}
\end{equation*}
$$

28. Uniplanar Motion of a Liquid due to the Passage of a Cylinder through it.-A streamfunction $\psi$ must be determined to satisfy the conditions

$$
\begin{gather*}
\nabla^{2} \psi=0 \text {, throughout the liquid; }  \tag{1}\\
\psi=\text { constant, over any fixed boundary; }  \tag{2}\\
\mathrm{d} \psi / \mathrm{ds}=\text { normal velocity reversed over a solid boundary, } \tag{3}
\end{gather*}
$$

so that, if the solid is moving with velocity $U$ in the direction $O x, d \psi / d s=-U d y / d s$, or $\psi+U y=$ constant over the moving cylinder; and $\psi+U y=\psi^{\prime}$ is the stream function of the relative motion of the liquid past the cylinder, and similarly $\psi-\mathrm{Vx}$ for the component velocity V along Oy; and generally

$$
\begin{equation*}
\psi^{\prime}=\psi+U y-V x \tag{4}
\end{equation*}
$$

is the relative stream-function, constant over a solid boundary moving with components U and V of velocity.

If the liquid is stirred up by the rotation R of a cylindrical body,

$$
\begin{align*}
\mathrm{d} \psi / \mathrm{ds} & =\text { normal velocity reversed } \\
& =-\mathrm{Rx} \frac{\mathrm{dx}}{\mathrm{ds}}-\mathrm{Ry} \frac{\mathrm{dy}}{\mathrm{ds}},  \tag{5}\\
& \psi+1 / 2 \mathrm{R}\left(\mathrm{x}^{2}+\mathrm{y}^{2}\right)=\psi^{\prime}, \tag{6}
\end{align*}
$$

a constant over the boundary; and $\psi^{\prime}$ is the current-function of the relative motion past the cylinder, but now

$$
\begin{equation*}
\mathrm{V}^{2} \psi^{\prime}+2 \mathrm{R}=0 \tag{7}
\end{equation*}
$$

throughout the liquid.
Inside an equilateral triangle, for instance, of height $h$,

$$
\begin{equation*}
\psi^{\prime}=-2 \mathrm{R} \alpha \beta \gamma / \mathrm{h}, \tag{8}
\end{equation*}
$$

where $\alpha, \beta, \gamma$ are the perpendiculars on the sides of the triangle.
In the general case $\psi^{\prime}=\psi+U y-V x+1 / 2 R\left(x^{2}+y^{2}\right)$ is the relative stream function for velocity components, $\mathrm{U}, \mathrm{V}, \mathrm{R}$.
29. Example 1.-Liquid motion past a circular cylinder.

Consider the motion given by

$$
\begin{equation*}
\omega=\mathrm{U}\left(\mathrm{z}+\mathrm{a}^{2} / \mathrm{z}\right) \tag{1}
\end{equation*}
$$

so that

$$
\begin{align*}
& \psi=\mathrm{U}\left(\mathrm{r}+\frac{\mathrm{a}^{2}}{\mathrm{r}}\right) \cos \theta=\mathrm{U}\left(1+\frac{\mathrm{a}^{2}}{\mathrm{r}^{2}}\right) \mathrm{x}  \tag{2}\\
& \varphi=\mathrm{U}\left(\mathrm{r}+\frac{\mathrm{a}^{2}}{\mathrm{r}}\right) \sin \theta=\mathrm{U}\left(1+\frac{\mathrm{a}^{2}}{\mathrm{r}^{2}}\right) \mathrm{y} \tag{2}
\end{align*}
$$

Then $\psi=0$ over the cylinder $\mathrm{r}=\mathrm{a}$, which may be considered a fixed post; and a stream line past it along which $\psi=\mathrm{Uc}$, a constant, is the curve

$$
\begin{equation*}
\left(r-\frac{a^{2}}{r}\right) \sin \theta=c,\left(x^{2}+y^{2}\right)(y-c)-a^{2} y=0 \tag{3}
\end{equation*}
$$

a cubic curve $\left(\mathrm{C}_{3}\right)$.
Over a concentric cylinder, external or internal, of radius $r=b$,

$$
\begin{equation*}
\psi^{\prime}=\psi+\mathrm{U}_{1} \mathrm{y}=\left[\mathrm{U}\left(1-\frac{\mathrm{a}^{2}}{\mathrm{~b}^{2}}\right)+\mathrm{U}_{1}\right] \mathrm{y} \tag{4}
\end{equation*}
$$

and $\psi^{\prime}$ is zero if

$$
\begin{equation*}
\mathrm{U}_{1} / \mathrm{U}=\left(\mathrm{a}^{2}-\mathrm{b}^{2}\right) / \mathrm{b}^{2} \tag{5}
\end{equation*}
$$

so that the cylinder may swim for an instant in the liquid without distortion, with this velocity $\mathrm{U}_{1}$, and $\omega$ in (1) will give the liquid motion in the interspace between the fixed cylinder $\mathrm{r}=\mathrm{a}$ and the concentric cylinder $\mathrm{r}=\mathrm{b}$, moving with velocity $\mathrm{U}_{1}$.

When $\mathrm{b}=0, \mathrm{U}_{1}=\infty$; and when $\mathrm{b}=\infty, \mathrm{U}_{1}=-\mathrm{U}$, so that at infinity the liquid is streaming in the direction xO with velocity U .

If the liquid is reduced to rest at infinity by the superposition of an opposite stream given by $\omega=-U z$, we are left with

$$
\begin{gather*}
\omega=\mathrm{Ua}^{2} / \mathrm{z}  \tag{6}\\
\varphi=\mathrm{U}\left(\mathrm{a}^{2} / \mathrm{r}\right) \cos \theta=\mathrm{Ua}^{2} \mathrm{x} /\left(\mathrm{x}^{2}+\mathrm{y}^{2}\right),  \tag{7}\\
\psi=-\mathrm{U}\left(\mathrm{a}^{2} / \mathrm{r}\right) \sin \theta=-\mathrm{Ua}^{2} \mathrm{y} /\left(\mathrm{x}^{2}+\mathrm{y}^{2}\right), \tag{8}
\end{gather*}
$$

giving the motion due to the passage of the Cylinder $\mathrm{r}=\mathrm{a}$ with velocity U through the origin O in the direction Ox .

If the direction of motion makes an angle $\theta^{\prime}$ with Ox ,

$$
\begin{equation*}
\tan \theta^{\prime}=\frac{d \varphi}{d y} / \frac{d \varphi}{d x}=\frac{2 x y}{x^{2}-y^{2}}=\tan 2 \theta, \quad \theta=1 / 2 \theta^{\prime} \tag{9}
\end{equation*}
$$

and the velocity is $\mathrm{Ua}^{2} / \mathrm{r}^{2}$.
Along the path of a particle, defined by the $\mathrm{C}_{3}$ of (3),

$$
\begin{gather*}
\sin ^{2} 1 / 2 \theta^{\prime}=\frac{y^{2}}{x^{2}+y^{2}}=\frac{y(y-c)}{a^{2}}  \tag{10}\\
1 / 2 \sin \theta^{\prime} \frac{d \theta^{\prime}}{d s}=\frac{2 y-c}{a^{2}} \frac{d y}{d s} \tag{11}
\end{gather*}
$$

on the radius of curvature is $1 / 4 \mathrm{a}^{2} /(\mathrm{y}-1 / 2 \mathrm{c})$, which shows that the curve is an Elastica or Lintearia. (J. C. Maxwell, Collected Works, ii. 208.)

If $\varphi_{1}$ denotes the velocity function of the liquid filling the cylinder $r=b$, and moving bodily with it with velocity $U_{1}$,

$$
\begin{equation*}
\varphi_{1}=-\mathrm{U}_{1} \mathrm{x}, \tag{12}
\end{equation*}
$$

and over the separating surface $r=b$

$$
\begin{equation*}
\frac{\varphi}{\varphi_{1}}=-\frac{\mathrm{U}}{\mathrm{U}_{1}}\left(1+\frac{\mathrm{a}^{2}}{\mathrm{~b}^{2}}\right)=\frac{\mathrm{a}^{2}+\mathrm{b}^{2}}{\mathrm{a}^{2}-\mathrm{b}^{2}}, \tag{13}
\end{equation*}
$$

and this, by $\S 36$, is also the ratio of the kinetic energy in the annular interspace between the two cylinders to the kinetic energy of the liquid moving bodily inside $\mathrm{r}=\mathrm{b}$.

Consequently the inertia to overcome in moving the cylinder $r=b$, solid or liquid, is its own inertia, increased by the inertia of liquid $\left(a^{2}+b^{2}\right) /\left(a^{2} \sim b^{2}\right)$ times the volume of the cylinder $r=$ b ; this total inertia is called the effective inertia of the cylinder $\mathrm{r}=\mathrm{b}$, at the instant the two cylinders are concentric.

With liquid of density $\rho$, this gives rise to a kinetic reaction to acceleration $\mathrm{dU} / \mathrm{dt}$, given by

$$
\begin{equation*}
\Pi \rho b^{2} \frac{a^{2}+b^{2}}{a^{2}-b^{2}} \frac{d U}{d t}=\frac{a^{2}+b^{2}}{a^{2}-b^{2}} M^{\prime} \frac{d U}{d t}, \tag{14}
\end{equation*}
$$

if $\mathrm{M}^{\prime}$ denotes the mass of liquid displaced by unit length of the cylinder $\mathrm{r}=\mathrm{b}$. In particular, when $\mathrm{a}=\infty$, the extra inertia is $\mathrm{M}^{\prime}$.

When the cylinder $r=a$ is moved with velocity $U$ and $r=b$ with velocity $U_{1}$ along $O x$,

$$
\begin{align*}
\varphi & =U \frac{a^{2}}{b^{2}-a^{2}}\left(\frac{b^{2}}{r}+r\right) \cos \theta-U_{1} \frac{b^{2}}{b^{2}-a^{2}}\left(r+\frac{a^{2}}{r}\right) \cos \theta,  \tag{15}\\
\psi & =-U \frac{a^{2}}{b^{2}-a^{2}}\left(\frac{b^{2}}{r}-r\right) \sin \theta-U_{1} \frac{b^{2}}{b^{2}-a^{2}}\left(r-\frac{a^{2}}{r}\right) \sin \theta, \tag{16}
\end{align*}
$$

and similarly, with velocity components V and $\mathrm{V}_{1}$ along Oy

$$
\begin{align*}
\varphi & =V \frac{a^{2}}{b^{2}-a^{2}}\left(\frac{b^{2}}{r}+r\right) \cos \theta-V_{1} \frac{b^{2}}{b^{2}-a^{2}}\left(r+\frac{a^{2}}{r}\right) \cos \theta,  \tag{17}\\
\psi & =V \frac{a^{2}}{b^{2}-a^{2}}\left(\frac{b^{2}}{r}-r\right) \sin \theta+V_{1} \frac{b^{2}}{b^{2}-a^{2}}\left(r-\frac{a^{2}}{r}\right) \sin \theta, \tag{18}
\end{align*}
$$

and then for the resultant motion

$$
\begin{array}{r}
\mathrm{w}=\left(\mathrm{U}^{2}+\mathrm{V}^{2}\right) \frac{a^{2}}{b^{2}-a^{2}} \frac{z}{U+V i}+\frac{a^{2} b^{2}}{b^{2}-a^{2}} \frac{U+V i}{z} \\
-\left(U_{1}^{2}+V_{1}^{2}\right) \frac{b^{2}}{b^{2}-a^{2}} \frac{z}{U_{1}+V_{1} i}-\frac{a^{2} b^{2}}{b^{2}-a^{2}} \frac{U_{1}+V_{1} i}{z} . \tag{19}
\end{array}
$$

The resultant impulse of the liquid on the cylinder is given by the component, over $\mathrm{r}=\mathrm{a}$ (§ 36),

$$
\begin{equation*}
\mathrm{X}=\int \rho \varphi \cos \theta \cdot \mathrm{a} d \theta=\pi \rho \mathrm{a}^{2}\left(\mathrm{U} \frac{\mathrm{~b}^{2}+\mathrm{a}^{2}}{\mathrm{~b}^{2}-\mathrm{a}^{2}}-\mathrm{U}_{1} \frac{2 \mathrm{~b}^{2}}{\mathrm{~b}^{2}-\mathrm{a}^{2}}\right) ; \tag{20}
\end{equation*}
$$

and over $\mathrm{r}=\mathrm{b}$

$$
\begin{equation*}
\mathrm{X}_{1}=\int \rho \varphi \cos \theta \cdot \mathrm{b} d \theta=\pi \rho \mathrm{b}^{2}\left(\mathrm{U} \frac{2 \mathrm{a}^{2}}{\mathrm{~b}^{2}-\mathrm{a}^{2}}-\mathrm{U}_{1} \frac{\mathrm{~b}^{2}+\mathrm{a}^{2}}{\mathrm{~b}^{2}-\mathrm{a}^{2}}\right), \tag{21}
\end{equation*}
$$

and the difference $\mathrm{X}-\mathrm{X}_{1}$ is the component momentum of the liquid in the interspace; with similar expressions for Y and $\mathrm{Y}_{1}$.
Then, if the outside cylinder is free to move

$$
\begin{equation*}
\mathrm{X}_{1}=0, \quad \frac{\mathrm{~V}_{1}}{\mathrm{U}}=\frac{2 \mathrm{a}^{2}}{\mathrm{~b}^{2}+\mathrm{a}^{2}}, \quad \mathrm{X}=n \rho \mathrm{a}^{2} \mathrm{U} \frac{\mathrm{~b}^{2}-\mathrm{a}^{2}}{\mathrm{~b}^{2}+\mathrm{a}^{2}} . \tag{22}
\end{equation*}
$$

But if the outside cylinder is moved with velocity $U_{1}$, and the inside cylinder is solid or filled with liquid of density $\sigma$,

$$
\begin{gather*}
\mathrm{X}=-п \rho \mathrm{a}^{2} \mathrm{U}, \quad \frac{\mathrm{U}_{1}}{\mathrm{U}}=\frac{2 \rho \mathrm{~b}^{2}}{\rho\left(\mathrm{~b}^{2}+\mathrm{a}^{2}\right)+\sigma\left(\mathrm{b}^{2}-\mathrm{a}^{2}\right)} \\
\frac{\mathrm{U}-\mathrm{U}_{1}}{\mathrm{U}_{1}}=\frac{(\rho-\sigma)\left(b^{2}-a^{2}\right)}{\rho\left(b^{2}+\mathrm{a}^{2}\right)+\sigma\left(\mathrm{b}^{2}-\mathrm{a}^{2}\right)} \tag{23}
\end{gather*}
$$

and the inside cylinder starts forward or backward with respect to the outside cylinder, according as $\rho>$ or $<\sigma$.
30. The expression for $\omega$ in (1) § 29 may be increased by the addition of the term

$$
\begin{equation*}
i m \log z=-m \theta+i m \log r, \tag{1}
\end{equation*}
$$

representing vortex motion circulating round the annulus of liquid.
Considered by itself, with the cylinders held fixed, the vortex sets up a circumferential velocity $\mathrm{m} / \mathrm{r}$ on a radius r , so that the angular momentum of a circular filament of annular cross section $d A$ is $\rho m d A$, and of the whole vortex is $\rho m \pi\left(b^{2}-a^{2}\right)$.

Any circular filament can be started from rest by the application of a circumferential impulse $п \rho m ~ d r ~ a t ~ e a c h ~ e n d ~ o f ~ a ~ d i a m e t e r ; ~ s o ~ t h a t ~ a ~ m e c h a n i s m ~ a t t a c h e d ~ t o ~ t h e ~ c y l i n d e r s, ~ w h i c h ~ c a n ~ s e t ~$ up a uniform distributed impulse $п \rho \mathrm{~m}$ across the two parts of a diameter in the liquid, will generate the vortex motion, and react on the cylinder with an impulse couple - $\rho m \operatorname{an~}^{2}$ and $\rho m \pi b^{2}$, having resultant $\rho m п\left(b^{2}-a^{2}\right)$, and this couple is infinite when $b=\infty$, as the angular momentum of the vortex is infinite. Round the cylinder $r=a$ held fixed in the $U$ current the liquid streams past with velocity

$$
\begin{equation*}
\mathrm{q}^{\prime}=2 \mathrm{U} \sin \theta+\mathrm{m} / \mathrm{a} ; \tag{2}
\end{equation*}
$$

and the loss of head due to this increase of velocity from $U$ to $q^{\prime}$ is

$$
\begin{equation*}
\frac{\mathrm{q}^{2}-\mathrm{U}^{2}}{2 \mathrm{~g}}=\frac{(2 \mathrm{U} \sin \theta+\mathrm{m} / \mathrm{a})^{2}-\mathrm{U}^{2}}{2 \mathrm{~g}} \tag{3}
\end{equation*}
$$

so that cavitation will take place, unless the head at a great distance exceeds this loss.
The resultant hydrostatic thrust across any diametral plane of the cylinder will be modified, but the only term in the loss of head which exerts a resultant thrust on the whole cylinder is $2 \mathrm{mU} \sin \theta / \mathrm{ga}$, and its thrust is $2 \pi \rho \mathrm{mU}$ absolute units in the direction $C y$, to be counteracted by a support at the centre C; the liquid is streaming past $r=a$ with velocity $U$ reversed, and the cylinder is surrounded by a vortex. Similarly, the streaming velocity V reversed will give rise to a thrust $2 \Pi \rho m V$ in the direction $x C$.

Now if the cylinder is released, and the components $U$ and $V$ are reversed so as to become the velocity of the cylinder with respect to space filled with liquid, and at rest at infinity, the cylinder will experience components of force per unit length
(i.) $-2 \pi \rho m V, 2 п \rho m U$, due to the vortex motion;
(ii.) $-п \rho \mathrm{a}^{2} \mathrm{dU} / \mathrm{dt},-п \rho \mathrm{a}^{2} \mathrm{dV} / \mathrm{dt}$, due to the kinetic reaction of the liquid;
(iii.) $0,-\Pi(\sigma-\rho) a^{2} g$, due to gravity,
taking Oy vertically upward, and denoting the density of the cylinder by $\sigma$; so that the equations of motion are

$$
\begin{gather*}
п \rho a^{2} \frac{d U}{d t}=-п \rho a^{2} \frac{d U}{d t}-2 п \rho m V,  \tag{4}\\
п \rho a^{2} \frac{d V}{d t}=-п \rho a^{2} \frac{d V}{d t}+2 \pi \rho m V-п(\sigma-\rho) a^{2} g, \tag{5}
\end{gather*}
$$

or, putting $m=a^{2} \omega$, so that the vortex velocity is due to an angular velocity $\omega$ at a radius $a$,

$$
\begin{gather*}
(\sigma+\rho) d U / d t+2 \rho \omega V=0  \tag{6}\\
(\sigma+\rho) d V / d t-2 \rho \omega U+(\sigma-\rho) g=0 \tag{7}
\end{gather*}
$$

Thus with $\mathrm{g}=0$, the cylinder will describe a circle with angular velocity $2 \rho \omega /(\sigma+\rho)$, so that the radius is $(\sigma+\rho) v / 2 \rho \omega$, if the velocity is v . With $\sigma=0$, the angular velocity of the cylinder is $2 \omega$; in this way the velocity may be calculated of the propagation of ripples and waves on the surface of a vertical whirlpool in a sink.
Restoring $\sigma$ will make the path of the cylinder a trochoid; and so the swerve can be explained of the ball in tennis, cricket, baseball, or golf.

Another explanation may be given of the sidelong force, arising from the velocity of liquid past a cylinder, which is encircled by a vortex. Taking two planes $x= \pm b$, and considering the increase of momentum in the liquid between them, due to the entry and exit of liquid momentum, the increase across dy in the direction Oy , due to elements at P and $\mathrm{P}^{\prime}$ at opposite ends of the diameter $\mathrm{PP}^{\prime}$, is

$$
\begin{aligned}
& \rho d y\left(U-U a^{2} r^{-2} \cos 2 \theta+m r^{-1} \sin \theta\right)\left(U a^{2} r^{-2} \sin 2 \theta+m r^{-1} \cos \theta\right) \\
+ & \rho \operatorname{dy}\left(-U+U a^{2} r^{-2} \cos 2 \theta+m r^{-1} \sin \theta\right)\left(U a^{2} r^{-2} \sin 2 \theta-m r^{-1} \cos \theta\right) \\
= & 2 \rho d y m U r^{-1}\left(\cos \theta-a^{2} r^{-2} \cos 3 \theta\right),
\end{aligned}
$$

and with $\mathrm{y}=\mathrm{b} \tan \theta, \mathrm{r}=\mathrm{b} \sec \theta$, this is

$$
\begin{equation*}
2 \rho m \mathrm{~d} \theta\left(1-\mathrm{a}^{2} \mathrm{~b}^{-2} \cos 3 \theta \cos \theta\right), \tag{9}
\end{equation*}
$$

and integrating between the limits $\theta= \pm 1 / 2 \pi$, the resultant, as before, is $2 \pi \rho m U$.
31. Example 2.-Confocal Elliptic Cylinders.-Employ the elliptic coordinates $\eta$, $\xi$, and $\zeta=\eta+$ $\xi \mathrm{i}$, such that

$$
\begin{equation*}
z=c \operatorname{ch} \zeta, x=c \operatorname{ch} \eta \cos \xi, y=c \operatorname{sh} \eta \sin \zeta ; \tag{1}
\end{equation*}
$$

then the curves for which $\eta$ and $\xi$ are constant are confocal ellipses and hyperbolas, and

$$
\begin{gather*}
J=\frac{d(x, y)}{d(\eta, \xi)}=c^{2}\left(\operatorname{ch}^{2} \eta-\cos ^{2} \xi\right) \\
=(1 / 2) c^{2}(\operatorname{ch} 2 \eta-\cos 2 \xi)=r_{1} r_{2}=O D^{2}, \tag{2}
\end{gather*}
$$

if OD is the semi-diameter conjugate to OP, and $r_{1}, r_{2}$ the focal distances,

$$
\begin{gather*}
r_{1}, r_{2}=c(\operatorname{ch} \eta \pm \cos \xi)  \tag{3}\\
r^{2}=x^{2}+y^{2}=c^{2}\left(\operatorname{ch}^{2} \eta-\sin ^{2} \xi\right) \\
=1 / 2 c^{2}(\operatorname{ch} 2 \eta+\cos 2 \xi) \tag{4}
\end{gather*}
$$

Consider the streaming motion given by

$$
\begin{gather*}
w=m \operatorname{ch}(\zeta-\gamma), \gamma=\alpha+\beta i  \tag{5}\\
\varphi=m \operatorname{ch}(\eta-\alpha) \cos (\xi-\beta), \psi=m \operatorname{sh}(\eta-\alpha) \sin (\xi-\beta) . \tag{6}
\end{gather*}
$$

Then $\psi=0$ over the ellipse $\eta=\alpha$, and the hyperbola $\xi=\beta$, so that these may be taken as fixed boundaries; and $\psi$ is a constant on a $\mathrm{C}_{4}$.

Over any ellipse $\eta$, moving with components $U$ and $V$ of velocity,

$$
\begin{gather*}
\psi^{\prime}=\psi+U y-V x=[m \operatorname{sh}(\eta-\alpha) \cos \beta+U c \operatorname{sh} \eta] \sin \xi \\
-[m \operatorname{sh}(\eta-\alpha) \sin \beta+V c \operatorname{ch} \eta] \cos \xi ; \tag{7}
\end{gather*}
$$

so that $\psi^{\prime}=0$, if

$$
\begin{equation*}
U=-\frac{m}{c} \frac{\operatorname{sh}(\eta-\alpha)}{\operatorname{sh} \eta} \cos \beta, V=-\frac{m}{c} \frac{\operatorname{sh}(\eta-\alpha)}{\operatorname{ch} \eta} \sin \beta, \tag{8}
\end{equation*}
$$

having a resultant in the direction PO, where $P$ is the intersection of an ellipse $\eta$ with the hyperbola $\beta$; and with this velocity the ellipse $\eta$ can be swimming in the liquid, without distortion for an instant.

At infinity

$$
\begin{align*}
& U=-\frac{m}{c} e^{-a} \cos \beta=-\frac{m}{a-b} \cos \beta \\
& V=-\frac{m}{c} e^{-a} \sin \beta=-\frac{m}{a-b} \sin \beta \tag{9}
\end{align*}
$$

a and b denoting the semi-axes of the ellipse $\alpha$; so that the liquid is streaming at infinity with velocity $\mathrm{Q}=\mathrm{m} /(\mathrm{a}+\mathrm{b})$ in the direction of the asymptote of the hyperbola $\beta$.

An ellipse interior to $\eta=\alpha$ will move in a direction opposite to the exterior current; and when $\eta=0, U=\infty$, but $V=(m / c) \operatorname{sh} \alpha \sin \beta$.

Negative values of $\eta$ must be interpreted by a streaming motion on a parallel plane at a level slightly different, as on a double Riemann sheet, the stream passing from one sheet to the other across a cut SS' joining the foci S, S'. A diagram has been drawn by Col. R. L. Hippisley.

The components of the liquid velocity $q$, in the direction of the normal of the ellipse $\eta$ and hyperbola $\xi$, are

$$
\begin{equation*}
-m J^{-1} \operatorname{sh}(\eta-\alpha) \cos (\xi-\beta), m J^{-1} \operatorname{ch}(\eta-\alpha) \sin (\xi-\beta) . \tag{10}
\end{equation*}
$$

The velocity $q$ is zero in a corner where the hyperbola $\beta$ cuts the ellipse $\alpha$; and round the ellipse $\alpha$ the velocity $q$ reaches a maximum when the tangent has turned through a right angle, and then

$$
\begin{equation*}
\mathrm{q}=\mathrm{Qe}^{\mathrm{a}} \frac{\sqrt{ }(\operatorname{ch} 2 \alpha-\cos 2 \beta)}{\operatorname{sh} 2 \alpha} ; \tag{11}
\end{equation*}
$$

and the condition can be inferred when cavitation begins.
With $\beta=0$, the stream is parallel to $\mathrm{x}_{0}$, and

$$
\begin{gather*}
\varphi=m \operatorname{ch}(\eta-\alpha) \cos \xi \\
=-U c \operatorname{ch}(\eta-\alpha) \operatorname{sh} \eta \cos \xi / \operatorname{sh}(\eta-\alpha) \tag{12}
\end{gather*}
$$

over the cylinder $\eta$, and as in (12) § 29,

$$
\begin{equation*}
\varphi_{1}=-\mathrm{Ux}=-\mathrm{Uc} \operatorname{ch} \eta \cos \xi \tag{13}
\end{equation*}
$$

for liquid filling the cylinder; and

$$
\begin{equation*}
\frac{\varphi}{\varphi_{1}}=\frac{\operatorname{th} \eta}{\operatorname{th}(\eta-\alpha)} \tag{14}
\end{equation*}
$$

over the surface of $\eta$; so that parallel to $O x$, the effective inertia of the cylinder $\eta$, displacing $M^{\prime}$ liquid, is increased by $M^{\prime}$ th $\eta / \operatorname{th}(\eta-\alpha)$, reducing when $\alpha=\infty$ to $M^{\prime}$ th $\eta=M^{\prime}(b / a)$.

Similarly, parallel to Oy, the increase of effective inertia is $M^{\prime} / \operatorname{th} \eta$ th ( $\eta-\alpha$ ), reducing to $M^{\prime} /$ th $\eta=M^{\prime}(a / b)$, when $\alpha=\infty$, and the liquid extends to infinity.
32. Next consider the motion given by

$$
\begin{equation*}
\varphi=m \operatorname{ch} 2(\eta-\alpha) \sin 2 \xi, \psi=-m \operatorname{sh} 2(\eta-\alpha) \cos 2 \xi \tag{1}
\end{equation*}
$$

in which $\psi=0$ over the ellipse $\alpha$, and

$$
\begin{equation*}
\psi^{\prime}=\psi+1 / 2 R\left(x^{2}+y^{2}\right)=\left[-m \operatorname{sh} 2(\eta-\alpha)+1 / 4 R c^{2}\right] \cos 2 \xi+1 / 4 R c^{2} \operatorname{ch} 2 \eta \tag{2}
\end{equation*}
$$

which is constant over the ellipse $\eta$ if

$$
\begin{equation*}
1 / 4 R^{2}=m \operatorname{sh} 2(\eta-\alpha) \tag{3}
\end{equation*}
$$

so that this ellipse can be rotating with this angular velocity R for an instant without distortion, the ellipse $\alpha$ being fixed.

For the liquid filling the interior of a rotating elliptic cylinder of cross section

$$
\begin{gather*}
x^{2} / a^{2}+y^{2} / b^{2}=1  \tag{4}\\
\Psi_{1}^{\prime}=m_{1}\left(x^{2} / a^{2}+y^{2} / b^{2}\right) \tag{5}
\end{gather*}
$$

with

$$
\begin{gather*}
\nabla^{2} \Psi_{1}^{\prime}=-2 R=-2 m_{1}\left(1 / a^{2}+1 / b^{2}\right) \\
\psi_{1}=m_{1}\left(x^{2} / \mathrm{a}^{2}+\mathrm{y}^{2} / \mathrm{b}^{2}\right)-1 / 2 \mathrm{R}\left(\mathrm{x}^{2}+\mathrm{y}^{2}\right) \\
=-1 / 2 \mathrm{R}\left(\mathrm{x}^{2}-\mathrm{y}^{2}\right)\left(\mathrm{a}^{2}-\mathrm{b}^{2}\right) /\left(\mathrm{a}^{2}+\mathrm{b}^{2}\right)  \tag{6}\\
\varphi_{1}=\operatorname{Rxy}\left(\mathrm{a}^{2}-\mathrm{b}^{2}\right) /\left(\mathrm{a}^{2}+\mathrm{b}^{2}\right) \\
\mathrm{w}_{1}=\varphi_{1}+\Psi_{1} \mathrm{i}=-1 / 2 \mathrm{iR}(\mathrm{x}+\mathrm{yi})^{2}\left(\mathrm{a}^{2}-\mathrm{b}^{2}\right) /\left(\mathrm{a}^{2}+\mathrm{b}^{2}\right)
\end{gather*}
$$

The velocity of a liquid particle is thus $\left(a^{2}-b^{2}\right) /\left(a^{2}+b^{2}\right)$ of what it would be if the liquid was frozen and rotating bodily with the ellipse; and so the effective angular inertia of the liquid is $\left(a^{2}-b^{2}\right)^{2} /\left(a^{2}+b^{2}\right)^{2}$ of the solid; and the effective radius of gyration, solid and liquid, is given by

$$
\begin{equation*}
\mathrm{k}^{2}=1 / 4\left(\mathrm{a}^{2}+\mathrm{b}^{2}\right), \text { and } 1 / 4\left(\mathrm{a}^{2}-\mathrm{b}^{2}\right)^{2} /\left(\mathrm{a}^{2}+\mathrm{b}^{2}\right) \tag{7}
\end{equation*}
$$

For the liquid in the interspace between $\alpha$ and $\eta$,

$$
\begin{gather*}
\frac{\varphi}{\varphi_{1}}=\frac{m \operatorname{ch} 2(\eta-\alpha) \sin 2 \xi}{1 / 4 R^{2} \operatorname{sh} 2 \eta \sin 2 \xi\left(a^{2}-b^{2}\right) /\left(a^{2}+b^{2}\right)} \\
=1 / \operatorname{th} 2(\eta-\alpha) \operatorname{th} 2 \eta \tag{8}
\end{gather*}
$$

and the effective $\mathrm{k}^{2}$ of the liquid is reduced to

$$
\begin{equation*}
1 / 4 c^{2} / \operatorname{th} 2(\eta-\alpha) \operatorname{sh} 2 \eta \tag{9}
\end{equation*}
$$

which becomes $1 / 4 c^{2} /$ sh $2 \eta=1 / 8\left(a^{2}-b^{2}\right) / a b$, when $\alpha=\infty$, and the liquid surrounds the ellipse $\eta$ to infinity.

An angular velocity $R$, which gives components - Ry, $R x$ of velocity to a body, can be resolved into two shearing velocities, -R parallel to Ox , and R parallel to Oy ; and then $\psi$ is resolved into $\psi_{1}+\psi_{2}$, such that $\psi_{1}+1 / 2 R x^{2}$ and $\psi_{2}+1 / 2 \operatorname{Ry}^{2}$ is constant over the boundary.

Inside a cylinder

$$
\begin{gather*}
\varphi_{1}+\psi_{1} \mathrm{i}=-1 / 2 \operatorname{iR}(\mathrm{x}+\mathrm{yi})^{2} \mathrm{a}^{2} /\left(\mathrm{a}^{2}+\mathrm{b}^{2}\right),  \tag{10}\\
\varphi_{2}+\psi_{2} \mathrm{i}=1 / 2 \mathrm{iR}(\mathrm{x}+\mathrm{yi})^{2} \mathrm{~b}^{2} /\left(\mathrm{a}^{2}+\mathrm{b}^{2}\right) \tag{11}
\end{gather*}
$$

and for the interspace, the ellipse $\alpha$ being fixed, and $\alpha_{1}$ revolving with angular velocity $R$

$$
\begin{gather*}
\varphi_{1}+\psi_{1} \mathrm{i}=-1 / 8 \operatorname{iRc}^{2} \operatorname{sh} 2(\eta-\alpha+\xi \mathrm{i})(\operatorname{ch} 2 \alpha+1) / \operatorname{sh} 2\left(\alpha_{1}-\alpha\right),  \tag{12}\\
\varphi_{2}+\psi_{2} \mathrm{i}=1 / 8 \operatorname{iRc}^{2} \operatorname{sh} 2(\eta-\alpha+\xi \mathrm{i})(\operatorname{ch} 2 \alpha-1) / \operatorname{sh} 2\left(\alpha_{1}-\alpha\right), \tag{13}
\end{gather*}
$$

satisfying the condition that $\psi_{1}$ and $\psi_{2}$ are zero over $\eta=\alpha$, and over $\eta=\alpha_{1}$

$$
\begin{align*}
& \psi_{1}+1 / 2 R x^{2}=1 / 8 R^{2}\left(\operatorname{ch} 2 \alpha_{1}+1\right),  \tag{14}\\
& \Psi_{2}+1 / 2 R y^{2}=1 / 8 R^{2}\left(\operatorname{ch} 2 \alpha_{1}-1\right), \tag{15}
\end{align*}
$$

constant values.
In a similar way the more general state of motion may be analysed, given by

$$
\begin{equation*}
\mathrm{w}=\mathrm{m} \operatorname{ch} 2(\zeta-\gamma), \gamma=\alpha+\beta i, \tag{16}
\end{equation*}
$$

as giving a homogeneous strain velocity to the confocal system; to which may be added a circulation, represented by an additional term $\mathrm{m} \zeta$ in w .

Similarly, with

$$
\begin{equation*}
x+y i=c \sqrt{ }[\sin (\xi+\eta i)] \tag{17}
\end{equation*}
$$

the function

$$
\begin{equation*}
\psi=\operatorname{Qc} \operatorname{sh} 1 / 2(\eta-\alpha) \sin 1 / 2(\xi-\beta) \tag{18}
\end{equation*}
$$

will give motion streaming past the fixed cylinder $\eta=\alpha$, and dividing along $\xi=\beta$; and then

$$
\begin{equation*}
x^{2}-y^{2}=c^{2} \sin \xi c h \eta, 2 x y=c^{2} \cos \xi \operatorname{sh} \eta . \tag{19}
\end{equation*}
$$

In particular, with sh $\alpha=1$, the cross-section of $\eta=\alpha$ is

$$
\begin{equation*}
x^{4}+6 x^{2} y^{2}+y^{4}=2 c^{4}, \text { or } x^{4}+y^{4}=c^{4} \tag{20}
\end{equation*}
$$

when the axes are turned through $45^{\circ}$.
33. Example 3.-Analysing in this way the rotation of a rectangle filled with liquid into the two components of shear, the stream function $\psi_{1}$ is to be made to satisfy the conditions
(i.) $\nabla^{2} \Psi_{1}=0$,
(ii.) $\psi_{1}+\frac{1}{2} \mathrm{Rx}^{2}=\frac{1}{2} \mathrm{Ra}^{2}$, or $\Psi_{1}=0$ when $\mathrm{x}= \pm \mathrm{a}$,
(iii.) $\psi_{1}+1 / 2 R x^{2}=1 / 2 R^{2}, \Psi_{1}=1 / 2 R\left(a^{2}-x^{2}\right)$, when $y= \pm b$.

Expanded in a Fourier series,

$$
\begin{equation*}
a^{2}-x^{2}=\frac{32}{\pi^{3}} a^{2} \sum \frac{\cos (2 n+1)^{1 / 2} \pi x / a}{(2 n+1)^{3}}, \tag{1}
\end{equation*}
$$

so that

$$
\begin{align*}
& \psi_{1}=R \frac{16}{\Pi^{3}} a^{2} \sum \frac{\left.\cos (2 n+1)^{1 / 2 \pi x} / a \cdot \operatorname{ch}(2 n+1)^{1 / 2 \Pi y / a}\right)}{(2 n+1)^{3} \cdot \operatorname{ch}(2 n+1)^{1 / 2 \pi b / a}} \\
& \mathrm{w}_{1}=\varphi_{1}+\psi_{1} \mathrm{i}=i R \frac{16}{\Pi^{3}} a^{2} \sum \frac{\cos (2 n+1)^{1 / 2 \Pi z / a}}{(2 n+1)^{3} \operatorname{ch}(2 n+1)^{1 / 2 \Pi b / a}} \tag{2}
\end{align*}
$$

interchanged; and thence $\psi=\psi_{1}+\psi_{2}$.
Example 4.-Parabolic cylinder, axial advance, and liquid streaming past.
The polar equation of the cross-section being

$$
\begin{equation*}
\mathrm{r}^{1 / 2} \cos { }^{1 / 2} \theta=\mathrm{a}^{1 / 2}, \text { or } r+\mathrm{x}=2 \mathrm{a} \tag{3}
\end{equation*}
$$

the conditions are satisfied by

$$
\begin{gather*}
\Psi^{\prime}=\mathrm{Ur} \sin \theta-2 \mathrm{Ua}^{1 / 2} \mathrm{r}^{1 / 2} \sin 1 / 2 \theta=2 \mathrm{Ur}^{1 / 2} \sin 1 / 2 \theta\left(\mathrm{r}^{1 / 2} \cos 1 / 2 \theta-\mathrm{a}^{1 / 2}\right)  \tag{4}\\
\psi=2 \mathrm{Ua}^{1 / 2} \mathrm{r}^{1 / 2} \sin 1 / 2 \theta=-\mathrm{U} \sqrt{ }[2 \mathrm{a}(\mathrm{r}-\mathrm{x})]  \tag{5}\\
\mathrm{w}=-2 \mathrm{Ua}^{1 / 2} \mathrm{z}^{1 / 2} \tag{6}
\end{gather*}
$$

and the resistance of the liquid is $2 \Pi \rho a V^{2} / 2 g$.
A relative stream line, along which $\psi^{\prime}=\mathrm{Uc}$, is the quartic curve

$$
\begin{equation*}
y-c=\sqrt{ }[2 a(r-x)], \quad x=\frac{\left(4 a^{2} y^{2}-(y-c)^{4}\right.}{4 a(y-c)^{2}}, \quad r=\frac{4 a^{2} y^{2}+(y-c)^{4}}{4 a(y-c)^{2}} \tag{7}
\end{equation*}
$$

and in the absolute space curve given by $\psi$,

$$
\begin{equation*}
\frac{d y}{d x}=-\frac{(y-c)^{2}}{2 a y}, x=\frac{2 a c}{y-c}-2 a \log (y-c) \tag{8}
\end{equation*}
$$

34. Motion symmetrical about an Axis.-When the motion of a liquid is the same for any plane passing through Ox , and lies in the plane, a function $\psi$ can be found analogous to that employed in plane motion, such that the flux across the surface generated by the revolution of any curve AP from A to P is the same, and represented by $2 \Pi$ ( $\psi-\psi_{0}$ ); and, as before, if $\mathrm{d} \psi$ is the increase in $\psi$ due to a displacement of P to $\mathrm{P}^{\prime}$, then k the component of velocity normal to the surface swept out by $\mathrm{PP}^{\prime}$ is such that $2 \pi d \psi=2 \pi y \mathrm{k} \cdot \mathrm{PP}^{\prime}$; and taking $\mathrm{PP}^{\prime}$ parallel to Oy and Ox ,

$$
\begin{equation*}
u=-d \psi / y d y, \quad v=d \psi / y d x \tag{1}
\end{equation*}
$$

and $\psi$ is called after the inventor, "Stokes's stream or current function," as it is constant along a stream line (Trans. Camb. Phil. Soc., 1842; "Stokes's Current Function," R. A. Sampson, Phil. Trans., 1892); and $\mathrm{d} \psi / \mathrm{yds}$ is the component velocity across ds in a direction turned through a right angle forward.

In this symmetrical motion

$$
\begin{gather*}
\xi=0, \eta=0,2 \zeta=\frac{d}{d x}\left(\frac{1}{y} \frac{d \psi}{d x}\right)+\frac{d}{d y}\left(\frac{1}{y} \frac{d \psi}{d y}\right) \\
=\frac{1}{y}\left(\frac{d^{2} \psi}{d x^{2}}+\frac{d^{2} \psi}{d y^{2}}-\frac{1}{y} \frac{d \psi}{d y}\right)=-\frac{1}{y} \nabla^{2} \psi \tag{2}
\end{gather*}
$$

suppose; and in steady motion,

$$
\begin{equation*}
\frac{d H}{d x}+\frac{1}{y^{2}} \frac{d \psi}{d x} \nabla^{2} \psi=0, \frac{d H}{d y}+\frac{1}{y^{2}} \frac{d \psi}{d y} \nabla^{2} \psi=0 \tag{3}
\end{equation*}
$$

so that

$$
\begin{equation*}
2 \zeta / \mathrm{y}=-\mathrm{y}^{-2} \nabla^{2} \psi=\mathrm{dH} / \mathrm{d} \psi \tag{4}
\end{equation*}
$$

is a function of $\psi$, say $f^{\prime}(\psi)$, and constant along a stream line;

$$
\begin{equation*}
\mathrm{dH} / \mathrm{dv}=2 \mathrm{q} \zeta, \quad \mathrm{H}-\mathrm{f}(\psi)=\text { constant } \tag{5}
\end{equation*}
$$

throughout the liquid.
When the motion is irrotational,

$$
\begin{gather*}
\zeta=0, \quad u=-\frac{d \varphi}{d x}=-\frac{1}{y} \frac{d \psi}{d y}, \quad v=-\frac{d \varphi}{d y}=\frac{1}{y} \frac{d \psi}{d x}  \tag{6}\\
\nabla^{2} \psi=0, \text { or } \frac{d^{2} \psi}{d x^{2}}+\frac{d^{2} \psi}{d y^{2}}-\frac{1}{y} \quad \frac{d \psi}{d y}=0 \tag{7}
\end{gather*}
$$

Changing to polar coordinates, $x=r \cos \theta, y=r \sin \theta$, the equation (2) becomes, with $\cos \theta=$

$$
\begin{equation*}
r^{2} \overline{d r^{2}}+\left(1-\mu^{2}\right) \overline{d \mu}=2 \zeta r^{3} \sin \theta \tag{8}
\end{equation*}
$$

of which a solution, when $\zeta=0$, is

$$
\begin{gather*}
\psi=\left(A r^{\mathrm{n}+1}+\frac{\mathrm{B}}{\mathrm{r}^{\mathrm{n}}}\right)\left(1-\mu^{2}\right) \frac{\mathrm{dP}_{\mathrm{n}}}{\mathrm{~d} \mu}=\left(\mathrm{Ar}^{\mathrm{n}-1}+\frac{\mathrm{B}}{\mathrm{r}^{\mathrm{n}+2}}\right) \mathrm{y}^{2} \frac{\mathrm{dP}}{\mathrm{n}} \mathrm{~d} \mathrm{\mu}  \tag{9}\\
\varphi=\left\{(\mathrm{n}+1) \mathrm{Ar}^{\mathrm{n}}-\mathrm{nBr}^{-\mathrm{n}-1}\right\} \mathrm{P}_{\mathrm{n}} \tag{10}
\end{gather*}
$$

where $P_{n}$ denotes the zonal harmonic of the nth order; also, in the exceptional case of

$$
\begin{align*}
& \psi=A_{0} \cos \theta, \varphi=A_{0} / r \\
& \psi=B_{0} r, \varphi=-B_{0} \log \tan 1 / 2 \theta=-1 / 2 B_{0} \operatorname{sh}^{-1} \mathrm{x} / \mathrm{y} \tag{11}
\end{align*}
$$

Thus $\cos \theta$ is the Stokes' function of a point source at $O$, and $P A-P B$ of a line source $A B$.
The stream function $\psi$ of the liquid motion set up by the passage of a solid of revolution, moving with axial velocity U , is such that

$$
\begin{equation*}
\frac{1}{\mathrm{y}} \frac{\mathrm{~d} \psi}{\mathrm{ds}}=-\mathrm{U} \frac{\mathrm{dy}}{\mathrm{ds}}, \Psi+1 / 2 \mathrm{Uy}^{2}=\text { constant } \tag{12}
\end{equation*}
$$

over the surface of the solid; and $\psi$ must be replaced by $\psi^{\prime}=\psi+1 / 2 \mathrm{Uy}^{2}$ in the general equations of steady motion above to obtain the steady relative motion of the liquid past the solid.

For instance, with $n=1$ in equation (9), the relative stream function is obtained for a sphere of radius a, by making it

$$
\begin{equation*}
\psi^{\prime}=\psi+1 / 2 U y^{2}=1 / 2 U\left(r^{2}-a^{3} / r\right) \sin ^{2} \theta, \psi=-1 / 2 \mathrm{Ua}^{3} \sin ^{2} \theta / \mathrm{r} \tag{13}
\end{equation*}
$$

and then

$$
\begin{align*}
\varphi^{\prime} & =\mathrm{Ux}\left(1+1 / 2 \mathrm{a}^{3} / \mathrm{r}^{2}\right), \varphi=1 / 2 \mathrm{Ua}^{3} \cos \theta / \mathrm{r}^{2} \\
-\frac{\mathrm{d} \varphi}{\mathrm{dr}} & =\mathrm{U} \frac{\mathrm{a}^{3}}{\mathrm{r}^{3}} \cos \theta, \quad-\frac{\mathrm{d} \varphi}{\mathrm{rd} \theta}=1 / 2 \mathrm{U} \frac{\mathrm{a}^{3}}{\mathrm{r}^{3}} \sin \theta \tag{14}
\end{align*}
$$

so that, if the direction of motion makes an angle $\psi$ with Ox ,

$$
\begin{equation*}
\tan (\psi-\theta)=1 / 2 \tan \theta, \tan \psi=3 \tan \theta /\left(2-\tan ^{2} \theta\right) \tag{16}
\end{equation*}
$$

Along the path of a liquid particle $\psi^{\prime}$ is constant, and putting it equal to $1 / 2 \mathrm{Uc}^{2}$,

$$
\begin{equation*}
\left(r^{2}-a^{3} / r\right) \sin ^{2} \theta=c^{2}, \sin ^{2} \theta=c^{2} r /\left(r^{3}-a^{3}\right) \tag{17}
\end{equation*}
$$

the polar equation; or

$$
\begin{equation*}
\mathrm{y}^{2}=\mathrm{c}^{2} \mathrm{r}^{3} /\left(\mathrm{r}^{3}-\mathrm{a}^{3}\right), \mathrm{r}^{3}=\mathrm{a}^{3} \mathrm{y}^{2} /\left(\mathrm{y}^{2}-\mathrm{c}^{2}\right) \tag{18}
\end{equation*}
$$

a curve of the 10 th degree $\left(\mathrm{C}_{10}\right)$.
In the absolute path in space

$$
\begin{equation*}
\cos \psi=\left(2-3 \sin ^{2} \theta\right) / \sqrt{ }\left(4-\sin ^{2} \theta\right), \text { and } \sin ^{3} \theta=\left(y^{3}-c^{2} y\right) / a^{3} \tag{19}
\end{equation*}
$$

which leads to no simple relation.
The velocity past the surface of the sphere is

$$
\begin{equation*}
\frac{1}{r \sin \theta} \frac{d \psi^{\prime}}{d r}=1 / 2 U\left(2 r+\frac{a^{3}}{r^{2}}\right) \frac{\sin ^{2} \theta}{r \sin \theta}=3 / 2 U \sin \theta, \text { when } r=a \tag{20}
\end{equation*}
$$

so that the loss of head is

$$
\begin{equation*}
\left(9 / 4 \sin ^{2} \theta-1\right) \mathrm{U}^{2} / 2 \mathrm{~g}, \text { having a maximum } 5 / 4 \mathrm{U}^{2} / 2 \mathrm{~g} \tag{21}
\end{equation*}
$$

which must be less than the head at infinite distance to avoid cavitation at the surface of the sphere.

With $\mathrm{n}=2$, a state of motion is given by

$$
\begin{equation*}
\psi=-1 / 2 U y^{2} a^{4} \mu / r^{4}, \quad \psi^{\prime}=1 / 2 U y^{2}\left(1-a^{4} \mu / r^{4}\right) \tag{22}
\end{equation*}
$$

$$
\begin{equation*}
\varphi^{\prime}=\mathrm{Ux}+\varphi, \quad \varphi=-1 / 3 \mathrm{U}\left(\mathrm{a}^{4} / \mathrm{r}^{3}\right) \mathrm{P}_{2}, \quad \mathrm{P}_{2}=3 / 2 \mu^{2}-1 / 2, \tag{23}
\end{equation*}
$$

representing a stream past the surface $r^{4}=a^{4} \mu$.
35. A circular vortex, such as a smoke ring, will set up motion symmetrical about an axis, and provide an illustration; a half vortex ring can be generated in water by drawing a semicircular blade a short distance forward, the tip of a spoon for instance. The vortex advances with a certain velocity; and if an equal circular vortex is generated coaxially with the first, the mutual influence can be observed. The first vortex dilates and moves slower, while the second contracts and shoots through the first; after which the motion is reversed periodically, as if in a game of leap-frog. Projected perpendicularly against a plane boundary, the motion is determined by an equal opposite vortex ring, the optical image; the vortex ring spreads out and moves more slowly as it approaches the wall; at the same time the molecular rotation, inversely as the cross-section of the vortex, is seen to increase. The analytical treatment of such vortex rings is the same as for the electro-magnetic effect of a current circulating in each ring.
36. Irrotational Motion in General.-Liquid originally at rest in a singly-connected space cannot be set in motion by a field of force due to a single-valued potential function; any motion set up in the liquid must be due to a movement of the boundary, and the motion will be irrotational; for any small spherical element of the liquid may be considered a smooth solid sphere for a moment, and the normal pressure of the surrounding liquid cannot impart to it any rotation.

The kinetic energy of the liquid inside a surface $S$ due to the velocity function $\varphi$ is given by

$$
\begin{gather*}
T=1 / 2 \rho \iiint\left[\left(\frac{d \varphi}{d x}\right)^{2}+\left(\frac{d \varphi}{d y}\right)^{2}+\left(\frac{d \varphi}{d z}\right)^{2}\right] d x d y d z \\
=1 / 2 \rho \iint \varphi \frac{d \varphi}{d \nu} d S \tag{1}
\end{gather*}
$$

by Green's transformation, $\mathrm{d} \nu$ denoting an elementary step along the normal to the exterior of the surface; so that $\mathrm{d} \varphi / \mathrm{d} \nu=0$ over the surface makes $\mathrm{T}=0$, and then

$$
\begin{equation*}
\left(\frac{d \varphi}{d x}\right)^{2}+\left(\frac{d \varphi}{d y}\right)^{2}+\left(\frac{d \varphi}{d z}\right)^{2}=0, \frac{d \varphi}{d x}=0, \frac{d \varphi}{d y}=0, \frac{d \varphi}{d z}=0 . \tag{2}
\end{equation*}
$$

If the actual motion at any instant is supposed to be generated instantaneously from rest by the application of pressure impulse over the surface, or suddenly reduced to rest again, then, since no natural forces can act impulsively throughout the liquid, the pressure impulse $\tilde{\omega}$ satisfies the equations

$$
\begin{gather*}
\frac{1}{\rho} \frac{d \tilde{\omega}}{d x}=-u, \quad \frac{1}{\rho} \quad \frac{d \tilde{\omega}}{d y}=-v, \quad \frac{1}{\rho} \quad \frac{d \tilde{\omega}}{d z}=\tilde{\omega},  \tag{3}\\
\tilde{\omega}=\rho \varphi+\text { a constant }, \tag{4}
\end{gather*}
$$

and the constant may be ignored; and Green's transformation of the energy T amounts to the theorem that the work done by an impulse is the product of the impulse and average velocity, or half the velocity from rest.

In a multiply connected space, like a ring, with a multiply valued velocity function $\varphi$, the liquid can circulate in the circuits independently of any motion of the surface; thus, for example,

$$
\begin{equation*}
\varphi=m \theta=m \tan ^{-1} y / x \tag{5}
\end{equation*}
$$

will give motion to the liquid, circulating in any ring-shaped figure of revolution round Oz .
To find the kinetic energy of such motion in a multiply connected space, the channels must be supposed barred, and the space made acyclic by a membrane, moving with the velocity of the liquid; and then if $k$ denotes the cyclic constant of $\varphi$ in any circuit, or the value by which $\varphi$ has increased in completing the circuit, the values of $\varphi$ on the two sides of the membrane are taken as differing by $k$, so that the integral over the membrane

$$
\begin{equation*}
\iint \varphi \frac{\mathrm{d} \varphi}{\mathrm{~d} \nu} \mathrm{dS}=\mathrm{k} \iint \frac{\mathrm{~d} \varphi}{\mathrm{~d} \nu} \mathrm{dS}, \tag{6}
\end{equation*}
$$

and this term is to be added to the terms in (1) to obtain the additional part in the kinetic energy; the continuity shows that the integral is independent of the shape of the barrier membrane, and its position. Thus, in (5), the cyclic constant $\mathrm{k}=2 \pi \mathrm{~m}$.

In plane motion the kinetic energy per unit length parallel to Oz

$$
\begin{align*}
T=1 / 2 \rho \iint[(\overline{d x}) & +\left(\frac{d y}{d y}\right] d x d y=1 / 2 \rho \iint[(\overline{d x})+(\overline{d y})] d x d y \\
& =1 / 2 \rho \int \varphi \frac{d \varphi}{d \nu} d s=1 / 2 \rho \int \psi \frac{d \psi}{d \nu} d s . \tag{7}
\end{align*}
$$

For example, in the equilateral triangle of (8) § 28, referred to coordinate axes made by the base and height,

$$
\begin{gather*}
\psi^{\prime}=-2 R \alpha \beta \gamma / h=-1 / 2 R y\left[(h-y)^{2}-3 x^{2}\right] / h  \tag{8}\\
\psi=\psi^{\prime}-1 / 2 R\left[(1 / 3 h-y)^{2}+x^{2}\right] \\
\left.=-1 / 2 R\left[1 / 2 h^{3}+1 / 3 h^{2} y+h\right)\left(x^{2}-y^{2}\right)-3 x^{2} y+y^{3}\right] / h \tag{9}
\end{gather*}
$$

and over the base $\mathrm{y}=0$,

$$
\begin{equation*}
d x / d \nu=-d x / d y=+1 / 2 R\left(1 / 3 h^{2}-3 x^{2}\right) / h, \psi=-1 / 2 R\left(1 / 9 h^{2}+x^{2}\right) \tag{10}
\end{equation*}
$$

Integrating over the base, to obtain one-third of the kinetic energy T,

$$
1 / 3 T=1 / 2 \rho \int \begin{gather*}
h / \sqrt{ } 3  \tag{11}\\
-h / \sqrt{ } 3
\end{gather*}{ }^{1 / 4} R^{2}\left(3 x^{4}-1 / 27 h^{4}\right) d x / h=\rho R^{2} h^{4} / 135 \sqrt{ } 3
$$

so that the effective $\mathrm{k}^{2}$ of the liquid filling the triangle is given by

$$
\begin{align*}
& \mathrm{k}^{2}=\mathrm{T} / 1 / 1 / 2 \rho \mathrm{R}^{2} \mathrm{~A}=2 \mathrm{~h}^{2} / 45 \\
& =2 / 5 \text { (radius of the inscribed circle) }{ }^{2} \tag{12}
\end{align*}
$$

or two-fifths of the $\mathrm{k}^{2}$ for the solid triangle.
Again, since

$$
\begin{gather*}
d \varphi / d \nu=d \psi / d s, \quad d \varphi / d s=-d \psi / d \nu  \tag{13}\\
T=1 / 2 \rho \int \varphi d \psi=-1 / 2 \rho \int \psi d \varphi \tag{14}
\end{gather*}
$$

With the Stokes' function $\psi$ for motion symmetrical about an axis.

$$
\begin{equation*}
\mathrm{T}=1 / 2 \rho \int \varphi \frac{\mathrm{~d} \psi}{\mathrm{yds}} 2 \pi y \mathrm{ds}=\Pi \rho \int \varphi \mathrm{d} \psi \tag{15}
\end{equation*}
$$

37. Flow, Circulation, and Vortex Motion.-The line integral of the tangential velocity along a curve from one point to another, defined by

$$
\begin{equation*}
\int\left(u \frac{d x}{d s}+v \frac{d y}{d s}+w \frac{d z}{d s}\right) d s=\int(u d x+v d y+z d z) \tag{1}
\end{equation*}
$$

is called the "flux" along the curve from the first to the second point; and if the curve closes in on itself the line integral round the curve is called the "circulation" in the curve.

With a velocity function $\varphi$, the flow

$$
\begin{equation*}
-\int \mathrm{d} \varphi=\varphi_{1}-\varphi_{2} \tag{2}
\end{equation*}
$$

so that the flow is independent of the curve for all curves mutually reconcilable; and the circulation round a closed curve is zero, if the curve can be reduced to a point without leaving a region for which $\varphi$ is single valued.
If through every point of a small closed curve the vortex lines are drawn, a tube is obtained, and the fluid contained is called a vortex filament.

By analogy with the spin of a rigid body, the component spin of the fluid in any plane at a point is defined as the circulation round a small area in the plane enclosing the point, divided by twice the area. For in a rigid body, rotating about Oz with angular velocity $\zeta$, the circulation round a curve in the plane xy is

$$
\begin{equation*}
\int \zeta\left(x \frac{d y}{d s}-y \frac{d x}{d s}\right) d s=\zeta \text { times twice the area. } \tag{3}
\end{equation*}
$$

In a fluid, the circulation round an elementary area dxdy is equal to

$$
\begin{equation*}
u d x+\left(v+\frac{d v}{d x} d x\right) d y-\left(u+\frac{d u}{d y} d y\right) d x-v d y=\left(\frac{d v}{d x}-\frac{d u}{d y}\right) d x d y \tag{4}
\end{equation*}
$$

so that the component spin is

$$
\begin{equation*}
1 / 2\left(\frac{d v}{d x}-\frac{d u}{d y}\right)=\zeta \tag{5}
\end{equation*}
$$

in the previous notation of $\S 24$; so also for the other two components $\xi$ and $\eta$.
Since the circulation round any triangular area of given aspect is the sum of the circulation round the projections of the area on the coordinate planes, the composition of the components of spin, $\xi, \eta, \zeta$, is according to the vector law. Hence in any infinitesimal part of the fluid the circulation is zero round every small plane curve passing through the vortex line; and consequently the circulation round any curve drawn on the surface of a vortex filament is zero.

If at any two points of a vortex line the cross-section $\mathrm{ABC}, \mathrm{A}^{\prime} \mathrm{B}^{\prime} \mathrm{C}^{\prime}$ is drawn of the vortex filament, joined by the vortex line $\mathrm{AA}^{\prime}$, then, since the flow in $\mathrm{AA}^{\prime}$ is taken in opposite directions in the complete circuit $A B C A A^{\prime} B^{\prime} C^{\prime} A^{\prime} A$, the resultant flow in ${A A^{\prime}}^{\prime}$ cancels, and the circulation in $A B C, A^{\prime} B^{\prime} C^{\prime}$ is the same; this is expressed by saying that at all points of a vortex filament $\omega \alpha$ is constant where $\alpha$ is the cross-section of the filament and $\omega$ the resultant spin (W. K. Clifford, Kinematic, book iii.).

So far these theorems on vortex motion are kinematical; but introducing the equations of motion of § 22,

$$
\begin{gather*}
\frac{\mathrm{Du}}{\mathrm{dt}}+\frac{\mathrm{dQ}}{\mathrm{dx}}=0, \quad \frac{\mathrm{Dv}}{\mathrm{dt}}+\frac{\mathrm{dQ}}{\mathrm{dy}}=0, \quad \frac{\mathrm{Dw}}{\mathrm{dt}}+\frac{\mathrm{dQ}}{\mathrm{dz}}=0  \tag{6}\\
\mathrm{Q}=\int \mathrm{dp} / \rho+\mathrm{V}, \tag{7}
\end{gather*}
$$

and taking $\mathrm{dx}, \mathrm{dy}, \mathrm{dz}$ in the direction of $\mathrm{u}, \mathrm{v}, \mathrm{w}$, and

$$
\begin{gather*}
d x: d y: d z=u: v: w \\
\frac{D}{d t}(u d x+v d y+w d z)=\frac{D u}{d t} d x+u \frac{D d x}{d t}+\ldots=-d Q+1 / 2 d q^{2} \tag{8}
\end{gather*}
$$

and integrating round a closed curve

$$
\begin{equation*}
\frac{\mathrm{D}}{\mathrm{dt}} \int(\mathrm{udx}+\mathrm{vdy}+\mathrm{wdz})=0 \tag{9}
\end{equation*}
$$

and the circulation in any circuit composed of the same fluid particles is constant; and if the motion is differential irrotational and due to a velocity function, the circulation is zero round all reconcilable paths. Interpreted dynamically the normal pressure of the surrounding fluid on a tube cannot create any circulation in the tube.

The circulation being always zero round a small plane curve passing through the axis of spin in vortical motion, it follows conversely that a vortex filament is composed always of the same fluid particles; and since the circulation round a cross-section of a vortex filament is constant, not changing with the time, it follows from the previous kinematical theorem that $\alpha \omega$ is constant for all time, and the same for every cross-section of the vortex filament.

A vortex filament must close on itself, or end on a bounding surface, as seen when the tip of a spoon is drawn through the surface of water.

Denoting the cross-section $\alpha$ of a filament by dS and its mass by dm, the quantity $\omega \mathrm{dS} / \mathrm{dm}$ is called the vorticity; this is the same at all points of a filament, and it does not change during the motion; and the vorticity is given by $\omega$ cosedS/dm, if dS is the oblique section of which the normal makes an angle $\varepsilon$ with the filament, while the aggregate vorticity of a mass M inside a surface $S$ is

$$
\mathrm{M}^{-1} \int \omega \cos \varepsilon \mathrm{dS} .
$$

Employing the equation of continuity when the liquid is homogeneous,

$$
\begin{equation*}
2\left(\frac{d \zeta}{d y}-\frac{d \eta}{d z}\right)=\nabla^{2} u, \ldots, \nabla^{2}=-\frac{d^{2}}{d x^{2}}-\frac{d^{2}}{d y^{2}}-\frac{d^{2}}{d z^{2}} \tag{10}
\end{equation*}
$$

which is expressed by

$$
\begin{equation*}
\nabla^{2}(u, v, w)=2 \operatorname{curl}(\xi, \eta, \zeta),(\xi, \eta, \zeta)=1 / 2 \operatorname{curl}(u, v, w) \tag{11}
\end{equation*}
$$

38. Moving Axes in Hydrodynamics.-In many problems, such as the motion of a solid in liquid, it is convenient to take coordinate axes fixed to the solid and moving with it as the movable trihedron frame of reference. The components of velocity of the moving origin are denoted by $\mathrm{U}, \mathrm{V}, \mathrm{W}$, and the components of angular velocity of the frame of reference by $\mathrm{P}, \mathrm{Q}$, $R$; and then if $u, v, w$ denote the components of fluid velocity in space, and $u^{\prime}, v^{\prime}, w^{\prime}$ the components relative to the axes at a point ( $\mathrm{x}, \mathrm{y}, \mathrm{z}$ ) fixed to the frame of reference, we have

$$
\begin{align*}
& \mathrm{u}=\mathrm{U}+\mathrm{u}^{\prime}-\mathrm{yR}+\mathrm{zQ} \\
& \mathrm{v}=\mathrm{V}+\mathrm{v}^{\prime}-\mathrm{zP}+\mathrm{xR}, \\
& \mathrm{w}=\mathrm{W}+\mathrm{w}^{\prime}-\mathrm{xQ}+\mathrm{yP} . \tag{1}
\end{align*}
$$

Now if k denotes the component of absolute velocity in a direction fixed in space whose direction cosines are $\mathrm{l}, \mathrm{m}, \mathrm{n}$,

$$
\begin{equation*}
\mathrm{k}=\mathrm{lu}+\mathrm{mv}+\mathrm{nw} ; \tag{2}
\end{equation*}
$$

and in the infinitesimal element of time dt, the coordinates of the fluid particle at ( $\mathrm{x}, \mathrm{y}, \mathrm{z}$ ) will have changed by ( $u^{\prime}, v^{\prime}, w^{\prime}$ )dt; so that

$$
\begin{gather*}
\frac{\mathrm{Dk}}{\mathrm{dt}}=\frac{d l}{d t} u+\frac{d m}{d t} v+\frac{d n}{d t} w \\
+1\left(\frac{d u}{d t}+u^{\prime} \frac{d u}{d x}+v^{\prime} \frac{d u}{d y}+w^{\prime} \frac{d u}{d z}\right) \\
+m\left(\frac{d v}{d t}+u^{\prime} \frac{d v}{d x}+v^{\prime} \frac{d v}{d y}+w^{\prime} \frac{d v}{d z}\right) \\
+n\left(\frac{d w}{d t}+u^{\prime} \frac{d w}{d x}+v^{\prime} \frac{d w}{d y}+w^{\prime} \frac{d w}{d z}\right) \tag{3}
\end{gather*}
$$

But as $\mathrm{l}, \mathrm{m}, \mathrm{n}$ are the direction cosines of a line fixed in space,

$$
\begin{equation*}
\frac{\mathrm{dl}}{\mathrm{dt}}=\mathrm{mR}-\mathrm{nQ}, \frac{\mathrm{dm}}{\mathrm{dt}}=\mathrm{nP}-\mathrm{lR}, \frac{\mathrm{dn}}{\mathrm{dt}}=1 \mathrm{Q}-\mathrm{mP} ; \tag{4}
\end{equation*}
$$

so that

$$
\begin{align*}
\frac{D k}{d t} & =l\left(\frac{d u}{d t}-v R+w Q+u^{\prime} \frac{d u}{d x}+v^{\prime} \frac{d u}{d y}+w^{\prime} \frac{d u}{d z}\right)+m(\ldots)+n(\ldots) \\
& =l\left(X-\frac{1}{p} \frac{d p}{d x}\right)+m\left(Y-\frac{1}{p} \frac{d p}{d y}\right)+n\left(Z-\frac{1}{p} \frac{d p}{d z}\right) \tag{5}
\end{align*}
$$

for all values of $\mathrm{l}, \mathrm{m}, \mathrm{n}$, leading to the equations of motion with moving axes.
When the motion is such that

$$
\begin{equation*}
u=-\frac{d \varphi}{d x}-m \frac{d \psi}{d x}, v=-\frac{d \varphi}{d y}-m \frac{d \psi}{d y}, w=-\frac{d \varphi}{d z}-m \frac{d \psi}{d z} \tag{6}
\end{equation*}
$$

as in § 25 (1), a first integral of the equations in (5) may be written

$$
\begin{align*}
& \int \frac{d p}{\rho}+V+1 / 2 q^{2}-\frac{d \varphi}{d t}-m \frac{d \psi}{d t}+\left(u-u^{\prime}\right)\left(\frac{d \varphi}{d x}+m \frac{d \psi}{d x}\right) \\
& +\left(v-v^{\prime}\right)\left(\frac{d \varphi}{d y}+m \frac{d \psi}{d y}\right)+\left(w-w^{\prime}\right)\left(\frac{d \varphi}{d z}+m \frac{d \psi}{d z}\right)=F(t) \tag{7}
\end{align*}
$$

in which

$$
\begin{gather*}
\frac{d \varphi}{d t}-\left(u-u^{\prime}\right) \frac{d \varphi}{d x}-\left(v-v^{\prime}\right) \frac{d \varphi}{d y}-\left(w-w^{\prime}\right) \frac{d \varphi}{d z} \\
=\frac{d \varphi}{d t}-(U-y R+z Q) \frac{d \varphi}{d x}-(V-z P+x R) \frac{d \varphi}{d y}-(W-x Q+y P) \frac{d \varphi}{d z} \tag{8}
\end{gather*}
$$

is the time-rate of change of $\varphi$ at a point fixed in space, which is left behind with velocity components $u-u^{\prime}, v-v^{\prime}, w-w^{\prime}$.

In the case of a steady motion of homogeneous liquid symmetrical about Ox , where O is advancing with velocity $U$, the equation (5) of § 34

$$
\begin{equation*}
\mathrm{p} / \rho+\mathrm{V}+1 / 2 \mathrm{q}^{\prime 2}-\mathrm{f}\left(\psi^{\prime}\right)=\mathrm{constant} \tag{9}
\end{equation*}
$$

becomes transformed into

$$
\begin{gather*}
\frac{p}{\rho}+V+1 / 2 q^{2}-\frac{U}{y} \frac{d \psi}{d y}+1 / 2 U^{2}-f\left(\psi+1 / 2 U y^{2}\right)=\text { constant, }  \tag{10}\\
\psi^{\prime}=\psi+1 / 4 U y^{2} \tag{11}
\end{gather*}
$$

subject to the condition, from (4) § 34 ,

$$
\begin{equation*}
y^{-2} \nabla^{2} \psi^{\prime}=-f^{\prime}\left(\psi^{\prime}\right), \quad y^{-2} \nabla^{2} \psi=-f^{\prime}\left(\psi+1 / 2 U y^{2}\right) \tag{12}
\end{equation*}
$$

Thus, for example, with

$$
\begin{equation*}
\psi^{\prime}=3 / 4 \mathrm{U} \mathrm{y}^{2}\left(\mathrm{r}^{2} \mathrm{a}^{-2}-1\right), \mathrm{r}^{2}=\mathrm{x}^{2}+\mathrm{y}^{2}, \tag{13}
\end{equation*}
$$

for the space inside the sphere $\mathrm{r}=\mathrm{a}$, compared with the value of $\psi^{\prime}$ in § 34 (13) for the space outside, there is no discontinuity of the velocity in crossing the surface.

Inside the sphere

$$
\begin{equation*}
2 \zeta=\frac{d}{d x}\left(\frac{1}{y} \frac{d \psi^{\prime}}{d x}\right)+\frac{d}{d y}\left(\frac{1}{y} \frac{d \psi^{\prime}}{d y}\right)=\frac{15}{2} U \frac{y}{a^{2}} \tag{14}
\end{equation*}
$$

so that § 34 (4) is satisfied, with

$$
\begin{equation*}
f^{\prime}\left(\psi^{\prime}\right)=\frac{15}{2} U a^{-2}, f\left(\psi^{\prime}\right)=\frac{15}{2} U \psi^{\prime} a^{-2} ; \tag{15}
\end{equation*}
$$

and (10) reduces to

$$
\begin{equation*}
\frac{\mathrm{p}}{\rho}+\mathrm{V}-\frac{9}{8} \mathrm{U}\left\{\left(\frac{\mathrm{x}^{2}}{\mathrm{a}^{2}}-1\right)^{2}-\left(\frac{\mathrm{y}^{2}}{\mathrm{a}^{2}}-1 / 2\right)^{2}\right\}=\text { constant } \tag{16}
\end{equation*}
$$

this gives the state of motion in M. J. M. Hill's spherical vortex, advancing through the surrounding liquid with uniform velocity.
39. As an application of moving axes, consider the motion of liquid filling the ellipsoidal case

$$
\begin{equation*}
\frac{x^{2}}{\mathrm{a}^{2}}+\frac{\mathrm{y}^{2}}{\mathrm{~b}^{2}}+\frac{\mathrm{z}^{2}}{\mathrm{c}^{2}}=1 \tag{1}
\end{equation*}
$$

and first suppose the liquid to be frozen, and the ellipsoid to be rotating about the centre with components of angular velocity $\xi, \eta, \zeta$; then

$$
\begin{equation*}
u=-y \zeta+z \eta, v=-z \xi+x \zeta, w=-x \eta+y \xi . \tag{2}
\end{equation*}
$$

Now suppose the liquid to be melted, and additional components of angular velocity $\Omega_{1}, \Omega_{2}, \Omega_{3}$ communicated to the ellipsoidal case; the additional velocity communicated to the liquid will be due to a velocity-function

$$
\begin{equation*}
\varphi=-\Omega_{1} \frac{\mathrm{~b}^{2}-\mathrm{c}^{2}}{\mathrm{~b}^{2}+\mathrm{c}^{2}} \mathrm{yz}-\Omega_{2} \frac{\mathrm{c}^{2}-\mathrm{a}^{2}}{\mathrm{c}^{2}+\mathrm{a}^{2}} \mathrm{zx}-\Omega_{3} \frac{\mathrm{a}^{2}-\mathrm{b}^{2}}{\mathrm{a}^{2}+\mathrm{b}^{2}} \mathrm{xy}, \tag{3}
\end{equation*}
$$

as may be verified by considering one term at a time.
If $u^{\prime}, v^{\prime}, w^{\prime}$ denote the components of the velocity of the liquid relative to the axes,

$$
\begin{gather*}
u^{\prime}=u+y R-z Q=\frac{2 a^{2}}{a^{2}+b^{2}} \Omega_{3} y-\frac{2 a^{2}}{c^{2}+a^{2}} \Omega_{2} z,  \tag{4}\\
v^{\prime}=v+z P-x R=\frac{2 b^{2}}{b^{2}+c^{2}} \Omega_{1} z-\frac{2 b^{2}}{a^{2}+b^{2}} \Omega_{3} x,  \tag{5}\\
w^{\prime}=w+x Q-y P=\frac{2 c^{2}}{c^{2}+a^{2}} \Omega_{2} x-\frac{2 c^{2}}{b^{2}+c^{2}} \Omega_{1} y,  \tag{6}\\
P=\Omega_{1}+\xi, Q=\Omega_{2}+\eta, R=\Omega_{3}+\zeta . \tag{7}
\end{gather*}
$$

Thus

$$
\begin{equation*}
\mathrm{u}^{\prime} \frac{\mathrm{x}}{\mathrm{a}_{2}}+\mathrm{v}^{\prime} \frac{\mathrm{y}}{\mathrm{~b}_{2}}+\mathrm{w}^{\prime} \frac{\mathrm{z}}{\mathrm{c}_{2}}=0, \tag{8}
\end{equation*}
$$

so that a liquid particle remains always on a similar ellipsoid.
The hydrodynamical equations with moving axes, taking into account the mutual gravitation of the liquid, become

$$
\begin{equation*}
\frac{1}{\rho} \frac{d p}{d x}+4 \Pi \rho A x+\frac{d u}{d t}-v R+w Q+u^{\prime} \frac{d u}{d x}+v^{\prime} \frac{d u}{d y}+w^{\prime} \frac{d u}{d z}=0, \ldots, \ldots \tag{9}
\end{equation*}
$$

where

$$
\begin{gather*}
A, B, C=\int_{0}^{\infty} \frac{a b c d \lambda}{\left(a^{2}+\lambda, b^{2}+\lambda, c^{2}+\lambda\right) P} \\
P^{2}=4\left(a^{2}+\lambda\right)\left(b^{2}+\lambda\right)\left(c^{2}+\lambda\right) . \tag{10}
\end{gather*}
$$

With the values above of $u, v, w, u^{\prime}, v^{\prime}, w^{\prime}$, the equations become of the form

$$
\begin{align*}
& \bar{\rho} \frac{d x}{d x}+4 п \rho A x+\alpha x+h y+g z=0  \tag{11}\\
& \frac{1}{\rho} \frac{d p}{d y}+4 п \rho B y+h x+\beta y+f z=0  \tag{12}\\
& \frac{1}{\rho} \frac{d p}{d z}+4 \Pi \rho C z+g x+f y+\gamma z=0 \tag{13}
\end{align*}
$$

and integrating

$$
\begin{gather*}
p \rho^{-1}+2 n \rho\left(A x^{2}+B y^{2}+C z^{2}\right) \\
+1 / 2\left(\alpha x^{2}+\beta y^{2}+\gamma z^{2}+2 f y z+2 g z x+2 h x y\right)=\text { const., } \tag{14}
\end{gather*}
$$

so that the surfaces of equal pressure are similar quadric surfaces, which, symmetry and dynamical considerations show, must be coaxial surfaces; and $f, g$, $h$ vanish, as follows also by algebraical reduction; and

$$
\begin{align*}
& \alpha=\frac{4 c^{2}\left(c^{2}-a^{2}\right)}{\left(c^{2}+a^{2}\right)^{2}} \Omega_{2}^{2}-\left(\frac{c^{2}-a^{2}}{c^{2}+a^{2}} \Omega_{2}-\eta\right)^{2} \\
& -\frac{4 b^{2}\left(a^{2}-b^{2}\right)}{\left(a^{2}+b^{2}\right)^{2}} \Omega_{3}^{2}-\left(\frac{a^{2}-b^{2}}{a^{2}+b^{2}} \Omega_{3}-\zeta\right)^{2}, \tag{15}
\end{align*}
$$

with similar equations for $\beta$ and $\gamma$.
If we can make

$$
\begin{equation*}
(4 \Pi \rho A+\alpha) x^{2}=(4 \pi \rho B+\beta) b^{2}=(4 \pi \rho C+\gamma) c^{2} \tag{16}
\end{equation*}
$$

the surfaces of equal pressure are similar to the external case, which can then be removed without affecting the motion, provided $\alpha, \beta, \gamma$ remain constant.

This is so when the axis of revolution is a principal axis, say Oz; when

$$
\begin{equation*}
\Omega_{1}=0, \Omega_{2}=0, \xi=0, \eta=0 \tag{17}
\end{equation*}
$$

If $\Omega_{3}=0$ or $\theta_{3}=\zeta$ in addition, we obtain the solution of Jacobi's ellipsoid of liquid of three unequal axes, rotating bodily about the least axis; and putting $\mathrm{a}=\mathrm{b}$, Maclaurin's solution is obtained of the rotating spheroid.

In the general motion again of the liquid filling a case, when $\mathrm{a}=\mathrm{b}, \Omega_{3}$ may be replaced by zero, and the equations, hydrodynamical and dynamical, reduce to

$$
\begin{gather*}
\frac{d \xi}{d t}=-\frac{2 c^{2}}{a^{2}+c^{2}} \Omega_{2} \zeta, \frac{d \eta}{d t}=\frac{2 a^{2}}{a^{2}+c^{2}} \Omega_{1} \zeta, \frac{d \zeta}{d t}=\frac{2 c^{2}}{a^{2}+c^{2}}\left(\Omega_{2} \xi-\Omega_{2} \eta\right)  \tag{18}\\
\frac{d \Omega_{1}}{d t}=\Omega_{2} \zeta+\frac{a^{2}+c^{2}}{a^{2}-c^{2}} \eta \zeta, \frac{d \Omega_{2}}{d t}=-\Omega_{1} \zeta-\frac{a^{2}+c^{2}}{a^{2}-c^{2}} \xi \zeta \tag{19}
\end{gather*}
$$

of which three integrals are

$$
\begin{gather*}
\xi^{2}+\eta^{2}=L-\frac{a^{2}}{c^{2}} \zeta^{2}  \tag{20}\\
\Omega_{1}^{2}+\Omega_{2}^{2}=M+\frac{\left(a^{2}+c^{2}\right)^{2}}{2 c^{2}\left(a^{2}-c^{2}\right)} \zeta^{2}  \tag{21}\\
\Omega_{1} \xi+\Omega_{2} \eta N=+\frac{a^{2}+c^{2}}{4 c^{2}} \zeta^{2} \tag{22}
\end{gather*}
$$

and then

$$
\begin{gather*}
\left(\frac{d \zeta}{d t}\right)^{2}=\frac{4 c^{4}}{\left(a^{2}+c^{2}\right)}\left(\Omega_{2} \xi-\Omega_{1}{ }^{2} \eta\right)^{2} \\
=\frac{4 c^{4}}{\left(a^{2}+c^{2}\right)^{2}}\left[\left(\xi^{2}+\eta^{2}\right)\left(\Omega_{1}^{2}+\Omega_{2}^{2}\right)-\left(\Omega_{1} \xi+\Omega_{2} \eta\right)^{2}\right] \\
=\frac{4 c^{4}}{\left(a^{2}+c^{2}\right)^{2}}\left[L M-N^{2}+\left\{\frac{\left(a^{2}+c^{2}\right)^{2}}{2 c^{2}\left(a^{2}+c^{2}\right)}-M \frac{a^{2}}{c^{2}}-N \frac{a^{2}+c^{2}}{2 c^{2}}\right\} \zeta^{2}\right. \\
\left.-\frac{\left(a^{2}+c^{2}\right)\left(9 a^{2}-c^{2}\right)}{16 c^{4}\left(a^{2}-c^{2}\right)} \zeta^{4}\right]=Z, \tag{23}
\end{gather*}
$$

where Z is a quadratic in $\zeta^{2}$, so that $\zeta$ is an elliptic function of t , except when $\mathrm{c}=\mathrm{a}$, or 3 a .

$$
\text { Put } \Omega_{1}=\Omega \cos \varphi, \Omega_{2}=-\Omega \sin \varphi,
$$

$$
\begin{gather*}
\Omega_{2} \frac{\mathrm{~d} \varphi}{\mathrm{dt}}=\frac{\mathrm{d} \Omega_{1}}{\mathrm{dt}} \Omega_{2}-\Omega_{1} \frac{\mathrm{~d} \Omega_{2}}{\mathrm{dt}}=\Omega^{2} \zeta-\frac{\left(\mathrm{a}^{2}+\mathrm{c}^{2}\right)}{\left(\mathrm{a}^{2}-\mathrm{c}^{2}\right)}\left(\Omega_{1} \xi+\Omega_{2} \eta\right) \zeta  \tag{24}\\
\mathrm{a}^{2}+\mathrm{c}^{2}
\end{gather*}
$$

$$
\begin{gather*}
\frac{d \varphi}{d t}=\zeta-\frac{\left(a^{2}+c^{2}\right)}{\left(a^{2}-c^{2}\right)} \cdot \frac{N+\frac{4 c^{2}}{M+\frac{\left(a^{2}+c^{2}\right)^{2}}{2 c^{2}\left(a^{2}-c^{2}\right)} \zeta^{2}}}{\varphi=\int \frac{\zeta d \zeta}{\sqrt{ } Z}-\frac{a^{2}+c^{2}}{a^{2}-c^{2}} \int \frac{N+\frac{a^{2}+c^{2}}{4 c^{2}}}{M+\frac{\left(a^{2}+c^{2}\right)^{2}}{2 c^{2}\left(a^{2}-c^{2}\right)} \zeta^{2}} \cdot \frac{\zeta d \zeta}{\sqrt{ } Z},} \\
\varphi \tag{25}
\end{gather*}
$$

which, as Z is a quadratic function of $\zeta^{2}$, are non-elliptic integrals; so also for $\psi$, where $\xi=\omega$ $\cos \psi, \eta=-\omega \sin \psi$.

In a state of steady motion

$$
\begin{gather*}
\frac{d \zeta}{d t}=0, \frac{\Omega_{1}}{\xi}=\frac{\Omega_{2}}{\eta},  \tag{27}\\
\varphi=\psi=n t, \text { suppose, }  \tag{28}\\
\Omega_{1} \xi+\Omega_{2} \eta=\Omega \omega, \\
\frac{d \varphi}{d t}=\zeta-\frac{a^{2}+c^{2}}{a^{2}-c^{2}} \frac{\omega}{\Omega} \zeta  \tag{29}\\
\frac{d \psi}{d t}=-\frac{2 a^{2}}{a^{2}+c^{2}} \frac{\Omega}{\omega} \zeta  \tag{30}\\
1-\frac{a^{2}+c^{2}}{a^{2}-c^{2}} \frac{\omega}{\Omega}=-\frac{2 a^{2}}{a^{2}+c^{2}} \frac{\Omega}{\omega},  \tag{31}\\
\left(\frac{\omega}{\Omega}-1 / 2 \frac{a^{2}+c^{2}}{a^{2}-c^{2}}\right)^{2}=\frac{\left(a^{2}-c^{2}\right)\left(9 a^{2}-c^{2}\right)}{4\left(a^{2}+c^{2}\right)}, \tag{32}
\end{gather*}
$$

and a state of steady motion is impossible when $3 \mathrm{a}>\mathrm{c}>\mathrm{a}$.
An experiment was devised by Lord Kelvin for demonstrating this, in which the difference of steadiness was shown of a copper shell filled with liquid and spun gyroscopically, according as the shell was slightly oblate or prolate. According to the theory above the stability is regained when the length is more than three diameters, so that a modern projectile with a cavity more than three diameters long should fly steadily when filled with water; while the old-fashioned type, not so elongated, would be highly unsteady; and for the same reason the gas bags of a dirigible balloon should be over rather than under three diameters long.
40. A Liquid Jet.-By the use of the complex variable and its conjugate functions, an attempt can be made to give a mathematical interpretation of problems such as the efflux of water in a jet or of smoke from a chimney, the discharge through a weir, the flow of water through the piers of a bridge, or past the side of a ship, the wind blowing on a sail or aeroplane, or against a wall, or impinging jets of gas or water; cases where a surface of discontinuity is observable, more or less distinct, which separates the running stream from the dead water or air.

Uniplanar motion alone is so far amenable to analysis; the velocity function $\varphi$ and stream function $\psi$ are given as conjugate functions of the coordinates $\mathrm{x}, \mathrm{y}$ by

$$
\begin{equation*}
w=f(z) \text { where } z=x+y i, w=\varphi+\psi i \tag{1}
\end{equation*}
$$

and then

$$
\begin{equation*}
\frac{d w}{d z}=\frac{d \varphi}{d x}+i \frac{d \psi}{d x}=-u+v i \tag{2}
\end{equation*}
$$

so that, with $u=q \cos \theta, v=q \sin \theta$, the function

$$
\begin{equation*}
\zeta=-Q \frac{d z}{d w}=\frac{Q}{(u-v i)}=\frac{Q}{q^{2}}(u+v i)=\frac{Q}{q}(\cos \theta+i \sin \theta), \tag{3}
\end{equation*}
$$

gives $\zeta$ as a vector representing the reciprocal of the velocity $q$ in direction and magnitude, in terms of some standard velocity Q .

To determine the motion of a jet which issues from a vessel with plane walls, the vector $\zeta$ must be Constructed so as to have a constant direction $\theta$ along a plane boundary, and to give a constant skin velocity over the surface of a jet, where the pressure is constant.

It is convenient to introduce the function

$$
\begin{equation*}
\Omega=\log \zeta=\log (\mathrm{Q} / \mathrm{q})+\theta \mathrm{i} \tag{4}
\end{equation*}
$$

so that the polygon representing $\Omega$ conformally has a boundary given by straight lines parallel to the coordinate axes; and then to determine $\Omega$ and w as functions of a variable $u$ (not to be confused with the velocity component of q), such that in the conformal representation the boundary of the $\Omega$ and w polygon is made to coincide with the real axis of $u$.

It will be sufficient to give a few illustrations.


Fig. 4.

Consider the motion where the liquid is coming from an infinite distance between two parallel walls at a distance $\mathrm{xx}^{\prime}$ (fig. 4), and issues in a jet between two edges $A$ and $A^{\prime}$; the wall $x A$ being bent at a corner $B$, with the external angle $\beta=1 / 2 \pi / n$.

The theory of conformal representation shows that the motion is given by

$$
\begin{equation*}
\zeta=\left[\frac{\sqrt{ }\left(b-a^{\prime} \cdot u-a\right)+\sqrt{ }\left(b-a \cdot u-a^{\prime}\right)}{\sqrt{ }\left(a-a^{\prime} \cdot u-b\right)}\right]^{1 / n}, u=a e^{-\Pi w / m} ; \tag{5}
\end{equation*}
$$

where $\mathrm{u}=\mathrm{a}$, $\mathrm{a}^{\prime}$ at the edge $\mathrm{A}, \mathrm{A}^{\prime} ; \mathrm{u}=\mathrm{b}$ at a corner $\mathrm{B} ; \mathrm{u}=0$ across $\mathrm{xx}^{\prime}$ where $\varphi=\infty$; and $\mathrm{u}=\infty$, $\varphi=\infty$ across the end $\mathrm{JJ}^{\prime}$ of the jet, bounded by the curved lines APJ, A'P'J', over which the skin velocity is Q . The stream lines xBAJ, $\mathrm{xA}^{\prime} \mathrm{J}^{\prime}$ are given by $\psi=0, \mathrm{~m}$; so that if c denotes the ultimate breadth JJ' of the jet, where the velocity may be supposed uniform and equal to the skin velocity Q,

$$
\mathrm{m}=\mathrm{Qc}, \quad \mathrm{c}=\mathrm{m} / \mathrm{Q}
$$

If there are more $B$ corners than one, either on $x A$ or $x^{\prime} A^{\prime}$, the expression for $\zeta$ is the product of corresponding factors, such as in (5).

Restricting the attention to a single corner B,

$$
\begin{gather*}
\zeta^{\mathrm{n}}=\left(\frac{\mathrm{Q}}{\mathrm{q}}\right)^{\mathrm{n}}(\cos n \theta+i \sin n \theta)=\frac{\sqrt{ }\left(b-a^{\prime} \cdot u-a\right)+\sqrt{ }\left(b-a \cdot u-a^{\prime}\right)}{\sqrt{ }\left(a-a^{\prime} \cdot u-b\right)},  \tag{6}\\
\operatorname{ch} n \omega=\operatorname{ch} \log \left(\frac{Q}{q}\right)^{n} \cos n \theta+i \operatorname{sh} \log \left(\frac{Q}{q}\right)^{n} \sin n \theta \\
=1 / 2\left(\zeta^{n}+\zeta^{-n}\right)=\sqrt{\frac{b-a^{\prime}}{a-a^{\prime}}} \sqrt{\frac{u-a}{u-b}}  \tag{7}\\
\operatorname{sh} n \Omega=\operatorname{sh} \log \left(\frac{Q}{q}\right) \cos n \theta+i \operatorname{ch} \log \left(\frac{Q}{q}\right)^{n} \sin n \theta \\
=1 / 2\left(\zeta^{n}+\zeta^{-n}\right)=\sqrt{\frac{b-a}{a-a^{\prime}} \sqrt{ } \frac{u-a^{\prime}}{u-b}}  \tag{8}\\
\infty>a>b>0>a^{\prime}>-\infty \tag{9}
\end{gather*}
$$

and then

$$
\begin{equation*}
\frac{d \Omega}{d u}=-\frac{1}{2 n} \frac{\sqrt{ }\left(b-a^{\prime} \cdot b-a^{\prime}\right)}{(u-b) \sqrt{ }\left(a-a \cdot u-a^{\prime}\right)}, \frac{d w}{d u}=-\frac{m}{\Pi u} \tag{10}
\end{equation*}
$$

the formulas by which the conformal representation is obtained.
For the $\Omega$ polygon has a right angle at $u=a, a^{\prime}$, and a zero angle at $u=b$, where $\theta$ changes from 0 to $1 / 2 \pi / n$ and $\Omega$ increases by $1 / 2 i n / n$; so that

$$
\begin{equation*}
\frac{d \Omega}{d u}=\frac{A}{(u-b) \sqrt{ }\left(u-a \cdot u-a^{\prime}\right)}, \text { where } A=\frac{\sqrt{ }\left(b-a \cdot b-a^{\prime}\right)}{2 n} \tag{11}
\end{equation*}
$$

And the w polygon has a zero angle at $u=0, \infty$, where $\psi$ changes from 0 to m and back again, so that w changes by im, and

$$
\begin{equation*}
\frac{d w}{d u}=\frac{B}{u}, \text { where } B=-\frac{m}{\pi} . \tag{12}
\end{equation*}
$$

Along the stream line xBAPJ,

$$
\begin{equation*}
\psi=0, \quad u=a e^{-\Pi \varphi / m} \tag{13}
\end{equation*}
$$

and over the jet surface JPA, where the skin velocity is $Q$,

$$
\begin{equation*}
\frac{\mathrm{d} \varphi}{\mathrm{ds}}=-\mathrm{q}=-\mathrm{Q}, \quad \mathrm{u}=\mathrm{ae}^{\mathrm{HS} \mathrm{Q} / \mathrm{m}}=\mathrm{ae}^{\mathrm{Hs} / \mathrm{c}} \tag{14}
\end{equation*}
$$

denoting the arc AP by s, starting at $u=a$;

$$
\operatorname{ch} n \Omega=\cos n \theta=\sqrt{ } \frac{b-a^{\prime}}{a-a^{\prime}} \sqrt{ } \frac{u-a}{u-b},
$$

$$
\begin{gather*}
\operatorname{sh} n \Omega=i \sin n \theta=i \sqrt{ } \frac{a-b}{a-a^{\prime}} \sqrt{\frac{u-a^{\prime}}{u-b}},  \tag{16}\\
\infty>u=a e^{\mathrm{Hs} / \mathrm{c}}>a \tag{17}
\end{gather*}
$$

and this gives the intrinsic equation of the jet, and then the radius of curvature

$$
\begin{align*}
\rho=-\frac{d s}{d \theta} & =\frac{1}{Q} \frac{d \varphi}{d \theta}=\frac{i}{Q} \frac{d w}{d \Omega}=\frac{i}{Q} \frac{d w}{d u} / \frac{d \Omega}{d u} \\
& =\frac{c}{\pi} \cdot \frac{u-b}{u} \frac{\sqrt{ }\left(u-a \cdot u-a^{\prime}\right)}{\sqrt{ }\left(a-b \cdot b-a^{\prime}\right)}, \tag{18}
\end{align*}
$$

not requiring the integration of (11) and (12)
If $\theta=\alpha$ across the end JJ' of the jet, where $u=\infty, q=Q$,

$$
\begin{equation*}
\operatorname{ch} n \Omega=\cos n \alpha=\sqrt{ } \frac{b-a^{\prime}}{a-a^{\prime}}, \operatorname{sh} n \Omega=i \sin n \alpha=i \sqrt{ } \frac{a-b}{a-a^{\prime}}, \tag{19}
\end{equation*}
$$

Then

$$
\begin{gather*}
\cos 2 n \alpha-\cos 2 n \theta=2 \frac{a-b \cdot b-a^{\prime}}{a-a^{\prime} \cdot u-b}=1 / 2 \sin ^{2} 2 n \alpha \frac{a-a^{\prime}}{u-b} \\
\sin 2 n \theta=2 \frac{\sqrt{ }\left(a-b \cdot b-a^{\prime}\right) \sqrt{ }\left(u-a \cdot u-b^{\prime}\right)}{a-a^{\prime} \cdot u-b}  \tag{20}\\
=\sin 2 n \alpha \frac{\sqrt{ }\left(a-a \cdot b-a^{\prime}\right)}{u-b} ; \\
=\frac{a-a^{\prime}+\left(a+a^{\prime}\right) \cos 2 n \alpha-\left[a+a^{\prime}+\left(a-a^{\prime}\right) \cos 2 n \alpha\right] \cos 2 n \theta}{\left(a-a^{\prime}\right) \sin ^{2} 2 n \alpha} \times \frac{\cos 2 n \alpha-\cos 2 n \theta}{\sin 2 n \theta} . \tag{21}
\end{gather*}
$$

Along the wall $\mathrm{AB}, \cos \mathrm{n} \theta=0, \sin \mathrm{n} \theta=1$,

$$
\begin{gather*}
a>u>b, \\
\operatorname{ch} n \Omega=i \operatorname{sh} \log \left(\frac{Q}{q}\right)^{n}=i \sqrt{\frac{b-a^{\prime}}{a-a^{\prime}}} \sqrt{\frac{a-u}{u-b},}  \tag{22}\\
\operatorname{sh} n \Omega=i \operatorname{ch} \log \left(\frac{Q}{q}\right)^{n}=i \sqrt{\frac{a-b}{a-a^{\prime}}} \sqrt{\frac{u-a^{\prime}}{u-b},}  \tag{23}\\
\frac{d s}{d u}=\frac{d s}{d \varphi} \frac{d \varphi}{d t}=\frac{m}{\pi q u}=\frac{c}{\Pi} \frac{Q}{q u}  \tag{24}\\
\Pi \frac{A B}{c}=\int_{b}^{a} \frac{Q}{q} \frac{d u}{u} \int\left[\frac{\sqrt{ }(a-b) \sqrt{ }\left(u-a^{\prime}\right)+\sqrt{ }\left(b-a^{\prime}\right) \sqrt{ }(a-u)}{\sqrt{ }\left(a-a^{\prime}\right) \sqrt{ }\left(u-b^{\prime}\right)}\right]^{1 / n} \frac{d u}{u} . \tag{25}
\end{gather*}
$$

Along the wall $\mathrm{Bx}, \cos \mathrm{n} \theta=1, \sin \mathrm{n} \theta=0$,

$$
\begin{gather*}
b>u>0  \tag{27}\\
\operatorname{ch} n \Omega=\operatorname{ch} \log \left(\frac{Q}{q}\right)^{n}=\sqrt{ } \frac{b-a^{\prime}}{a-a^{\prime}} \sqrt{ } \frac{a-u}{b-u},  \tag{28}\\
\operatorname{sh} n \Omega=\operatorname{sh} \log \left(\frac{Q}{q}\right)^{n}=\sqrt{\frac{a-b}{a-a^{\prime}}} \sqrt{\frac{u-a^{\prime}}{b-u}} . \tag{29}
\end{gather*}
$$

At x where $\varphi=\infty, \mathrm{u}=0$, and $\mathrm{q}=\mathrm{q}_{0}$,

$$
\begin{equation*}
\left(\frac{Q}{q_{0}}\right)^{n}=\sqrt{ } \frac{b-a^{\prime}}{a-a^{\prime}} \sqrt{ } \frac{a}{b}+\sqrt{ } \frac{a-b}{a-a^{\prime}} \sqrt{ } \frac{-a^{\prime}}{q} . \tag{30}
\end{equation*}
$$

In crossing to the line of flow $\mathrm{x}^{\prime} \mathrm{A}^{\prime} \mathrm{P}^{\prime} \mathrm{J}^{\prime}, \psi$ changes from 0 to m , so that with $\mathrm{q}=\mathrm{Q}$ across $\mathrm{JJ}^{\prime}$, while across $\mathrm{xx}^{\prime}$ the velocity is $\mathrm{q}_{0}$, so that

$$
\begin{gather*}
\mathrm{m}=\mathrm{q}_{0} \cdot \mathrm{xx}^{\prime}=\mathrm{Q} \cdot \mathrm{JJ}^{\prime}  \tag{31}\\
\frac{\mathrm{JJ}^{\prime}}{\mathrm{xx}^{\prime}}=\frac{\mathrm{q}_{0}}{\mathrm{Q}}\left[\sqrt{ } \frac{\mathrm{~b}-\mathrm{a}^{\prime}}{\mathrm{a}-\mathrm{a}^{\prime}} \sqrt{\frac{a}{b}}-\sqrt{\left.\frac{a-b}{a-\mathrm{a}^{\prime}} \sqrt{ } \frac{-\mathrm{a}^{\prime}}{\mathrm{b}}\right]^{1 / n},}\right. \tag{32}
\end{gather*}
$$

giving the contraction of the jet compared with the initial breadth of the stream.
Along the line of flow $\mathrm{x}^{\prime} \mathrm{A}^{\prime} \mathrm{P}^{\prime} \mathrm{J}^{\prime}, \psi=\mathrm{m}, \mathrm{u}=\mathrm{a}^{\prime} \mathrm{e}^{-\Pi \varphi / \mathrm{m}}$, and from $\mathrm{x}^{\prime}$ to $\mathrm{A}^{\prime}, \cos \mathrm{n} \theta=1, \sin \mathrm{n} \theta=0$,

$$
\begin{equation*}
\operatorname{ch} n \Omega=\operatorname{ch} \log \left(\frac{Q}{q}\right)^{n}=\sqrt{ } \frac{b-a^{\prime}}{a-a^{\prime}} \sqrt{ } \frac{a-u}{b-u}, \tag{33}
\end{equation*}
$$

$$
\begin{gather*}
\operatorname{sh} n \Omega=\operatorname{sh} \log \left(\frac{Q}{q}\right)^{n}=\sqrt{ } \frac{a-b}{a-a^{\prime}} \sqrt{ } \frac{u-a^{\prime}}{b-u} .  \tag{34}\\
0>u>a^{\prime} . \tag{35}
\end{gather*}
$$

Along the jet surface $A^{\prime} J^{\prime}, q=Q$,

$$
\begin{align*}
\operatorname{ch} n \Omega=\cos n \theta & =\sqrt{ } \frac{b-a^{\prime}}{a-a^{\prime}} \sqrt{ } \frac{a-u}{b-u},  \tag{36}\\
\operatorname{sh} n \Omega=i \sin n \theta & =i \sqrt{ } \frac{a-b}{a-a^{\prime}} \sqrt{\frac{u-a^{\prime}}{b-u}} .  \tag{37}\\
a^{\prime}>u & =a^{\prime} e^{n / s c}>-\infty, \tag{38}
\end{align*}
$$

giving the intrinsic equation.
41. The first problem of this kind, worked out by H. v. Helmholtz, of the efflux of a jet between two edges $A$ and $A_{1}$ in an infinite wall, is obtained by the symmetrical duplication of the above, with $\mathrm{n}=1, \mathrm{~b}=0, \mathrm{a}^{\prime}=-\infty$, as in fig. 5 ,

$$
\begin{equation*}
\operatorname{ch} \Omega=\sqrt{ } \frac{\mathrm{u}-\mathrm{a}}{\mathrm{u}}, \operatorname{sh} \Omega=\sqrt{ } \frac{-\mathrm{a}}{\mathrm{u}} ; \tag{1}
\end{equation*}
$$

and along the jet APJ, $\infty>\mathrm{u}=\mathrm{ae}^{\mathrm{ms} / \mathrm{c}}>\mathrm{a}$,

$$
\begin{gather*}
\operatorname{sh} \Omega=i \sin \theta-i \sqrt{ } \frac{\mathrm{a}}{\mathrm{u}}=\mathrm{ie}^{-1 / 2 \pi s / c},  \tag{2}\\
\mathrm{PM}=\int_{\mathrm{s}}^{\infty} \sin \theta \mathrm{ds}=\int \mathrm{e}^{-1 / 2 \pi \mathrm{~s} / \mathrm{c}} \mathrm{ds}=\frac{\mathrm{c}}{1 / 2 \pi} \mathrm{e}^{-1 / 2 \pi \mathrm{~m} / \mathrm{c}}=\frac{\mathrm{c}}{1 / 2 \Pi} \sin \theta, \tag{3}
\end{gather*}
$$

so that $\mathrm{PT}=\mathrm{c} / 112 \pi$, and the curve AP is the tractrix; and the coefficient of contraction, or

$$
\begin{equation*}
\frac{\text { breadth of the jet }}{\text { breadth of the orifice }}=\frac{\Pi}{\Pi+2} . \tag{4}
\end{equation*}
$$

A change of $\Omega$ and $\theta$ into $\mathrm{n} \Omega$ and $\mathrm{n} \theta$ will give the solution for two walls converging symmetrically to the orifice $\mathrm{AA}_{1}$ at an angle $\pi / \mathrm{n}$. With $\mathrm{n}=1 / 2$, the reentrant walls are given of Borda's mouthpiece, and the coefficient of contraction becomes $1 / 2$. Generally, by making $a^{\prime}=$ $-\infty$, the line $x^{\prime} A^{\prime}$ may be taken as a straight stream line of infinite length, forming an axis of symmetry; and then by duplication the result can be obtained, with assigned $n$, $a$, and $b$, of the efflux from a symmetrical converging mouthpiece, or of the flow of water through the arches of a bridge, with wedge-shaped piers to divide the stream.


Fig. 5.


Fig. 6.
42. Other arrangements of the constants $n, a, b, a^{\prime}$ will give the results of special problems considered by J. M. Michell, Phil. Trans. 1890.

Thus with $\mathrm{a}^{\prime}=0$, a stream is split symmetrically by a wedge of angle $n / \mathrm{n}$ as in Bobyleff's problem; and, by making $\mathrm{a}=\infty$, the wedge extends to infinity; then

$$
\begin{equation*}
\operatorname{ch} \mathrm{n} \Omega=\sqrt{ } \frac{\mathrm{b}}{\mathrm{~b}-\mathrm{u}}, \operatorname{sh} \mathrm{n} \Omega=\sqrt{ } \frac{\mathrm{n}}{\mathrm{~b}-\mathrm{u}} . \tag{1}
\end{equation*}
$$

Over the jet surface $\psi=m, q=Q$,

$$
\begin{gather*}
u=-e^{\Pi \varphi / m}=-b e^{\Pi^{2} / c} \\
\operatorname{ch} \Omega=\cos n \theta=\sqrt{ } \frac{1}{e^{\Pi^{2} / c}+1}, \operatorname{sh} \Omega=i \sin n \theta=i \sqrt{ } \frac{e^{\mathrm{n}^{2} / c}}{e^{\Pi^{2} / c}+1}  \tag{2}\\
e^{1 / 2 \pi^{2} / c}=\tan n \theta, \frac{1 / 2 \Pi}{c} \frac{d s}{d \theta}=\frac{2 n}{\sin 2 n \theta} . \tag{3}
\end{gather*}
$$

For a jet impinging normally on an infinite plane, as in fig. $6, \mathrm{n}=1$,

$$
\begin{align*}
& \mathrm{e}^{1 / 2 \pi^{2} / \mathrm{c}}=\tan \theta, \operatorname{ch}(1 / 2 \pi \mathrm{~s} / \mathrm{c}) \sin 2 \theta=1,  \tag{4}\\
& \operatorname{sh} 1 / 2 \pi \mathrm{x} / \mathrm{c}=\cot \theta, \operatorname{sh} 1 / 2 \pi y / \mathrm{c}=\tan \theta,
\end{align*}
$$

$$
\begin{equation*}
\operatorname{sh} 1 / 2 \pi \mathrm{x} / \mathrm{c} \operatorname{sh} 1 / 2 \pi \mathrm{y} / \mathrm{c}=1, \mathrm{e}^{1 / 2 \pi(\mathrm{x}+\mathrm{y}) / \mathrm{c}}=\mathrm{e}^{1 / 2 \pi \mathrm{xx} / \mathrm{c}}+\mathrm{e}^{1 / 2 \pi \mathrm{yy} / \mathrm{c}}+1 . \tag{5}
\end{equation*}
$$

With $\mathrm{n}=1 / 2$, the jet is reversed in direction, and the profile is the catenary of equal strength. In Bobyleff's problem of the wedge of finite breadth,

$$
\begin{align*}
\operatorname{ch} n \Omega=\sqrt{ } \frac{b}{a} \sqrt{ } \frac{u-a}{u-b}, \operatorname{sh} n \Omega=\sqrt{ } \frac{b-a}{a} \sqrt{ } \frac{u}{u-b},  \tag{6}\\
\cos n \alpha=\sqrt{\frac{b}{a}}, \sin n \alpha=\sqrt{ } \frac{a-b}{a}, \tag{7}
\end{align*}
$$

and along the free surface APJ, $q=Q, \psi=0, u=e^{-\Pi \varphi / m}=\mathrm{ae}^{\mathrm{ns} / \mathrm{c}}$,

$$
\begin{gather*}
\cos n \theta=\cos n \alpha \sqrt{ } \frac{e^{\mathrm{n}^{2} / \mathrm{c}}-1}{\mathrm{e}^{\mathrm{T}^{2} / \mathrm{c}}-\cos ^{2} \mathrm{n} \alpha} \\
\mathrm{e}^{\mathrm{T}^{2} / \mathrm{c}}=\frac{\cos ^{2} n \alpha \sin ^{2} \mathrm{n} \theta}{\sin ^{2} \mathrm{n} \theta-\sin ^{2} \mathrm{n} \alpha} \tag{8}
\end{gather*}
$$

the intrinsic equation, the other free surface $A^{\prime} \mathrm{P}^{\prime} \mathrm{J}^{\prime}$ being given by

$$
\begin{equation*}
\mathrm{e}^{\mathrm{T}^{2} / \mathrm{c}}=\frac{\cos ^{2} \mathrm{n} \alpha \sin ^{2} \mathrm{n} \theta}{\sin ^{2} \mathrm{n} \alpha-\sin ^{2} \mathrm{n} \theta} \tag{9}
\end{equation*}
$$

Putting $\mathrm{n}=1$ gives the case of a stream of finite breadth disturbed by a transverse plane, a particular case of Fig. 7.

When $\mathrm{a}=\mathrm{b}, \alpha=0$, and the stream is very broad compared with the wedge or lamina; so, putting $\mathrm{w}=\mathrm{w}^{\prime}(\mathrm{a}-\mathrm{b}) / \mathrm{a}$ in the penultimate case, and

$$
\begin{gather*}
u=\mathrm{ae}^{-\mathrm{w}} \approx \mathrm{a}-(\mathrm{a}-\mathrm{b}) \mathrm{w}^{\prime},  \tag{10}\\
\operatorname{ch} \mathrm{n} \Omega=\sqrt{ } \frac{\mathrm{w}^{\prime}+1}{\mathrm{w}^{\prime}}, \operatorname{sh} \mathrm{n} \Omega=\sqrt{ } \frac{1}{\sqrt{ } \mathrm{w}^{\prime}}, \tag{11}
\end{gather*}
$$

in which we may write

$$
\begin{equation*}
w^{\prime}=\varphi+\psi i . \tag{12}
\end{equation*}
$$

Along the stream line xABPJ, $\psi=0$; and along the jet surface APJ, $-1>\varphi>-\infty$; and putting $\varphi=-\pi \mathrm{s} / \mathrm{c}-1$, the intrinsic equation is

$$
\begin{equation*}
\pi s / c=\cot ^{2} n \theta, \tag{13}
\end{equation*}
$$

which for $\mathrm{n}=1$ is the evolute of a catenary.
43. When the barrier $A A^{\prime}$ is held oblique to the current, the stream line xB is curved to the branch point $B$ on $A A^{\prime}$ (fig. 7), and so must be excluded from the boundary of $u$; the conformal representation is made now with

$$
\begin{align*}
& \frac{d \Omega}{d u}=-\frac{\sqrt{ }\left(b-a \cdot b-a^{\prime}\right)}{(u-b) \sqrt{ }\left(u-a \cdot u-a^{\prime}\right)}  \tag{1}\\
& \frac{d w}{d u}=-\frac{m}{\Pi} \frac{1}{u-j}-\frac{m^{\prime}}{\Pi} \frac{1}{u-j} \\
&=-\frac{m+m^{\prime}}{\Pi} \cdot \frac{u-b}{u-j \cdot u-j^{\prime}} \\
& b=\frac{m j^{\prime}+m^{\prime} j}{m+m^{\prime}}
\end{align*}
$$



Fig. 7.
taking $u=\infty$ at the source where $\varphi=\infty, u=b$ at the branch point $B, u=j, j^{\prime}$ at the end of the two diverging streams where $\varphi=-\infty$; while $\psi=0$ along the stream line which divides at B and passes through $\mathrm{A}, \mathrm{A}^{\prime}$; and $\psi=\mathrm{m},-\mathrm{m}^{\prime}$ along the outside boundaries, so that $\mathrm{m} / \mathrm{Q}, \mathrm{m}^{\prime} / \mathrm{Q}$ is the final breadth of the jets, and $\left(m+m^{\prime}\right) / Q$ is the initial breadth, $c_{1}$ of the impinging stream. Then

$$
\begin{gather*}
\operatorname{ch} \frac{1}{2} \Omega=\sqrt{ } \frac{b-a^{\prime}}{a-a^{\prime}} \sqrt{\frac{u-b}{u-b}}, \operatorname{sh}{ }^{1 / 2 \Omega}=\sqrt{ } \frac{b-a}{a-a^{\prime}} \sqrt{\frac{u-a^{\prime}}{u-b}},  \tag{3}\\
2 b-a-a^{\prime} \quad N
\end{gather*}
$$

$$
\begin{gather*}
\operatorname{ch} \Omega=\frac{a-a^{\prime}}{}-\overline{u-b}, \\
\operatorname{sh} \Omega=\sqrt{ } N \frac{\sqrt{ }\left(2 \cdot a-u \cdot u-a^{\prime}\right)}{u-b}, \\
N=2 \frac{a-b \cdot b-a^{\prime}}{a-a^{\prime}} . \tag{4}
\end{gather*}
$$

Along a jet surface, $q=Q$, and

$$
\begin{equation*}
\operatorname{ch} \Omega=\cos \theta=\cos \alpha-1 / 2 \sin ^{2} \alpha\left(a-a^{\prime}\right) /(u-b) \tag{5}
\end{equation*}
$$

if $\theta=\alpha$ at the source $x$ of the jet $x B$, where $u=\infty$; and supposing $\theta=\beta, \beta^{\prime}$ at the end of the streams where $u=j, j^{\prime}$,

$$
\begin{align*}
\frac{u-b}{a-a^{\prime}}= & \frac{1 / 2 \sin ^{2} \alpha}{\cos \alpha-\cos \theta}, \frac{u-j}{a-a^{\prime}} 1_{2}^{2} \sin ^{2} \alpha \frac{\cos \theta-\cos \beta}{(\cos \alpha-\cos \beta)(\cos \alpha-\cos \theta)}, \\
& \frac{u-j^{\prime}}{a-a^{\prime}}=1 / 2 \sin ^{2} \alpha \frac{\cos \theta-\cos \beta^{\prime}}{\left(\cos \alpha-\cos \beta^{\prime}\right)(\cos \alpha-\cos \theta)} \tag{6}
\end{align*}
$$

and $\psi$ being constant along a stream line

$$
\begin{gather*}
\frac{d \varphi}{d u}=\frac{d w}{d u}, Q \frac{d s}{d \theta}=\frac{d \varphi}{d \theta}=\frac{d w}{d u} \frac{d u}{d \theta} \\
\frac{\Pi Q}{m+m^{\prime}} \frac{d s}{d \theta}=\frac{\Pi}{c} \frac{d s}{d \theta}=\frac{(\cos \alpha-\cos \beta)\left(\cos \alpha-\cos \beta^{\prime}\right) \sin \theta}{(\cos \alpha-\cos \theta)(\cos \theta-\cos \beta)\left(\cos \theta-\cos \alpha^{\prime}\right)} \\
=\frac{\sin \theta}{\cos \alpha-\cos \theta}+\frac{\cos \alpha-\cos \beta^{\prime}}{\cos \beta-\cos \beta^{\prime}} \cdot \frac{\sin \theta}{\cos \theta-\cos \beta} \\
\frac{\cos \alpha-\cos \beta}{\cos \beta-\cos \beta^{\prime}} \cdot \frac{\sin \theta}{\cos \theta-\cos \beta^{\prime}} \tag{7}
\end{gather*}
$$

giving the intrinsic, equation of the surface of a jet, with proper attention to the sign.
From $A$ to $B, a>u>b, \theta=0$,

$$
\begin{gather*}
\operatorname{ch} \Omega=\operatorname{ch} \log \frac{Q}{q}=\cos \alpha-1 / 2 \sin ^{2} \alpha \frac{a-a^{\prime}}{a-b} \\
\operatorname{sh} \Omega=\operatorname{sh} \log \frac{Q}{q}=\frac{\sqrt{ }\left(a-u \cdot u-a^{\prime}\right)}{u-b} \sin \alpha \\
\frac{Q}{q}=\frac{(u-b) \cos \alpha-1 / 2\left(a-a^{\prime}\right) \sin ^{2} \alpha+\sqrt{ }\left(a-u \cdot u-a^{\prime}\right) \sin \alpha}{u-b}  \tag{8}\\
Q \frac{d s}{d u}=Q \frac{d s}{d \varphi} \frac{d \varphi}{d u}=-\frac{Q}{q} \frac{d w}{d u} \\
\pi \frac{A B}{c}=\int_{b}^{a} \frac{\left(2 b-a-a^{\prime}\right)(u-b)-2(a-b)\left(b-a^{\prime}\right)+2 \sqrt{\prime}\left(a-b \cdot b-a^{\prime} \cdot a-u \cdot u-a^{\prime}\right)}{a-a^{\prime} \cdot j-u \cdot u-j^{\prime}} d u, d_{10}
\end{gather*}
$$

with a similar expression for $\mathrm{BA}^{\prime}$.
The motion of a jet impinging on an infinite barrier is obtained by putting $j=a, j^{\prime}=a^{\prime}$; duplicated on the other side of the barrier, the motion reversed will represent the direct collision of two jets of unequal breadth and equal velocity. When the barrier is small compared with the jet, $\alpha=\beta=\beta^{\prime}$, and G. Kirchhoff's solution is obtained of a barrier placed obliquely in an infinite stream.

Two corners $B_{1}$ and $B_{2}$ in the wall $x A$, with $\mathrm{a}^{\prime}=-\infty$, and $n=1$, will give the solution, by duplication, of a jet issuing by a reentrant mouthpiece placed symmetrically in the end wall of the channel; or else of the channel blocked partially by a diaphragm across the middle, with edges turned back symmetrically, problems discussed by J. H. Michell, A. E. H. Love and M. Réthy.

When the polygon is closed by the walls joining, instead of reaching back to infinity at $\mathrm{xx}^{\prime}$, the liquid motion must be due to a source, and this modification has been worked out by $B$. Hopkinson in the Proc. Lond. Math. Soc., 1898.

Michell has discussed also the hollow vortex stationary inside a polygon (Phil. Trans., 1890); the solution is given by

$$
\begin{equation*}
\operatorname{ch} \mathrm{n} \Omega=\mathrm{sn} \mathrm{w}, \operatorname{sh} \mathrm{n} \Omega=\mathrm{i} \mathrm{cn} \mathrm{w} \tag{11}
\end{equation*}
$$

so that, round the boundary of the polygon, $\psi=K^{\prime}, \sin n \theta=0$; and on the surface of the vortex $\psi=0, q=Q$, and

$$
\begin{equation*}
\cos n \theta=\operatorname{sn} \varphi, n \theta=1 / 2 \Pi-\mathrm{am} \mathrm{~s} / \mathrm{c} \tag{12}
\end{equation*}
$$

the intrinsic equation of the curve.
This is a closed Sumner line for $\mathrm{n}=1$, when the boundary consists of two parallel walls; and n $=1 / 2$ gives an Elastica.
44. The Motion of a Solid through a Liquid.-An important problem in the motion of a liquid is the determination of the state of velocity set up by the passage of a solid through it; and thence of the pressure and reaction of the liquid on the surface of the solid, by which its motion is influenced when it is free.

Beginning with a single body in liquid extending to infinity, and denoting by U, V, W, P, Q, R the components of linear and angular velocity with respect to axes fixed in the body, the velocity function takes the form

$$
\begin{equation*}
\varphi=\mathrm{U}_{\varphi 1}+\mathrm{V}_{\varphi 2}+\mathrm{W}_{\varphi 3}+\mathrm{P}_{\chi 1}+\mathrm{Q}_{\chi 2}+\mathrm{R}_{\chi 3} \tag{1}
\end{equation*}
$$

where the $\varphi$ 's and $\chi$ 's are functions of $x, y, z$, depending on the shape of the body; interpreted dynamically, $C-\rho \varphi$ represents the impulsive pressure required to stop the motion, or $C+\rho \varphi$ to start it again from rest.

The terms of $\varphi$ may be determined one at a time, and this problem is purely kinematical; thus to determine $\varphi_{1}$, the component U alone is taken to exist, and then $\mathrm{l}, \mathrm{m}, \mathrm{n}$, denoting the direction cosines of the normal of the surface drawn into the exterior liquid, the function $\varphi_{1}$ must be determined to satisfy the conditions
(i.) $\nabla^{2} \varphi_{1}=0$. throughout the liquid;
(ii.) $\mathrm{d} \varphi_{1} / \mathrm{d} v=-\mathrm{l}$, the gradient of $\varphi$ down the normal at the surface of the moving solid;
(iii.) $d \varphi_{1} / d v=0$, over a fixed boundary, or at infinity;
similarly for $\varphi_{2}$ and $\varphi_{3}$.
To determine $\chi_{1}$ the angular velocity $P$ alone is introduced, and the conditions to be satisfied are
(i.) $\nabla^{2} \chi_{1}=0$, throughout the liquid;
(ii.) $d \chi_{1} / d v=m z-n y$, at the surface of the moving body, but zero over a fixed surface, and at infinity; the same for $\chi_{2}$ and $\chi_{3}$.

For a cavity filled with liquid in the interior of the body, since the liquid inside moves bodily for a motion of translation only,

$$
\begin{equation*}
\varphi_{1}=-x, \varphi_{2}=-y, \varphi_{3}=-z ; \tag{2}
\end{equation*}
$$

but a rotation will stir up the liquid in the cavity, so that the $\chi$ 's depend on the shape of the surface.

The ellipsoid was the shape first worked out, by George Green, in his Research on the Vibration of a Pendulum in a Fluid Medium (1833); the extension to any other surface will form an important step in this subject.

A system of confocal ellipsoids is taken

$$
\begin{equation*}
\frac{x^{2}}{a^{2}+\lambda}+\frac{y^{2}}{b^{2}+\lambda}+\frac{z^{2}}{c^{2}+\lambda}=1 \tag{3}
\end{equation*}
$$

and a velocity function of the form

$$
\begin{equation*}
\varphi=\mathrm{x} \psi, \tag{4}
\end{equation*}
$$

where $\psi$ is a function of $\lambda$ only, so that $\psi$ is constant over an ellipsoid; and we seek to determine the motion set up, and the form of $\psi$ which will satisfy the equation of continuity.

Over the ellipsoid, p denoting the length of the perpendicular from the centre on a tangent plane,

$$
\begin{gather*}
\mathrm{l}=\frac{\mathrm{px}}{\mathrm{a}^{2}+\lambda}, \quad \mathrm{m}=\frac{\mathrm{py}}{\mathrm{~b}^{2}+\lambda}, \quad \mathrm{n}=\frac{\mathrm{pz}}{\mathrm{c}^{2}+\lambda}  \tag{5}\\
1=\frac{\mathrm{p}^{2} \mathrm{x}^{2}}{\left(\mathrm{a}^{2}+\lambda\right)^{2}}+\frac{\mathrm{p}^{2} \mathrm{y}^{2}}{\left(\mathrm{~b}^{2}+\lambda\right)^{2}}+\frac{\mathrm{p}^{2} z^{2}}{\left(c^{2}+\lambda\right)^{2}}, \\
\mathrm{p}^{2}=\left(\mathrm{a}^{2}+\lambda\right) \mathrm{l}^{2}+\left(b^{2}+\lambda\right) m^{2}+\left(c^{2}+\lambda\right) n^{2}, \\
=\mathrm{a}^{2} \mathrm{l}^{2}+\mathrm{b}^{2} \mathrm{~m}^{2}+\mathrm{c}^{2} \mathrm{n}^{2}+\lambda,
\end{gather*}
$$

$$
\begin{equation*}
2 p^{\frac{d p}{}}=\frac{\mathrm{d} \lambda}{} ; \tag{7}
\end{equation*}
$$

Thence

$$
\begin{gather*}
\frac{d \varphi}{d s}=\frac{d x}{d s} \psi+x \frac{d \psi}{d s} \\
=\frac{d x}{d s} \psi+2\left(a^{2}+\lambda\right) \frac{d \psi}{d \lambda} l \frac{d p}{d s} \tag{9}
\end{gather*}
$$

so that the velocity of the liquid may be resolved into a component $-\psi$ parallel to Ox , and $-2\left(\mathrm{a}^{2}\right.$ $+\lambda) l \mathrm{~d} \psi / \mathrm{d} \lambda$ along the normal of the ellipsoid; and the liquid flows over an ellipsoid along a line of slope with respect to Ox, treated as the vertical.

Along the normal itself

$$
\begin{equation*}
\frac{d \varphi}{d s}\left\{\psi+2\left(a^{2}+\lambda\right) \frac{d \psi}{d \lambda}\right\} \tag{10}
\end{equation*}
$$

so that over the surface of an ellipsoid where $\lambda$ and $\psi$ are constant, the normal velocity is the same as that of the ellipsoid itself, moving as a solid with velocity parallel to Ox

$$
\begin{equation*}
\mathrm{U}=-\psi-2\left(\mathrm{a}^{2}+\lambda\right) \frac{\mathrm{d} \psi}{\mathrm{~d} \lambda} \tag{11}
\end{equation*}
$$

and so the boundary condition is satisfied; moreover, any ellipsoidal surface $\lambda$ may be supposed moving as if rigid with the velocity in (11), without disturbing the liquid motion for the moment.

The continuity is secured if the liquid between two ellipsoids $\lambda$ and $\lambda_{1}$, moving with the velocity $U$ and $U_{1}$ of equation (11), is squeezed out or sucked in across the plane $x=0$ at a rate equal to the integral flow of the velocity $\psi$ across the annular area $\alpha_{1}-\alpha$ of the two ellipsoids made by $x=0$; or if

$$
\begin{gather*}
\alpha U-\alpha_{1} U_{1}=\int_{\lambda}^{\lambda_{1}} \psi \frac{d \alpha}{d \lambda} d \lambda  \tag{12}\\
\alpha=\pi \sqrt{ }\left(b^{2}+\lambda \cdot c^{2}+\lambda\right) . \tag{13}
\end{gather*}
$$

Expressed as a differential relation, with the value of $U$ from (11),

$$
\begin{gather*}
\frac{d}{d \lambda}\left[\alpha \psi+2\left(a^{2}+\lambda\right) \alpha \frac{d \psi}{d \lambda}\right]-\psi \frac{d \alpha}{d \lambda}=0  \tag{14}\\
3 \alpha \frac{d \psi}{d \lambda}+2\left(a^{2}+\lambda\right) \frac{d}{d \lambda}\left(\alpha \frac{d \psi}{d \lambda}\right)=0 \tag{15}
\end{gather*}
$$

and integrating

$$
\begin{equation*}
\left(a^{2}+\lambda\right)^{3 / 2} \alpha \frac{d \psi}{d \lambda}=\mathrm{a} \text { constant } \tag{16}
\end{equation*}
$$

so that we may put

$$
\begin{gather*}
\psi=\int \frac{\mathrm{Md} \mathrm{\lambda}}{\left(\mathrm{a}^{2}+\lambda\right) \mathrm{P}},  \tag{17}\\
\mathrm{P}^{2}=4\left(\mathrm{a}^{2}+\lambda\right)\left(\mathrm{b}^{2}+\lambda\right)\left(\mathrm{c}^{2}+\lambda\right), \tag{18}
\end{gather*}
$$

where $M$ denotes a constant; so that $\psi$ is an elliptic integral of the second kind.
The quiescent ellipsoidal surface, over which the motion is entirely tangential, is the one for which

$$
\begin{equation*}
2\left(a^{2}+\lambda\right) \frac{d \psi}{d \lambda}+\psi=0 \tag{19}
\end{equation*}
$$

and this is the infinite boundary ellipsoid if we make the upper limit $\lambda_{1}=\infty$.
The velocity of the ellipsoid defined by $\lambda=0$ is then

$$
\begin{gather*}
U=-2 a^{2} \frac{d \psi_{0}}{d \lambda}-\psi_{0} \\
=\frac{M}{a b c}-\int_{0}^{\infty} \frac{M d \lambda}{\left(a^{2}+\lambda\right) P} \\
=\frac{M}{a b c}\left(1-A_{0}\right), \tag{20}
\end{gather*}
$$

with the notation

$$
\begin{align*}
& A \text { or } A_{\lambda}=\int_{\lambda}^{\infty} \frac{a b c d \lambda}{\left(a^{2}+\lambda\right) P} \\
& =-2 a b c \frac{d}{d a^{2}} \int_{\lambda}^{\infty} \frac{d \lambda}{P} \tag{21}
\end{align*}
$$

so that in (4)

$$
\begin{equation*}
\varphi=\frac{\mathrm{M}}{\mathrm{abc}} \mathrm{xA}=\frac{\mathrm{UxA}}{1-\mathrm{A}_{0}}, \quad \varphi_{1}=\frac{\mathrm{xA}_{\lambda}}{1-\mathrm{A}_{0}} \tag{22}
\end{equation*}
$$

in (1) for an ellipsoid.
The impulse required to set up the motion in liquid of density $\rho$ is the resultant of an impulsive pressure $\rho \varphi$ over the surface $S$ of the ellipsoid, and is therefore

$$
\begin{equation*}
\iint \rho \varphi \mathrm{l} d S=\rho \psi_{0} \iint \mathrm{xl} d S=\rho \psi_{0} \text { (volume of the ellipsoid) }=\psi_{0} \mathrm{~W}^{\prime}, \tag{23}
\end{equation*}
$$

where $\mathrm{W}^{\prime}$ denotes the weight of liquid displaced.
Denoting the effective inertia of the liquid parallel to Ox by $\alpha \mathrm{W}^{\prime}$. the momentum

$$
\begin{gather*}
\alpha \mathrm{W}^{\prime} \mathrm{U}=\psi_{0} \mathrm{~W}^{\prime}  \tag{24}\\
\alpha=\frac{\psi_{0}}{\mathrm{U}}=\frac{\mathrm{A}_{0}}{1-\mathrm{A}_{0}} ; \tag{25}
\end{gather*}
$$

in this way the air drag was calculated by Green for an ellipsoidal pendulum.
Similarly, the inertia parallel to Oy and Oz is

$$
\begin{gather*}
\beta \mathrm{W}^{\prime}=\frac{\mathrm{B}_{0}}{1-\mathrm{B}_{0}} \mathrm{~W}^{\prime}, \quad \gamma \mathrm{W}^{\prime}=\frac{\mathrm{C}_{0}}{1-\mathrm{C}_{0}} \mathrm{~W}^{\prime},  \tag{26}\\
\mathrm{B}_{\lambda}, \mathrm{C}_{\lambda}=\int_{\lambda}^{\infty} \frac{\mathrm{abc} \mathrm{~d} \lambda}{\left(\mathrm{~b}^{2}+\lambda, \mathrm{c}^{2}+\lambda\right) \mathrm{P}} \tag{27}
\end{gather*}
$$

and

$$
\begin{equation*}
\mathrm{A}+\mathrm{B}+\mathrm{C}=\mathrm{abc} / 1 / 2 \mathrm{P}, \quad \mathrm{~A}_{0}+\mathrm{B}_{0}+\mathrm{C}_{0}=1 \tag{28}
\end{equation*}
$$

For a sphere

$$
\begin{equation*}
a=b=c, \quad A_{0}=B_{0}=C_{0}=1 / 3, \quad \alpha=\beta=\gamma=1 / 2, \tag{29}
\end{equation*}
$$

so that the effective inertia of a sphere is increased by half the weight of liquid displaced; and in frictionless air or liquid the sphere, of weight W , will describe a parabola with vertical acceleration

$$
\begin{equation*}
\frac{\mathrm{W}-\mathrm{W}^{\prime}}{\mathrm{W}+1 / 2 \mathrm{~W}^{\prime}} \mathrm{g} . \tag{30}
\end{equation*}
$$

Thus a spherical air bubble, in which $\mathrm{W} / \mathrm{W}^{\prime}$ is insensible, will begin to rise in water with acceleration 2 g .
45. When the liquid is bounded externally by the fixed ellipsoid $\lambda=\lambda_{1}$, a slight extension will give the velocity function $\varphi$ of the liquid in the interspace as the ellipsoid $\lambda=0$ is passing with velocity $U$ through the confocal position; $\varphi$ must now take the form $\mathrm{x}(\psi+\mathrm{N})$, and will satisfy the conditions in the shape

$$
\begin{equation*}
\varphi=U x \frac{A+B_{1}+C_{1}}{B_{0}+C_{0}-B_{1}-C_{1}}=U x \frac{\frac{a b c}{a_{1} b_{1} c_{1}}+\int_{\lambda}^{\lambda_{1}} \frac{a b c d \lambda}{\left(a^{2}+\lambda\right) P}}{1-\frac{a b c}{a_{1} b_{1} c_{1}}-\int_{0}^{\lambda_{1}} \frac{a b c d \lambda}{\left(a^{2}+\lambda\right) P}} \tag{1}
\end{equation*}
$$

and any confocal ellipsoid defined by $\lambda$, internal or external to $\lambda=\lambda_{1}$, may be supposed to swim with the liquid for an instant, without distortion or rotation, with velocity along Ox

$$
\mathrm{U} \frac{\mathrm{~B}_{\lambda}+\mathrm{C}_{\lambda}-\mathrm{B}_{1}-\mathrm{C}_{1}}{\mathrm{~B}_{0}+\mathrm{C}_{0}-\mathrm{B}_{1}-\mathrm{C}_{1}} .
$$

Since - Ux is the velocity function for the liquid $W^{\prime}$ filling the ellipsoid $\lambda=0$, and moving bodily with it, the effective inertia of the liquid in the interspace is

$$
\begin{equation*}
\frac{\mathrm{A}_{0}+\mathrm{B}_{1}+\mathrm{C}_{1}}{\mathrm{~B}_{0}+\mathrm{C}_{0}-\mathrm{B}_{1}-\mathrm{C}_{1}} \mathrm{~W}^{\prime} . \tag{2}
\end{equation*}
$$

If the ellipsoid is of revolution, with $b=c$,

$$
\begin{equation*}
\varphi=1 / 2 \mathrm{Ux} \frac{\mathrm{~A}+2 \mathrm{~B}_{1}}{\mathrm{~B}_{0}-\mathrm{B}_{1}} \tag{3}
\end{equation*}
$$

and the Stokes' current function $\psi$ can be written down

$$
\begin{equation*}
\psi=-1 / 2 U y^{2} \frac{B-B_{1}}{B_{0}-B_{1}} ; \tag{4}
\end{equation*}
$$

reducing, when the liquid extends to infinity and $B_{1}=0$, to

$$
\begin{equation*}
\varphi=1 / 2 \mathrm{Ux} \frac{\mathrm{~A}}{\mathrm{~B}_{0}}, \quad \psi=-1 / 2 \mathrm{Uy}^{2} \frac{\mathrm{~B}}{\mathrm{~B}_{0}} ; \tag{5}
\end{equation*}
$$

so that in the relative motion past the body, as when fixed in the current $U$ parallel to xO ,

$$
\begin{equation*}
\varphi^{\prime}=1 / 2 \mathrm{Ux}\left(1+\frac{\mathrm{A}}{\mathrm{~B}_{0}}\right), \quad \psi^{\prime}=1 / 2 U y^{2}\left(1-\frac{\mathrm{B}}{\mathrm{~B}_{0}}\right) . \tag{6}
\end{equation*}
$$

Changing the origin from the centre to the focus of a prolate spheroid, then putting $b^{2}=p a, \lambda$ $=\lambda^{\prime} \mathrm{a}$, and proceeding to the limit where $\mathrm{a}=\infty$, we find for a paraboloid of revolution

$$
\begin{gather*}
B=\frac{1 / 2}{2} \frac{p}{p+\lambda^{\prime}}, \quad \frac{B}{B_{0}}=\frac{p}{p+\lambda^{\prime}},  \tag{7}\\
\frac{y^{2}}{p+\lambda^{\prime}}=p+\lambda^{\prime}-2 x, \tag{8}
\end{gather*}
$$

with $\lambda^{\prime}=0$ over the surface of the paraboloid; and then

$$
\begin{gather*}
\psi^{\prime}=1 / 2 U\left[y^{2}-p \sqrt{ }\left(x^{2}+y^{2}\right)+p x\right] ;  \tag{9}\\
\psi=-1 / 2 U p\left[\sqrt{ }\left(x^{2}+y^{2}\right)-x\right] ;  \tag{10}\\
\varphi=-1 / 2 \operatorname{Up} \log \left[\sqrt{ }\left(x^{2}+y^{2}\right)+x\right] \tag{11}
\end{gather*}
$$

The relative path of a liquid particle is along a stream line

$$
\begin{gather*}
\psi^{\prime}=1 / 2 U c^{2} \text {, a constant, } \\
x=\frac{p^{2} y^{2}-\left(y^{2}-c^{2}\right)^{2}}{2 p\left(y^{2}-c^{2}\right)}, \quad V\left(x^{2}+y^{2}\right)=\frac{p^{2} y^{2}-\left(y^{2}-c^{2}\right)^{2}}{2 p\left(y^{2}-c^{2}\right)} \tag{12}
\end{gather*}
$$

a $\mathrm{C}_{4}$; while the absolute path of a particle in space will be given by

$$
\begin{gather*}
\frac{d y}{d x}=-\frac{r-x}{y}=\frac{y^{2}-c^{2}}{2 p y}  \tag{14}\\
y^{2}-c^{2}=a^{2} e^{-x / p} \tag{15}
\end{gather*}
$$

46. Between two concentric spheres, with

$$
\begin{gather*}
a^{2}+\lambda=r^{2}, a^{2}+\lambda_{1}=a_{1}{ }^{2},  \tag{1}\\
A=B=C=a^{3} / 3 r^{3}, \\
\varphi=1 / 2 U x \frac{a^{3} / r^{3}+2 a^{3} / a_{1}{ }^{3}}{1-a^{4} / a_{1}{ }^{2}}, \quad \psi=1 / 2 U y^{2} \frac{a^{3} / r^{3}-a^{3} / a_{1}{ }^{3}}{1-a^{3} / a_{1}{ }^{3}} ; \tag{2}
\end{gather*}
$$

and the effective inertia of the liquid in the interspace is

$$
\begin{equation*}
\frac{\mathrm{A}_{0}+2 \mathrm{~A}_{1}}{2 \mathrm{~A}_{0}-2 \mathrm{~A}_{1}} \mathrm{~W}^{\prime}=1 / 2 \frac{\mathrm{a}_{1}^{3}+2 \mathrm{a}^{3}}{\mathrm{a}_{1}^{3}-\mathrm{a}^{3}} \mathrm{~W}^{\prime} \tag{3}
\end{equation*}
$$

When the spheres are not concentric, an expression for the effective inertia can be found by the method of images (W. M. Hicks, Phil. Trans., 1880).

The image of a source of strength $\mu$ at $S$ outside a sphere of radius a is a source of strength $\mu \mathrm{a} / \mathrm{f}$ at H , where $\mathrm{OS}=\mathrm{f}, \mathrm{OH}=\mathrm{a}^{2} / \mathrm{f}$, and a line sink reaching from the image H to the centre O of line strength $-\mu / a$; this combination will be found to produce no flow across the surface of the sphere.

Taking Ox along OS, the Stokes' function at $P$ for the source $S$ is $\mu$ cos PSx, and of the source H and line sink OH is $\mu(\mathrm{a} / \mathrm{f}) \cos \mathrm{PHx}$ and $-(\mu / \mathrm{a})(\mathrm{PO}-\mathrm{PH})$; so that

$$
\begin{equation*}
\psi=\mu\left(\cos P S x+\frac{a}{f} \cos P H x-\frac{P O-P H}{a}\right) \tag{4}
\end{equation*}
$$

and $\psi=-\mu$, a constant, over the surface of the sphere, so that there is no flow across.
When the source $S$ is inside the sphere and $H$ outside, the line sink must extend from $H$ to infinity in the image system; to realize physically the condition of zero flow across the sphere, an equal sink must be introduced at some other internal point $S^{\prime}$.

When S and $\mathrm{S}^{\prime}$ lie on the same radius, taken along Ox , the Stokes' function can be written down; and when S and $\mathrm{S}^{\prime}$ coalesce a doublet is produced, with a doublet image at H .

For a doublet at S , of moment m , the Stokes' function is

$$
\begin{equation*}
m \frac{d}{d f} \cos P S x=-m \frac{y^{2}}{P S S^{3}} \tag{5}
\end{equation*}
$$

and for its image at H the Stokes' function is

$$
\begin{equation*}
m \frac{d}{d f} \cos P H x=-m \frac{a^{3}}{f^{3}} \frac{y^{2}}{P H^{3}} ; \tag{6}
\end{equation*}
$$

so that for the combination

$$
\begin{equation*}
\psi=\mathrm{my}^{2}\left(\frac{\mathrm{a}^{3}}{f^{3}} \frac{1}{\mathrm{PH}^{3}}-\frac{1}{\mathrm{PS}^{3}}\right)=m \frac{\mathrm{y}^{2}}{f^{3}}\left(\frac{\mathrm{a}^{3}}{\mathrm{PH}^{3}}-\frac{f^{3}}{\mathrm{PS}^{3}}\right) \tag{7}
\end{equation*}
$$

and this vanishes over the surface of the sphere.
There is ao Stokes' function when the axis of the doublet at S does not pass through O ; the image system will consist of an inclined doublet at $H$, making an equal angle with OS as the doublet S , and of a parallel negative line doublet, extending from H to O , of moment varying as the distance from $O$.

A distribution of sources and doublets over a moving surface will enable an expression to be obtained for the velocity function of a body moving in the presence of a fixed sphere, or inside it.

The method of electrical images will enable the stream function $\psi^{\prime}$ to be inferred from a distribution of doublets, finite in number when the surface is composed of two spheres intersecting at an angle $\pi / \mathrm{m}$, where m is an integer (R. A. Herman, Quart. Jour. of Math. xxii.).

Thus for $m=2$, the spheres are orthogonal, and it can be verified that

$$
\begin{equation*}
\psi^{\prime}=1 / 2 U y^{2}\left(1-\frac{a_{1}^{3}}{r_{1}^{3}}-\frac{a_{2}^{3}}{r_{2}^{3}}+\frac{a^{3}}{r^{3}}\right) \tag{8}
\end{equation*}
$$

where $a_{1}, a_{2}, a=a_{1} a_{2} / \sqrt{ }\left(a_{1}^{2}+a_{2}^{2}\right)$ is the radius of the spheres and their circle of intersection, and $r_{1}, r_{2}, r$ the distances of a point from their centres.

The corresponding expression for two orthogonal cylinders will be

$$
\begin{equation*}
\psi^{\prime}=U y\left(1-\frac{a_{1}^{2}}{r_{1}^{2}}-\frac{a_{2}^{2}}{r_{2}^{2}}+\frac{a^{2}}{r^{2}}\right) \tag{8}
\end{equation*}
$$

With $\mathrm{a}_{2}=\infty$, these reduce to

$$
\begin{equation*}
\psi^{\prime}=1 / 2 U y^{2}\left(1-\frac{a^{5}}{r^{5}}\right) \frac{x}{a}, \text { or } U y\left(1-\frac{a^{4}}{r^{4}}\right) \frac{x}{a} \tag{10}
\end{equation*}
$$

for a sphere or cylinder, and a diametral plane.
Two equal spheres, intersecting at $120^{\circ}$, will require

$$
\begin{equation*}
\psi^{\prime}=1 / 2 U y^{2}\left[\frac{x}{a}-\frac{a^{3}}{2 r_{1}{ }^{3}}+\frac{a^{4}(a-2 x)}{2 r_{1}{ }^{5}}+\frac{a^{3}}{2 r_{2}{ }^{3}}-\frac{a^{4}(a+2 x)}{2 r_{2}{ }^{5}}\right] \tag{11}
\end{equation*}
$$

with a similar expression for cylinders; so that the plane $\mathrm{x}=0$ may be introduced as a boundary, cutting the surface at $60^{\circ}$. The motion of these cylinders across the line of centres is the equivalent of a line doublet along each axis.
47. The extension of Green's solution to a rotation of the ellipsoid was made by A. Clebsch, by taking a velocity function

$$
\begin{equation*}
\varphi=x y \chi \tag{1}
\end{equation*}
$$

for a rotation R about Oz ; and a similar procedure shows that an ellipsoidal surface $\lambda$ may be in rotation about Oz without disturbing the motion if

$$
\begin{equation*}
\mathrm{R}=-\frac{\left[1 /\left(\mathrm{a}^{2}+\lambda\right)+1 /\left(\mathrm{b}^{2}+\lambda\right)\right] \chi+2 \mathrm{dx} / \mathrm{d} \lambda}{1 /\left(\mathrm{b}^{2}+\lambda\right)-1 /\left(\mathrm{a}^{2}+\lambda\right)} \tag{2}
\end{equation*}
$$

and that the continuity of the liquid is secured if

$$
\begin{align*}
& \left(a^{2}+\lambda\right)^{3 / 2}\left(b^{2}+\lambda\right)^{3 / 2}\left(c^{2}+\lambda\right) \frac{1}{2} \frac{d \chi}{d \lambda}=\text { constant }  \tag{3}\\
& \chi=\int_{\lambda}^{\infty} \frac{N}{\left(a^{2}+\lambda\right)\left(b^{2}+\lambda\right) P}=\frac{N}{a b c} \cdot \frac{B_{\lambda}-A_{\lambda}}{a^{2}-b^{2}} \tag{4}
\end{align*}
$$

and at the surface $\lambda=0$,

$$
\begin{gather*}
R=-\frac{\left[\left(1 / a^{2}+1 / b^{2}\right) \cdot N / a b c \cdot\left(B_{0}-A_{0}\right) /\left(a^{2}-b^{2}\right)\right]-N / a b c \cdot 1 / a^{2} b^{2}}{1 / b^{2}-1 / a^{2}},  \tag{5}\\
\frac{N}{a b c}=R \frac{1 / b^{2}-1 / a^{2}}{1 / a^{2} b^{2}-\left[\left(1 / a^{2}+1 / b^{2}\right) \cdot\left(B_{0}-A_{0}\right) /\left(a^{2}-b^{2}\right)\right]}  \tag{6}\\
=R \frac{\left(a^{2}-b^{2}\right)^{2} /\left(a^{2}+b^{2}\right)}{\left(a^{2}-b^{2}\right) /\left(a^{2}+b^{2}\right)-\left(B_{0}-A_{0}\right)}
\end{gather*},
$$

The velocity function of the liquid inside the ellipsoid $\lambda=0$ due to the same angular velocity will be

$$
\begin{equation*}
\varphi_{1}=\operatorname{Rxy}\left(\mathrm{a}^{2}-\mathrm{b}^{2}\right) /\left(\mathrm{a}^{2}+\mathrm{b}^{2}\right), \tag{7}
\end{equation*}
$$

and on the surface outside

$$
\begin{equation*}
\varphi_{0}=x y \chi_{0}=x y \frac{N}{a b c} \frac{\mathrm{~B}_{0}-\mathrm{A}_{0}}{\mathrm{a}^{2}-\mathrm{b}^{2}} \tag{8}
\end{equation*}
$$

so that the ratio of the exterior and interior value of $\varphi$ at the surface is

$$
\begin{equation*}
\frac{\varphi_{0}}{\varphi_{1}}=\frac{\mathrm{B}_{0}-\mathrm{A}_{0}}{\left(\mathrm{a}^{2}-\mathrm{b}^{2}\right) /\left(\mathrm{a}^{2}+\mathrm{b}^{2}\right)-\left(\mathrm{B}_{0}-\mathrm{A}_{0}\right)} \tag{9}
\end{equation*}
$$

and this is the ratio of the effective angular inertia of the liquid, outside and inside the ellipsoid $\lambda=0$.

The extension to the case where the liquid is bounded externally by a fixed ellipsoid $\lambda=\lambda_{1}$ is made in a similar manner, by putting

$$
\begin{equation*}
\varphi=x y(\chi+M), \tag{10}
\end{equation*}
$$

and the ratio of the effective angular inertia in (9) is changed to

$$
\begin{equation*}
\frac{\left(B_{0}-A_{0}\right)-\left(B_{1}-A_{1}\right)+\frac{a_{1}{ }^{2}-b_{1}{ }^{2}}{a_{1}{ }^{2}+b_{1}{ }^{2}} \frac{a b c}{a_{1} b_{1} c_{1}}}{\frac{a^{2}-b^{2}}{a^{2}+b^{2}}-\frac{a_{1}{ }^{2}-b_{1}^{2}}{a_{1}{ }^{2}+b_{1}{ }^{2}} \frac{a b c}{a_{1} b_{1} c_{1}}-\left(B_{0}-A_{0}\right)+\left(B_{1}-A_{1}\right)} . \tag{11}
\end{equation*}
$$

Make $\mathrm{c}=\infty$ for confocal elliptic cylinders; and then

$$
\begin{gather*}
A \lambda=\int_{\lambda}^{\infty} \frac{a b}{\left(a^{2}+\lambda\right) \sqrt{ }\left(4 a^{2}+\lambda b^{2}+\lambda\right)}=\frac{a b}{a^{2}-b^{2}}\left(1-\sqrt{ } \frac{b^{2}+\lambda}{a^{2}+\lambda}\right),  \tag{12}\\
B \lambda=\frac{a b}{a^{2}-b^{2}}\left(\sqrt{ } \frac{a^{2}+\lambda}{b^{2}+\lambda}-1\right), \quad C \lambda=0
\end{gather*}
$$

and then as above in § 31, with

$$
\begin{equation*}
a=c \operatorname{ch} \alpha, b=c \operatorname{sh} \alpha, a_{1}=\sqrt{ }\left(a^{2}+\lambda\right)=c \operatorname{ch} \alpha_{1}, b_{1}=c \operatorname{sh} \alpha_{1} \tag{13}
\end{equation*}
$$

the ratio in (11) agrees with § 31 (6).
As before in § 31, the rotation may be resolved into a shear-pair, in planes perpendicular to Ox and Oy .

A torsion of the ellipsoidal surface will give rise to a velocity function of the form $\varphi=\operatorname{xyz} \Omega$, where $\Omega$ can be expressed by the elliptic integrals $\mathrm{A}_{\lambda}, \mathrm{B}_{\lambda}, \mathrm{C}_{\lambda}$, in a similar manner, since

$$
\Omega=\mathrm{L} \int_{\lambda}^{\infty} \mathrm{d} \lambda / \mathrm{P}^{3} .
$$

48. The determination of the $\varphi$ 's and $\chi^{\prime} s$ is a kinematical problem, solved as yet only for a few cases, such as those discussed above.

But supposing them determined for the motion of a body through a liquid, the kinetic energy T of the system, liquid and body, is expressible as a quadratic function of the components $\mathrm{U}, \mathrm{V}$, $\mathrm{W}, \mathrm{P}, \mathrm{Q}, \mathrm{R}$. The partial differential coefficient of T with respect to a component of velocity, linear or angular, will be the component of momentum, linear or angular, which corresponds.

Conversely, if the kinetic energy $T$ is expressed as a quadratic function of $\mathrm{x}_{1}, \mathrm{x}_{2}, \mathrm{x}_{3}, \mathrm{y}_{1}, \mathrm{y}_{2}, \mathrm{y}_{3}$, the components of momentum, the partial differential coefficient with respect to a momentum component will give the component of velocity to correspond.

These theorems, which hold for the motion of a single rigid body, are true generally for a flexible system, such as considered here for a liquid, with one or more rigid bodies swimming in it; and they express the statement that the work done by an impulse is the product of the impulse and the arithmetic mean of the initial and final velocity; so that the kinetic energy is the work done by the impulse in starting the motion from rest.

Thus if T is expressed as a quadratic function of $\mathrm{U}, \mathrm{V}, \mathrm{W}, \mathrm{P}, \mathrm{Q}, \mathrm{R}$, the components of momentum corresponding are

$$
\begin{align*}
& \mathrm{x}_{1}=\frac{\mathrm{dT}}{\mathrm{dU}}, \mathrm{x}_{2}=\frac{\mathrm{dT}}{\mathrm{dV}}, \mathrm{x}_{3}=\frac{\mathrm{dT}}{\mathrm{dW}}  \tag{1}\\
& \mathrm{y}_{1}=\frac{\mathrm{dT}}{\mathrm{dP}}, \mathrm{y}_{2}=\frac{\mathrm{dT}}{\mathrm{dQ}}, \mathrm{y}_{3}=\frac{\mathrm{dT}}{\mathrm{dR}}
\end{align*}
$$

but when it is expressed as a quadratic function of $\mathrm{x}_{1}, \mathrm{x}_{2}, \mathrm{x}_{3}, \mathrm{y}_{1}, \mathrm{y}_{2}, \mathrm{y}_{3}$,

$$
\begin{align*}
& \mathrm{U}=\frac{\mathrm{dT}}{\mathrm{dx}_{1}}, \mathrm{~V}=\frac{\mathrm{dT}}{\mathrm{dx}_{2}}, \mathrm{~W}=\frac{\mathrm{dT}}{\mathrm{dx}_{3}},  \tag{2}\\
& \mathrm{P}=\frac{\mathrm{dT}}{\mathrm{dy}_{1}}, \mathrm{Q}=\frac{\mathrm{dT}}{\mathrm{dy}_{2}}, \mathrm{R}=\frac{\mathrm{dT}}{\mathrm{dy}_{3}} .
\end{align*}
$$

The second system of expression was chosen by Clebsch and adopted by Halphen in his Fonctions elliptiques; and thence the dynamical equations follow

$$
\begin{gather*}
X=\frac{d x_{1}}{d t}-x_{2} \frac{d T}{d y_{3}}+x_{3} \frac{d T}{d y_{2}}, Y=\ldots, Z=\ldots,  \tag{3}\\
L=\frac{d y_{1}}{d t}-y_{2} \frac{d T}{d y_{3}}+y_{3} \frac{d T}{d y_{2}}-x_{2} \frac{d T}{d x_{3}}+x_{3} \frac{d T}{d x_{2}}, M=\ldots, N=\ldots, \tag{4}
\end{gather*}
$$

where X, Y, Z, L, M, N denote components of external applied force on the body.
These equations are proved by taking a line fixed in space, whose direction cosines are $l, m$, $n$, then

$$
\begin{equation*}
\frac{\mathrm{dl}}{\mathrm{dt}}=\mathrm{mR}-\mathrm{nQ}, \quad \frac{\mathrm{dm}}{\mathrm{dt}}=\mathrm{nP}-\mathrm{lR}, \quad \frac{\mathrm{dn}}{\mathrm{dt}}=1 \mathrm{Q}-\mathrm{mP} . \tag{5}
\end{equation*}
$$

If $P$ denotes the resultant linear impulse or momentum in this direction

$$
\begin{gather*}
P=\mathrm{lx}_{1}+\mathrm{mx}_{2}+\mathrm{nx}_{3},  \tag{6}\\
\frac{\mathrm{dP}}{\mathrm{dt}}=\frac{\mathrm{dl}}{\mathrm{dt}} \mathrm{x}_{1}+\frac{\mathrm{dm}}{\mathrm{dt}} \mathrm{x}_{2}+\frac{\mathrm{dn}}{\mathrm{dt}} \mathrm{x}_{3} \\
+\mathrm{l} \frac{\mathrm{dx}}{\mathrm{dt}}+\mathrm{m} \frac{d x_{2}}{\mathrm{dt}}+\mathrm{n} \frac{\mathrm{dx}}{\mathrm{dt}}, \\
=\mathrm{l}\left(\frac{d x_{1}}{\mathrm{dt}}-\mathrm{x}_{2} \mathrm{R}+\mathrm{x}_{3} \mathrm{Q}\right) \\
+\mathrm{m}\left(\frac{d x_{2}}{d t}-\mathrm{x}_{3} \mathrm{P}+\mathrm{x}_{1} \mathrm{R}\right) \\
+\mathrm{n}\left(\frac{d x_{3}}{\mathrm{dt}}-\mathrm{x}_{1} \mathrm{Q}+\mathrm{x}_{2} \mathrm{P}\right) \\
=\mathrm{lX}+m Y+\mathrm{nZ}, \tag{7}
\end{gather*}
$$

for all values of $\mathrm{l}, \mathrm{m}, \mathrm{n}$.
Next, taking a fixed origin $\Omega$ and axes parallel to $\mathrm{Ox}, \mathrm{Oy}, \mathrm{Oz}$ through O , and denoting by $\mathrm{x}, \mathrm{y}$, z the coordinates of O , and by G the component angular momentum about $\Omega$ in the direction ( $l$, $\mathrm{m}, \mathrm{n}$ )

$$
\begin{align*}
& G=1\left(y_{1}-x_{2} z+x_{3} y\right) \\
& +m\left(y_{2}-x_{3} x+x_{1} z\right) \\
& +n\left(y_{3}-x_{1} y+x_{2} x\right) . \tag{8}
\end{align*}
$$

Differentiating with respect to t , and afterwards moving the fixed origin up to the moving origin O , so that

$$
\begin{aligned}
& \mathrm{x}=\mathrm{y}=\mathrm{z}=0, \text { but } \frac{\mathrm{dx}}{\mathrm{dt}}=\mathrm{U}, \frac{\mathrm{dy}}{\mathrm{dt}}=\mathrm{V}, \frac{\mathrm{dz}}{\mathrm{dt}}=\mathrm{W}, \\
& \mathrm{dG} \quad \mathrm{dy}_{1}
\end{aligned}
$$

$$
\begin{gather*}
\overline{\mathrm{dt}}=1\left(\overline{\mathrm{dt}}-\mathrm{y}_{2} \mathrm{R}+\mathrm{y}_{3} \mathrm{Q}-\mathrm{x}_{2} \mathrm{~W}+\mathrm{x}_{3} \mathrm{~V}\right) \\
+\mathrm{m}\left(\frac{\mathrm{dy}}{2} \mathrm{dt}\right. \\
\left.+y_{3} P+y_{1} R-x_{3} \mathrm{U}+\mathrm{x}_{1} \mathrm{~W}\right) \\
\left(\frac{d y_{3}}{\mathrm{dt}}-\mathrm{y}_{1} \mathrm{Q}+\mathrm{y}_{2} \mathrm{P}-\mathrm{x}_{1} \mathrm{~V}+\mathrm{x}_{2} \mathrm{U}\right)  \tag{9}\\
=\mathrm{l} L+\mathrm{mM}+\mathrm{nN},
\end{gather*}
$$

for all values of $\mathrm{l}, \mathrm{m}, \mathrm{n}$.
When no external force acts, the case which we shall consider, there are three integrals of the equations of motion
(i.) $\mathrm{T}=$ constant,
(ii.) $\mathrm{x}_{1}^{2}+\mathrm{x}_{2}^{2}+\mathrm{x}_{3}^{2}=\mathrm{F}^{2}$, a constant,
(iii.) $\mathrm{x}_{1} \mathrm{y}_{1}+\mathrm{x}_{2} \mathrm{y}_{2}+\mathrm{x}_{3} \mathrm{y}_{3}=\mathrm{n}=\mathrm{GF}$, a constant;
and the dynamical equations in (3) express the fact that $x_{1}, x_{2}, x_{3}$ are the components of a constant vector having a fixed direction; while (4) shows that the vector resultant of $y_{1}, y_{2}, y_{3}$ moves as if subject to a couple of components

$$
\begin{equation*}
x_{2} W-x_{3} V, x_{3} U-x_{1} W, x_{1} V-x_{2} U, \tag{10}
\end{equation*}
$$

and the resultant couple is therefore perpendicular to $F$, the resultant of $\mathrm{x}_{1}, \mathrm{x}_{2}, \mathrm{x}_{3}$, so that the component along OF is constant, as expressed by (iii).

If a fourth integral is obtainable, the solution is reducible to a quadrature, but this is not possible except in a limited series of cases, investigated by H. Weber, F. Kötter, R. Liouville, Caspary, Jukovsky, Liapounoff, Kolosoff and others, chiefly Russian mathematicians; and the general solution requires the double-theta hyperelliptic function.
49. In the motion which can be solved by the elliptic function, the most general expression of the kinetic energy was shown by A. Clebsch to take the form

$$
\begin{align*}
& \mathrm{T}=1 / 2 \mathrm{p}\left(\mathrm{x}_{1}^{2}+\mathrm{x}_{2}^{2}\right)+1 / 2 \mathrm{p}^{\prime} \mathrm{x}_{3}{ }^{2} \\
& +\mathrm{q}\left(\mathrm{x}_{1} \mathrm{y}_{1}+\mathrm{x}_{2} \mathrm{y}_{2}\right)+\mathrm{q}^{\prime} \mathrm{x}_{3} \mathrm{y}_{3} \\
&  \tag{1}\\
& +1 / 2 \mathrm{r}\left(\mathrm{y}_{1}^{2}+\mathrm{y}_{2}^{2}\right)+1 / 2 \mathrm{r}^{\prime} \mathrm{y}_{3}^{2}
\end{align*}
$$

so that a fourth integral is given by

$$
\begin{gather*}
\mathrm{dy}_{3} / \mathrm{dt}=0, \mathrm{y}_{3}=\text { constant } ; \\
\frac{\mathrm{dx}_{3}}{\mathrm{dt}}=\mathrm{x}_{1}\left(\mathrm{qx}_{2}+\mathrm{ry}_{2}\right)-\mathrm{x}_{2}\left(\mathrm{qx}_{1}+\mathrm{ry}_{1}\right)=\mathrm{r}\left(\mathrm{x}_{1} \mathrm{y}_{2}-\mathrm{x}_{2} \mathrm{y}_{1}\right),  \tag{2}\\
\frac{1}{\mathrm{r}^{2}}\left(\frac{\mathrm{dx}}{\mathrm{~d}}{ }_{\mathrm{dt}}\right)^{2}=\left(\mathrm{x}_{1}{ }^{2}+\mathrm{x}_{2}^{2}\right)\left(\mathrm{y}_{1}{ }^{2}+\mathrm{y}_{2}^{2}\right)-\left(\mathrm{x}_{1} \mathrm{y}_{1}+\mathrm{x}_{2} \mathrm{y}_{2}\right)^{2}  \tag{3}\\
=\left(\mathrm{x}_{1}{ }^{2}+\mathrm{x}_{2}^{2}\right)\left(\mathrm{y}_{1}{ }^{2}+\mathrm{y}_{2}^{2}\right)-\left(\mathrm{FG}-\mathrm{x}_{3} \mathrm{y}_{3}\right)^{2} \\
=\left(\mathrm{x}_{1}{ }^{2}+\mathrm{x}_{2}^{2}\right)\left(\mathrm{y}_{1}{ }^{2}+\mathrm{y}_{2}{ }^{2}+\mathrm{y}_{3}^{2}-\mathrm{G}^{2}\right)-\left(\mathrm{Gx}_{3}-\mathrm{Fy}\right)^{2}, \tag{4}
\end{gather*}
$$

in which

$$
\begin{align*}
x_{1}^{2}+x_{2}^{2} & =F^{2}-x_{3}^{2}, x_{1} y_{1}+x_{2} y_{2}=F G-x_{3} y_{3}  \tag{5}\\
r\left(y_{1}^{2}+\right. & \left.y_{2}^{2}\right)=2 T-p\left(x_{1}^{2}+x_{2}^{2}\right)-p^{\prime} x_{3}^{2} \\
& -2 q\left(x_{1} y_{1}+x_{2} y_{2}\right)-2 q^{\prime} x_{3} y_{3}-r^{\prime} y_{3}^{2} \\
& =\left(p-p^{\prime}\right) x_{3}^{2}+2\left(q-q^{\prime}\right) x_{3} y_{3}+m_{1}  \tag{6}\\
m_{1} & -2 T-p F^{2}-2 q F G-r_{1} y_{3}^{2} \tag{7}
\end{align*}
$$

so that

$$
\begin{equation*}
\frac{1}{\mathrm{r}^{2}}\left(\frac{\mathrm{dx}_{3}}{\mathrm{dt}}\right)^{2}=\mathrm{X}_{3} \tag{8}
\end{equation*}
$$

where $X_{3}$ is a quartic function of $x_{3}$, and thus $t$ is given by an elliptic integral of the first kind; and by inversion $x_{3}$ is in elliptic function of the time $t$. Now

$$
\begin{equation*}
\left(\mathrm{x}_{1}-\mathrm{x}_{2} \mathrm{i}\right)\left(\mathrm{y}_{1}+\mathrm{y}_{2} \mathrm{i}\right)=\mathrm{x}_{1} \mathrm{y}_{1}+\mathrm{x}_{2} \mathrm{y}_{2}+\mathrm{i}\left(\mathrm{x}_{1} \mathrm{y}_{2}-\mathrm{x}_{2} \mathrm{y}_{1}\right)=\mathrm{FG}-\mathrm{xy}_{3} \mathrm{y}_{3}+\mathrm{i} \sqrt{ } \mathrm{X}_{3}, \tag{9}
\end{equation*}
$$

$$
\begin{gather*}
\frac{y_{1}+y_{2} \mathrm{i}}{x_{1}+x_{2} i}=\frac{F G-x_{3} y_{3}+i \sqrt{ } X_{3}}{x_{1}{ }^{2}+x_{2}{ }^{2}},  \tag{10}\\
\frac{d}{d t}\left(x_{1}+x_{2} i\right)=-i\left[\left(q^{\prime}-q\right) x_{3}+r^{\prime} y_{3}\right]+\operatorname{irx}_{3}\left(y_{1}+y_{2} i\right),  \tag{11}\\
\frac{d}{d t i} \log \left(x_{1}+x_{2} i\right)=-\left(q^{\prime}-q\right) x_{3}-r^{\prime} y_{3}+r x_{3} \frac{F G-x_{3} y_{3}+i \sqrt{ } x_{3}}{F^{2}-x_{3}{ }^{2}},  \tag{12}\\
\frac{d}{d t i} \log \sqrt{ } \frac{x_{1}+x_{2} i}{x_{1}-x_{2} i}=-\left(q^{\prime}-q\right) x_{3}-\left(r^{\prime}-r\right) y_{3}-F r \frac{F y_{3}-G x_{3}}{F^{2}-x_{3}{ }^{2}}, \tag{13}
\end{gather*}
$$

requiring the elliptic integral of the third kind; thence the expression of $\mathrm{x}_{1}+\mathrm{x}_{2} \mathrm{i}$ and $\mathrm{y}_{1}+\mathrm{y}_{2} \mathrm{i}$. Introducing Euler's angles $\theta, \varphi, \psi$,

$$
\begin{align*}
& \mathrm{x}_{1}=\mathrm{F} \sin \theta \sin \varphi, \quad \mathrm{x}_{2}=\mathrm{F} \sin \theta \cos \varphi \text {, } \\
& \mathrm{x}_{1}+\mathrm{x}_{2} \mathrm{i}=\mathrm{iF} \sin \theta \varepsilon^{-\psi \mathrm{i}}, \quad \mathrm{x}_{3}=\mathrm{F} \cos \theta \text {; }  \tag{14}\\
& \sin \theta \frac{d \psi}{d t}=P \sin \varphi+Q \cos \varphi,  \tag{15}\\
& F \sin ^{2} \theta \frac{d \psi}{d t}=\frac{d T}{d y_{1}} x_{1}+\frac{d T}{d y_{2}} x_{2} \\
& =\left(\mathrm{qx}_{1}+r \mathrm{y}_{1}\right) \mathrm{x}_{1}+\left(\mathrm{qx}_{2}+\mathrm{ry}_{2}\right) \mathrm{x}_{2} \\
& =q\left(x_{1}^{2}+x_{2}^{2}\right)+r\left(x_{1} y_{1}+x_{2} y_{2}\right) \\
& =g F^{2} \sin ^{2} \theta+r\left(F G-x_{3} y_{3}\right) \text {, }  \tag{16}\\
& \psi-\mathrm{qFt}=\int \frac{\mathrm{FG}-\mathrm{x}_{3} \mathrm{y}_{3}}{\mathrm{~F}^{2}-\mathrm{x}_{3}{ }^{2}} \frac{\mathrm{Frdx}}{\sqrt{2}} \mathrm{X}_{3}, \tag{17}
\end{align*}
$$

elliptic integrals of the third kind.
Employing G. Kirchhoff's expressions for X, Y, Z, the coordinates of the centre of the body,

$$
\begin{gather*}
\mathrm{FX}=\mathrm{y}_{1} \cos \overline{\mathrm{xY}}+\mathrm{y}_{2} \cos \overline{\mathrm{yY}}+\mathrm{y}_{3} \cos \overline{\mathrm{zY}},  \tag{18}\\
\mathrm{FY}=-\mathrm{y}_{1} \cos \overline{\mathrm{xX}}+\mathrm{y}_{2} \cos \overline{\mathrm{yX}}+\mathrm{y}_{3} \cos \overline{\mathrm{zX}},  \tag{19}\\
\mathrm{G}=\mathrm{y}_{1} \cos \overline{\mathrm{xZ}}+\mathrm{y}_{2} \cos \overline{\mathrm{yZ}}+\mathrm{y}_{3} \cos \overline{\mathrm{zZ}},  \tag{20}\\
\mathrm{~F}^{2}\left(\mathrm{X}^{2}+\mathrm{Y}^{2}\right)=\mathrm{y}_{1}^{2}+\mathrm{y}_{2}^{2}+\mathrm{y}_{3}^{2}-G^{2},  \tag{21}\\
\mathrm{~F}(\mathrm{X}+\mathrm{Yi})=\frac{\mathrm{Fy}_{3}-\mathrm{Gx}_{3}+\mathrm{i} \sqrt{ } \mathrm{X}_{3}}{\sqrt{ }\left(\mathrm{~F}^{2}-\mathrm{x}_{3}^{2}\right)} \varepsilon^{\psi \mathrm{i}} . \tag{22}
\end{gather*}
$$

Suppose $x 3-F$ is a repeated factor of $X_{3}$, then $y_{3}=G$, and

$$
\begin{equation*}
X_{3}=\left(x_{3}-F\right)^{2}\left[\frac{p^{\prime}-p}{r}\left(x_{3}+F\right)^{2}+2 \frac{q^{\prime}-q}{r} G\left(x_{3}+F\right)-G^{2}\right], \tag{23}
\end{equation*}
$$

and putting $\mathrm{x}_{3}-\mathrm{F}=\mathrm{y}$,

$$
\left(\frac{d y}{d t}\right)^{2}=r^{2} y^{2}\left[4 \frac{p^{\prime}-p}{r} F^{2}+4 \frac{q^{\prime}-q}{r} F G-G^{2}+2\left(2 \frac{p^{\prime}-p}{r} F+\frac{q^{\prime}-q}{r} G\right) y+\frac{p^{\prime}-p}{(24)} y^{2}\right]
$$

so that the stability of this axial movement is secured if

$$
\begin{equation*}
\mathrm{A}=4 \frac{\mathrm{p}^{\prime}-\mathrm{p}}{\mathrm{r}} \mathrm{~F}^{2}+4 \frac{\mathrm{q}^{\prime}-\mathrm{q}}{\mathrm{r}} \mathrm{FG}-\mathrm{G}^{2} \tag{25}
\end{equation*}
$$

is negative, and then the axis makes $\mathrm{r} \sqrt{ }(-\mathrm{A}) / \Pi$ nutations per second. Otherwise, if A is positive

$$
\begin{gather*}
r t=\int \frac{d y}{y \sqrt{ }\left(A+2 B y+C y^{2}\right)} \\
=\frac{1}{\sqrt{ } A} \mathrm{sh}^{-1}  \tag{26}\\
\mathrm{ch}^{-1}
\end{gather*} \frac{\sqrt{ } \mathrm{~A} \sqrt{ }\left(\mathrm{~A}+2 \mathrm{By}+\mathrm{Cy}^{2}\right)}{\mathrm{y} \sqrt{ }\left(\mathrm{~B}^{2} \sim \mathrm{AC}\right)}=\frac{1}{\sqrt{ } \mathrm{~A}} \quad \mathrm{ch}^{-1} \quad \operatorname{sh}^{-1} \quad \frac{\mathrm{~A}+\mathrm{By}}{\mathrm{y} \sqrt{ }\left(\mathrm{~B}^{2} \sim \mathrm{AC}\right)},
$$

and the axis falls away ultimately from its original direction.
A number of cases are worked out in the American Journal of Mathematics (1907), in which the motion is made algebraical by the use of the pseudo-elliptic integral. To give a simple instance, changing to the stereographic projection by putting $\tan 1 / 2 \theta=x$,

$$
\begin{equation*}
\left(\mathrm{Nx} \mathrm{e}^{\psi i}\right)^{3 / 2}=(\mathrm{x}+1) \sqrt{ } \mathrm{X}_{1}+\mathrm{i}(\mathrm{x}-1) \sqrt{ } \mathrm{X}_{2}, \tag{27}
\end{equation*}
$$

$$
\begin{gather*}
\frac{X_{1}}{X_{2}}= \pm a x^{4}+2 a x^{3} \pm 3(a+b) x^{2}+2 b x \pm b  \tag{28}\\
N^{3}=-8(a+b) \tag{29}
\end{gather*}
$$

will give a possible state of motion of the axis of the body; and the motion of the centre may then be inferred from (22).
50. The theory preceding is of practical application in the investigation of the stability of the axial motion of a submarine boat, of the elongated gas bag of an airship, or of a spinning rifled projectile. In the steady motion under no force of such a body in a medium, the centre of gravity describes a helix, while the axis describes a cone round the direction of motion of the centre of gravity, and the couple causing precession is due to the displacement of the medium.

In the absence of a medium the inertia of the body to translation is the same in all directions, and is measured by the weight W , and under no force the C.G. proceeds in a straight line, and the axis of rotation through the C.G. preserves its original direction, if a principal axis of the body; otherwise the axis describes a cone, right circular if the body has uniaxial symmetry, and a Poinsot cone in the general case.

But the presence of the medium makes the effective inertia depend on the direction of motion with respect to the external shape of the body, and on $\mathrm{W}^{\prime}$ the weight of fluid medium displaced.

Consider, for example, a submarine boat under water; the inertia is different for axial and broadside motion, and may be represented by

$$
\begin{equation*}
\mathrm{c}_{1}=\mathrm{W}+\mathrm{W}^{\prime} \alpha, \quad \mathrm{c}_{2}=\mathrm{W}+\mathrm{W}^{\prime} \beta, \tag{1}
\end{equation*}
$$

where $\alpha, \beta$ are numerical factors depending on the external shape; and if the C.G. is moving with velocity V at an angle $\varphi$ with the axis, so that the axial and broadside component of velocity is $\mathrm{u}=\mathrm{V} \cos \varphi, \mathrm{v}=\mathrm{V} \sin \varphi$, the total momentum F of the medium, represented by the vector OF at an angle $\theta$ with the axis, will have components, expressed in sec. H ,

$$
\begin{equation*}
F \cos \theta=c_{1} \frac{u}{g}=\left(W+W^{\prime} \alpha\right) \frac{V}{g} \cos \varphi, F \sin \theta=c_{2} \frac{v}{g}=\left(W+W^{\prime} \beta\right) \frac{V}{g} . \tag{2}
\end{equation*}
$$

Suppose the body is kept from turning as it advances; after $t$ seconds the C.G. will have moved from O to $\mathrm{O}^{\prime}$, where $\mathrm{OO}^{\prime}=\mathrm{Vt}$; and at $\mathrm{O}^{\prime}$ the momentum is the same in magnitude as before, but its vector is displaced from OF to $\mathrm{O}^{\prime} \mathrm{F}^{\prime}$.

For the body alone the resultant of the components of momentum

$$
\begin{equation*}
\mathrm{W} \frac{\mathrm{~V}}{\mathrm{~g}} \cos \varphi \text { and } \mathrm{W} \frac{\mathrm{~V}}{\mathrm{~g}} \sin \varphi \text { is } \mathrm{W} \frac{\mathrm{~V}}{\mathrm{~g}} \sec . \mathrm{t} \text {, } \tag{3}
\end{equation*}
$$

acting along $\mathrm{OO}^{\prime}$, and so is unaltered.
But the change of the resultant momentum F of the medium as well as of the body from the vector OF to $\mathrm{O}^{\prime} \mathrm{F}^{\prime}$ requires an impulse couple, tending to increase the angle FOO ', of magnitude, in sec. foot-pounds

$$
\begin{equation*}
\mathrm{F} \cdot \mathrm{OO}^{\prime} \cdot \sin \mathrm{FOO}^{\prime}=\mathrm{FVt} \sin (\theta-\varphi), \tag{4}
\end{equation*}
$$

equivalent to an incessant couple

$$
\begin{align*}
& N=F V \sin (\theta-\varphi) \\
& =(F \sin \theta \cos \varphi-F \cos \theta \sin \varphi) V \\
& =\left(c_{2}-c_{1}\right)\left(V^{2} / g\right) \sin \varphi \cos \varphi \\
& =W^{\prime}(\beta-\alpha) u v / g . \tag{5}
\end{align*}
$$

This N is the couple in foot-pounds changing the momentum of the medium, the momentum of the body alone remaining the same; the medium reacts on the body with the same couple N in the opposite direction, tending when $\mathrm{c}_{2}-\mathrm{c}_{1}$ is positive to set the body broadside to the advance.

An oblate flattened body, like a disk or plate, has $\mathrm{c}_{2}-\mathrm{c}_{1}$ negative, so that the medium steers the body axially; this may be verified by a plate dropped in water, and a leaf or disk or rocketstick or piece of paper falling in air. A card will show the influence of the couple N if projected with a spin in its plane, when it will be found to change its aspect in the air.
An elongated body like a ship has $\mathrm{c}_{2}-\mathrm{c}_{1}$ positive, and the couple N tends to disturb the axial movement and makes it unstable, so that a steamer requires to be steered by constant attention at the helm.

Consider a submarine boat or airship moving freely with the direction of the resultant momentum horizontal, and the axis at a slight inclination $\theta$. With no reserve of buoyancy $\mathrm{W}=$ $\mathrm{W}^{\prime}$, and the couple N , tending to increase $\theta$, has the effect of diminishing the metacentric height by h ft. vertical, where

$$
\begin{gather*}
\mathrm{Wh} \tan \theta=\mathrm{N}=\left(\mathrm{c}_{2}-\mathrm{c}_{1}\right) \frac{\mathrm{c}_{1}}{\mathrm{c}_{2}} \frac{\mathrm{u}^{2}}{\mathrm{~g}} \tan \theta,  \tag{6}\\
\mathrm{~h}=\frac{\mathrm{c}_{2}-\mathrm{c}_{1}}{\mathrm{~W}} \frac{\mathrm{c}_{1}}{\mathrm{c}_{2}} \frac{\mathrm{u}^{2}}{\mathrm{~g}}=(\beta-\alpha) \frac{1+\alpha}{1+\beta} \frac{\mathrm{u}^{2}}{\mathrm{~g}} . \tag{7}
\end{gather*}
$$

51. An elongated shot is made to preserve its axial flight through the air by giving it the spin sufficient for stability, without which it would turn broadside to its advance; a top in the same way is made to stand upright on the point in the position of equilibrium, unstable statically but dynamically stable if the spin is sufficient; and the investigation proceeds in the same way for the two problems (see Gyroscope).

The effective angular inertia of the body in the medium is now required; denote it by $\mathrm{C}_{1}$ about the axis of the figure, and by $\mathrm{C}_{2}$ about a diameter of the mean section. A rotation about the axis of a figure of revolution does not set the medium in motion, so that $\mathrm{C}_{1}$ is the moment of inertia of the body about the axis, denoted by $\mathrm{Wk}_{1}{ }^{2}$. But if $\mathrm{Wk}_{2}{ }^{2}$ is the moment of inertia of the body about a mean diameter, and $\omega$ the angular velocity about it generated by an impulse couple M , and $\mathrm{M}^{\prime}$ is the couple required to set the surrounding medium in motion, supposed of effective radius of gyration $\mathrm{k}^{\prime}$,

$$
\begin{gather*}
\mathrm{Wk}_{2}^{2} \omega=\mathrm{M}-\mathrm{M}^{\prime}, \mathrm{W}^{\prime} \mathrm{k}^{\prime 2} \omega=\mathrm{M}^{\prime}  \tag{1}\\
\left(\mathrm{Wk}_{2}^{2}+\mathrm{W}^{\prime} \mathrm{k}^{\prime 2}\right) \omega=\mathrm{M}  \tag{2}\\
\mathrm{C}_{2}=\mathrm{Wk}_{2}^{2}+\mathrm{W}^{\prime} \mathrm{k}^{\prime 2}=\left(\mathrm{W}+\mathrm{W}^{\prime} \varepsilon\right) \mathrm{k}_{2}^{2} \tag{3}
\end{gather*}
$$

in which we have put $\mathrm{k}^{\prime 2}=\varepsilon \mathrm{k}^{2}$, where $\varepsilon$ is a numerical factor depending on the shape.
If the shot is spinning about its axis with angular velocity $p$, and is preceding steadily at a rate $\mu$ about a line parallel to the resultant momentum F at an angle $\theta$, the velocity of the vector of angular momentum, as in the case of a top, is

$$
\begin{equation*}
\mathrm{C}_{1} \mathrm{p} \mu \sin \theta-\mathrm{C}_{2} \mu^{2} \sin \theta \cos \theta \tag{4}
\end{equation*}
$$

and equating this to the impressed couple (multiplied by g ), that is, to

$$
\begin{equation*}
\mathrm{gN}=\left(\mathrm{c}_{1}-\mathrm{c}_{2}\right) \frac{\mathrm{c}_{1}}{\mathrm{c}_{2}} \mathrm{u}^{2} \tan \theta, \tag{5}
\end{equation*}
$$

and dividing out $\sin \theta$, which equated to zero would imply perfect centring, we obtain

$$
\begin{equation*}
C_{2} \mu^{2} \cos \theta-C_{1} p \mu+\left(c_{2}-c_{1}\right) \frac{c_{1}}{c_{2}} u^{2} \sec \theta=0 . \tag{6}
\end{equation*}
$$

The least admissible value of $p$ is that which makes the roots equal of this quadratic in $\mu$, and then

$$
\begin{equation*}
\mu=1 / 2 \frac{C_{1}}{C_{2}} p \sec \theta, \tag{7}
\end{equation*}
$$

the roots would be imaginary for a value of p smaller than given by

$$
\begin{gather*}
\mathrm{C}_{1}^{2} \mathrm{p}^{2}-4\left(\mathrm{c}_{2}-\mathrm{c}_{1}\right) \frac{\mathrm{c}_{1}}{\mathrm{c}_{2}} \mathrm{C}_{2} \mathrm{u}^{2}=0  \tag{8}\\
\frac{\mathrm{p}^{2}}{\mathrm{u}^{2}}=4\left(\mathrm{c}_{2}-\mathrm{c}_{1}\right) \frac{\mathrm{c}_{1}}{\mathrm{c}_{2}} \frac{\mathrm{C}_{2}}{\mathrm{C}_{1}{ }^{2}} \tag{9}
\end{gather*}
$$

Table of Rifling for Stability of an Elongated Projectile, x Calibres long, giving $\delta$ the Angle of Rifling, and $n$ the Pitch of Rifling in Calibres.

|  |  | $\begin{gathered} \text { Cast-iron Common Shell } \\ f=2 / 3, \text { S.G. } 7.2 . \end{gathered}$ |  | Palliser Shell $f=1 / 2$, S.G. 8 . |  | Solid Steel Bullet $f=0$, S.G. 8 . |  | Solid Lead Bullet$\mathrm{f}=0 \text {, S.G. } 10.9 .$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| x | $\beta-\alpha$ | $\delta$ | n | $\delta$ | n | $\delta$ | n | $\delta$ | n |
| 1.0 | 0.0000 | $0^{\circ} 0^{\prime}$ | Infinity | $0^{\circ} 0^{\prime}$ | Infinity | $0^{\circ} \quad 0^{\prime}$ | Infinity | $0^{\circ} 0^{\prime}$ | Infinity |
| 2.0 | 0.4942 | 249 | 63.87 | 232 | 71.08 | 229 | 72.21 | 208 | 84.29 |
| 2.5 | 0.6056 | 346 | 47.91 | 323 | 53.32 | 319 | 54.17 | 251 | 63.24 |


| 3.0 | 0.6819 | 4 | 41 | 38.45 | 4 | 13 | 42.79 | 4 | 09 | 43.47 | 3 | 38 | 50.74 |
| :---: | :---: | ---: | :---: | :---: | ---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3.5 | 0.7370 | 5 | 35 | 32.13 | 5 | 02 | 35.75 | 4 | 58 | 36.33 | 4 | 15 | 42.40 |
| 4.0 | 0.7782 | 6 | 30 | 27.60 | 5 | 51 | 30.72 | 5 | 45 | 31.21 | 4 | 56 | 36.43 |
| 4.5 | 0.8100 | 7 | 24 | 24.20 | 6 | 40 | 26.93 | 6 | 32 | 27.36 | 5 | 37 | 31.94 |
| 5.0 | 0.8351 | 8 | 16 | 21.56 | 7 | 28 | 23.98 | 7 | 21 | 24.36 | 6 | 18 | 28.44 |
| 6.0 | 0.8721 | 10 | 05 | 17.67 | 9 | 04 | 19.67 | 8 | 56 | 19.98 | 7 | 40 | 23.33 |
| 10.0 | 0.9395 | 16 | 57 | 10.31 | 15 | 19 | 11.47 | 15 | 05 | 11.65 | 13 | 00 | 13.60 |
| Infinity | 1.0000 | 90 | 00 | 0.00 | 90 | 00 | 0.00 | 90 | 00 | 0.00 | 90 | 00 | 0.00 |

If the shot is moving as if fired from a gun of calibre d inches, in which the rifling makes one turn in a pitch of $n$ calibres or nd inches, so that the angle $\delta$ of the rifling is given by

$$
\begin{equation*}
\tan \delta=\pi d / n d=1 / 2 d p / u, \tag{10}
\end{equation*}
$$

which is the ratio of the linear velocity of rotation $1 / 2 \mathrm{dp}$ to $u$, the velocity of advance,

$$
\begin{gather*}
\tan ^{2} \delta=\frac{\mathrm{n}^{2}}{\mathrm{n}^{2}}=\frac{\mathrm{d}^{2} \mathrm{p}^{2}}{4 \mathrm{u}^{2}}=\left(\mathrm{c}_{2}-\mathrm{c}_{1}\right) \frac{\mathrm{c}_{1}}{\mathrm{c}_{2}} \frac{\mathrm{C}_{2} \mathrm{~d}^{2}}{\mathrm{C}_{1}{ }^{2}} \\
=\frac{\mathrm{W}^{\prime}}{\mathrm{W}}(\beta-\alpha) \frac{1+\frac{\mathrm{W}^{\prime}}{\mathrm{W}} \alpha}{1+\frac{\mathrm{W}^{\prime}}{\mathrm{W}} \beta} \cdot \frac{\left(1+\frac{\mathrm{W}^{\prime}}{\mathrm{W}} \varepsilon\right)\left(\frac{\mathrm{k}_{1}}{\mathrm{~d}}\right)^{2}}{\left(\frac{\mathrm{k}_{1}}{\mathrm{~W}}\right)^{4}} . \tag{11}
\end{gather*}
$$

For a shot in air the ratio $\mathrm{W}^{\prime} / \mathrm{W}$ is so small that the square may be neglected, and formula (11) can be replaced for practical purpose in artillery by

$$
\begin{equation*}
\tan ^{2} \delta=\frac{\Pi^{2}}{\mathrm{n}^{2}}=\frac{\mathrm{W}^{\prime}}{\mathrm{W}}(\beta-\alpha)\left(\frac{\mathrm{k}_{2}}{\mathrm{~d}}\right)^{2} /\left(\frac{\mathrm{k}_{1}}{\mathrm{~d}}\right)^{4}, \tag{12}
\end{equation*}
$$

if then we can calculate $\beta$, $\alpha$, or $\beta-\alpha$ for the external shape of the shot, this equation will give the value of $\delta$ and $n$ required for stability of flight in the air.

The ellipsoid is the only shape for which $\alpha$ and $\beta$ have so far been determined analytically, as shown already in § 44, so we must restrict our calculation to an egg-shaped bullet, bounded by a prolate ellipsoid of revolution, in which, with $b=c$,

$$
\begin{gather*}
A_{0}=\int_{0}^{\infty} \frac{a b^{2} d \lambda}{\left(a^{2}+\lambda\right) \sqrt{ }\left[4\left(a^{2}+\lambda\right)\left(b^{2}+\lambda\right)^{2}\right]}=\int_{0}^{\infty} \frac{a b^{2} d \lambda}{2\left(a^{2}+\lambda\right)^{3 / 2}\left(b^{2}+\lambda\right)}  \tag{13}\\
A_{0}+2 B_{0}=1 \\
a=\frac{A_{0}}{1-A_{0}}, \beta=\frac{B_{0}}{1-B_{0}}=\frac{1-A_{0}}{1+A_{0}}=\frac{1}{1+2 \alpha} . \tag{14}
\end{gather*}
$$

The length of the shot being denoted by l and the calibre by d , and the length in calibres by x

$$
\begin{gather*}
l / d=2 a / 2 b=x  \tag{16}\\
A_{0}=\frac{x}{\left(x^{2}-1\right)^{3 / 2}} \operatorname{ch}^{-1} x-\frac{1}{x^{2}-1},  \tag{17}\\
2 B_{0}=\frac{-x}{\left(x^{2}-1\right)^{3 / 2}} \operatorname{ch}^{-1} x+\frac{x^{2}}{x^{2}+1}, \\
x^{2} A_{0}+2 B_{0}=\frac{x \operatorname{sh}^{-1} \sqrt{ }\left(x^{2}-1\right)}{\sqrt{ }\left(x^{2}-1\right)}=\frac{x}{\sqrt{ }\left(x^{2}-1\right)} \log \left[x+\sqrt{ }\left(x^{2}-1\right)\right] . \tag{18}
\end{gather*}
$$

If $\sigma$ denotes the density of the metal, and if the shell has a cavity homothetic with the external ellipsoidal shape, a fraction $f$ of the linear scale; then the volume of a round shot being $1 / 6 \pi d^{3}$, and $1 / 6 \pi d^{3} x$ of a shot $x$ calibres long

$$
\begin{gather*}
\mathrm{W}=1 / 6 \Pi d^{3} \times\left(i-f^{3}\right) \sigma \\
\mathrm{Wk}_{1}^{2}=1 / 6 \pi d^{3} \times \frac{d^{2}}{10}\left(1-f^{5}\right) \sigma  \tag{20}\\
\mathrm{Wk}_{2}^{2}=1 / 6 \Pi d^{3} \times \frac{l^{2}+d^{2}}{20}\left(1-f^{5}\right) \sigma \tag{21}
\end{gather*}
$$

If $\rho$ denotes the density of the air or medium

$$
\begin{align*}
\mathrm{W}^{\prime} & =1 / 6 \pi d^{3} \mathrm{x} \rho \\
\frac{\mathrm{~W}^{\prime}}{\mathrm{W}} & =\frac{1}{1-f^{3}} \frac{\rho}{\sigma} \tag{23}
\end{align*}
$$

$$
\begin{gather*}
\frac{k 1^{2}}{d^{2}}=\frac{1}{10} \frac{1-f^{5}}{1-f^{3}}, \quad \frac{k 2^{2}=x^{2}+1}{k 1^{2}} \frac{2}{2}  \tag{25}\\
\tan ^{2} \delta=\frac{\rho}{\sigma}(\beta-\alpha) \frac{x^{2}+1}{1 / 5\left(1-f^{5}\right)}, \tag{26}
\end{gather*}
$$

in which $\sigma / \rho$ may be replaced by 800 times the S.G. of the metal, taking water as 800 times denser than air on the average, in round numbers, and formula (10) may be written $n \tan \delta=\pi$, or $n \delta=180$, when $\delta$ is a small angle, and given in degrees.

From this formula (26) the table following has been calculated by A. G. Hadcock, and the results are in agreement with practical experience.
52. In the steady motion the centre of the shot describes a helix, with axial velocity

$$
\begin{equation*}
u \cos \theta=v \sin \theta=\left(1+\frac{c_{1}}{c_{2}} \tan ^{2} \theta\right) u \cos \theta \approx u \sec \theta \tag{1}
\end{equation*}
$$

and transverse velocity

$$
\begin{equation*}
u \sin \theta-v \cos \theta=\left(1-\frac{c_{1}}{c_{2}}\right) u \sin \theta \approx(\beta-\alpha) u \sin \theta \tag{2}
\end{equation*}
$$

and the time of completing a turn of the spiral is $2 \pi / \mu$.
When $\mu$ has the critical value in (7),

$$
\begin{equation*}
\frac{2 \pi}{\mu}=\frac{4 \Pi}{p} \quad \frac{C_{2}}{C_{1}} \cos \theta=\frac{2 \Pi}{p}\left(x^{2}+1\right) \cos \theta, \tag{3}
\end{equation*}
$$

which makes the circumference of the cylinder on which the helix is wrapped

$$
\begin{align*}
\frac{2 \pi}{\mu}(u \sin \theta & -v \cos \theta=\frac{2 \pi u}{p}(\beta-\alpha)\left(x^{2}+1\right) \sin ^{2} \theta \cos \theta \\
& =n d(\beta-\alpha)\left(x^{2}+1\right) \sin \theta \cos \theta \tag{4}
\end{align*}
$$

and the length of one turn of the helix

$$
\begin{equation*}
\frac{2 \pi}{\mu}(u \cos \theta+v \sin \theta)=n d\left(x^{2}+1\right) ; \tag{5}
\end{equation*}
$$

thus for $\mathrm{x}=3$, the length is 10 times the pitch of the rifling.
53. The Motion of a Perforated Solid in Liquid.-In the preceding investigation, the liquid stops dead when the body is brought to rest; and when the body is in motion the surrounding liquid moves in a uniform manner with respect to axes fixed in the body, and the force experienced by the body from the pressure of the liquid on its surface is the opposite of that required to change the motion of the liquid; this has been expressed by the dynamical equations given above. But if the body is perforated, the liquid can circulate through a hole, in reentrant stream lines linked with the body, even while the body is at rest; and no reaction from the surface can influence this circulation, which may be supposed started in the ideal manner described in § 29, by the application of impulsive pressure across an ideal membrane closing the hole, by means of ideal mechanism connected with the body. The body is held fixed, and the reaction of the mechanism and the resultant of the impulsive pressure on the surface are a measure of the impulse, linear $\xi, \eta, \zeta$, and angular $\lambda, \mu, \nu$, required to start the circulation.

This impulse will remain of constant magnitude, and fixed relatively to the body, which thus experiences an additional reaction from the circulation which is the opposite of the force required to change the position in space of the circulation impulse; and these extra forces must be taken into account in the dynamical equations.

An article may be consulted in the Phil. Mag., April 1893, by G. H. Bryan, in which the analytical equations of motion are deduced of a perforated solid in liquid, from considerations purely hydrodynamical.

The effect of an external circulation of vortex motion on the motion of a cylinder has been investigated in § 29; a similar procedure will show the influence of circulation through a hole in a solid, taking as the simplest illustration a ring-shaped figure, with uniplanar motion, and denoting by $\xi$ the resultant axial linear momentum of the circulation.

As the ring is moved from $O$ to $O^{\prime}$ in time $t$, with velocity $Q$, and angular velocity $R$, the components of liquid momentum change from

$$
\alpha M^{\prime} \mathrm{U}+\xi \text { and } \beta \mathrm{M}^{\prime} \mathrm{V} \text { along } \mathrm{Ox} \text { and } \mathrm{Oy}
$$

$$
\begin{equation*}
\alpha M^{\prime} U^{\prime}+\xi \text { and } \beta M^{\prime} V^{\prime} \text { along } O^{\prime} x^{\prime} \text { and } O^{\prime} y^{\prime}, \tag{1}
\end{equation*}
$$

the axis of the ring changing from Ox to $\mathrm{O}^{\prime} \mathrm{x}^{\prime}$; and

$$
\begin{align*}
& U=Q \cos \theta, \quad V=Q \sin \theta, \\
& U^{\prime}=Q \cos (\theta-R t), \quad V^{\prime}=Q \sin (\theta-R t), \tag{2}
\end{align*}
$$

so that the increase of the components of momentum, $\mathrm{X}_{1}, \mathrm{Y}_{1}$, and $\mathrm{N}_{1}$, linear and angular, are

$$
\begin{align*}
& \mathrm{X}_{1}=\left(\alpha \mathrm{M}^{\prime} \mathrm{U}^{\prime}+\xi\right) \cos \mathrm{Rt}-\alpha \mathrm{M}^{\prime} \mathrm{U}-\xi-\beta \mathrm{M}^{\prime} \mathrm{V}^{\prime} \sin \mathrm{Rt} \\
& =(\alpha-\beta) M^{\prime} Q \sin (\theta-R t) \sin R t-\xi \text { ver Rt }  \tag{3}\\
& Y_{1}=\left(\alpha M^{\prime} U^{\prime}+\xi\right) \sin R t+\beta M^{\prime} V^{\prime} \cos R t-\beta M^{\prime} V \\
& =(\alpha-\beta) M^{\prime} Q \cos (\theta-R t) \sin R t+\xi \sin R T \text {, }  \tag{4}\\
& \mathrm{N}_{1}=\left[-\left(\alpha \mathrm{M}^{\prime} \mathrm{U}^{\prime}+\xi\right) \sin (\theta-\mathrm{Rt})+\beta \mathrm{M}^{\prime} \mathrm{V}^{\prime} \cos (\theta-\mathrm{Rt})\right] \mathrm{OO}^{\prime} \\
& =\left[-(\alpha-\beta) M^{\prime} Q \cos (\theta-R t) \sin (\theta-R t)-\xi \sin (\theta-R t)\right] Q t . \tag{5}
\end{align*}
$$

The components of force, $\mathrm{X}, \mathrm{Y}$, and N , acting on the liquid at O , and reacting on the body, are then

$$
\begin{gather*}
X=\text { lt. } X_{1} / t=(\alpha-\beta) M^{\prime} Q R \sin \theta=(\alpha-\beta) M^{\prime} V R,  \tag{6}\\
Y=\text { lt. } Y_{1} / t=(\alpha-\beta) M^{\prime} Q R \cos \theta+\xi R=(\alpha-\beta) M^{\prime} U R+\xi R, \\
Z=\text { lt. } Z_{1} / t=-(\alpha-\beta) M^{\prime} Q^{2} \sin \theta \cos \theta-\xi Q \sin \theta=\left[-(\alpha-\beta) M^{\prime} U+\xi\right] V . \tag{8}
\end{gather*}
$$

Now suppose the cylinder is free; the additional forces acting on the body are the components of kinetic reaction of the liquid

$$
\begin{equation*}
-\alpha \mathrm{M}^{\prime}\left(\frac{\mathrm{dU}}{\mathrm{dt}}-\mathrm{VR}\right), \quad-\beta \mathrm{M}^{\prime}\left(\frac{\mathrm{dV}}{\mathrm{dt}}+\mathrm{UR}\right), \quad \varepsilon \mathrm{C}^{\prime} \frac{\mathrm{dR}}{\mathrm{dt}}, \tag{9}
\end{equation*}
$$

so that its equations of motion are

$$
\begin{align*}
M\left(\frac{d U}{d t}-V R\right) & =-\alpha M^{\prime}\left(\frac{d U}{d t}-V R\right)-(\alpha-\beta) M^{\prime} V R,  \tag{10}\\
M\left(\frac{d V}{d t}+U R\right) & =-\beta M^{\prime}\left(\frac{d V}{d t}+U R\right)-(\alpha-\beta) M^{\prime} U R-\xi R,  \tag{11}\\
C \frac{d R}{d t} & =-\varepsilon C^{\prime} \frac{d R}{d t}+(\alpha-\beta) M^{\prime} U V+\xi V ; \tag{12}
\end{align*}
$$

and putting as before

$$
\begin{gather*}
M+\alpha M^{\prime}=c_{1}, \quad M+\beta M^{\prime}=c_{2}, \quad C+\varepsilon C^{\prime}=C_{3},  \tag{13}\\
c_{1} \frac{d U}{d t} c_{2} V R=0,  \tag{14}\\
c_{2} \frac{d V}{d t}+\left(c_{1} U+\xi\right) R=0,  \tag{15}\\
c_{3} \frac{d R}{d t}-\left(c_{1} U+\xi-c_{2} U\right) V=0 ; \tag{16}
\end{gather*}
$$

showing the modification of the equations of plane motion, due to the component $\xi$ of the circulation.

The integral of (14) and (15) may be written

$$
\begin{align*}
& \mathrm{c}_{1} \mathrm{U}+\xi=\mathrm{F} \cos \theta, \mathrm{c}_{2} \mathrm{~V}=-\mathrm{F} \sin \theta,  \tag{17}\\
& \frac{d x}{d t}=U \cos \theta-V \sin \theta=\frac{F \cos ^{2} \theta}{c_{1}}+\frac{F \sin ^{2} \theta}{\mathrm{c}_{2}}-\frac{\xi}{\mathrm{c}_{1}} \cos \theta,  \tag{18}\\
& \frac{\mathrm{~d} \mu}{\mathrm{dt}}=\mathrm{U} \sin \theta+\mathrm{V} \cos \theta=\left(\frac{\mathrm{F}}{\mathrm{C}_{1}}-\frac{\mathrm{F}}{\mathrm{C}_{2}}\right) \sin \theta \cos \theta-\frac{\xi}{\mathrm{C}_{1}} \sin \theta,  \tag{19}\\
& C_{3} \frac{d^{2} \theta}{\mathrm{dt}^{2}}=\left(\frac{\mathrm{F}^{2}}{\mathrm{C}_{1}}-\frac{\mathrm{F}^{2}}{\mathrm{C}_{2}}\right) \sin \theta \cos \theta-\frac{\mathrm{F} \xi}{\mathrm{C}_{1}} \sin \theta=\mathrm{F} \frac{\mathrm{~d} \mu}{\mathrm{dt}},  \tag{20}\\
& C_{3} \frac{d \theta}{d t}=F y=\sqrt{ }\left[-\frac{F^{2} \cos ^{2} \theta}{C_{1}}-\frac{F^{2} \sin ^{2} \theta}{C_{2}}+2 \frac{F \xi}{C_{1}} \cos \theta+H\right] ; \tag{21}
\end{align*}
$$

so that $\cos \theta$ and $y$ is an elliptic function of the time.
When $\xi$ is absent, $\mathrm{dx} / \mathrm{dt}$ is always positive, and the centre of the body cannot describe loops; but with $\xi$, the influence may be great enough to make dx/dt change sign, and so loops occur, as shown in A. B. Basset's Hydrodynamics, i. 192, resembling the trochoidal curves, which can be looped, investigated in $\S 29$ for the motion of a cylinder under gravity, when surrounded by a vortex.

The branch of hydrodynamics which discusses wave motion in a liquid or gas is given now in the articles Sound and Wave; while the influence of viscosity is considered under Hydraulics.

References.-For the history and references to the original memoirs see Report to the British Association, by G. G. Stokes (1846), and W. M. Hicks (1882). See also the Fortschritte der Mathematik, and A. E. H. Love, "Hydrodynamik" in the Encyklöpadie der mathematischen Wissenschaften (1901).
(A. G. G.)

HYDROMEDUSAE, a group of marine animals, recognized as belonging to the Hydrozoa (q.v.) by the following characters. (1) The polyp (hydropolyp) is of simple structure, typically much longer than broad, without ectodermal oesophagus or mesenteries, such as are seen in the anthopolyp (see article Anthozoa); the mouth is usually raised above the peristome on a short conical elevation or hypostome; the ectoderm is without cilia. (2) With very few exceptions, the polyp is not the only type of individual that occurs, but alternates in the lifecycle of a given species, with a distinct type, the medusa (q.v.), while in other cases the polypstage may be absent altogether, so that only medusa-individuals occur in the life-cycle.
The Hydromedusae represent, therefore, a sub-class of the Hydrozoa. The only other subclass is the Scyphomedusae (q.v.). The Hydromedusae contrast with the Scyphomedusae in the following points. (1) The polyp, when present, is without the strongly developed longitudinal retractor muscles, forming ridges (taeniolae) projecting into the digestive cavity, seen in the scyphistoma or scyphopolyp. (2) The medusa, when present, has a velum and is hence said to be craspedote; the nervous system forms two continuous rings running above and below the velum; the margin of the umbrella is not lobed (except in Narcomedusae) but entire; there are characteristic differences in the sense-organs (see below, and Scyphomedusae); and gastral filaments (phacellae), subgenital pits, \&c., are absent. (3) The gonads, whether formed in the polyp or the medusa, are developed in the ectoderm.

The Hydromedusae form a widespread, dominant and highly differentiated group of animals, typically marine, and found in all seas and in all zones of marine life. Fresh-water forms, however, are also known, very few as regards species or genera, but often extremely abundant as individuals. In the British fresh-water fauna only two genera, Hydra and Cordylophora, are found; in America occurs an additional genus, Microhydra. The paucity of fresh-water forms contrasts sharply, with the great abundance of marine genera common in all seas and on every shore. The species of Hydra, however, are extremely common and familiar inhabitants of ponds and ditches.

In fresh-water Hydromedusae the life-cycle is usually secondarily simplified, but in marine forms the life-cycle may be extremely complicated, and a given species often passes in the course of its history through widely different forms adapted to different habitats and modes of life. Apart from larval or embryonic forms there are found typically two types of person, as already stated, the polyp and the medusa, each of which may vary independently of the other, since their environment and life-conditions are usually quite different. Hence both polyp and medusa present characters for classification, and a given species, genus or other taxonomic category may be defined by polyp-characters or medusa-characters or by both combined. If our knowledge of the life-histories of these organisms were perfect, their polymorphism would present no difficulties to classification; but unfortunately this is far from being the case. In the majority of cases we do not know the polyp corresponding to a given medusa, or the medusa that arises from a given polyp. ${ }^{1}$ Even when a medusa is seen to be budded, from a polyp under observation in an aquarium, the difficulty is not always solved, since the freshly-liberated, immature medusa may differ greatly from the full-grown, sexually-mature medusa after several months of life on the high seas (see figs. 11, B, C, and 59, a, b, c). To establish the exact relationship it is necessary not only to breed but to rear the medusa, which cannot always be done in confinement. The alternative is to fish all stages of the medusa in its growth in the open sea, a slow and laborious method in which the chance of error is very great, unless the series of stages is very complete.

At present, therefore, classifications of the Hydromedusae have a more or less tentative character, and are liable to revision with increased knowledge of the life-histories of these organisms. Many groups bear at present two names, the one representing the group as defined by polyp-characters, the other as defined by medusa-characters. It is not even possible in all cases to be certain that the polyp-group corresponds exactly to the medusa-group, especially in minor systematic categories, such as families.

The following is the main outline of the classification that is Adopted in the present article. Groups founded on polyp-characters are printed in ordinary type, those founded on medusacharacters in italics. For definitions of the groups see below.

Sub-class Hydromedusae (Hydrozoa Craspedota). Order I. Eleutheroblastea.<br>II. Hydroidea (Leptolinae).<br>Sub-order 1. Gymnoblastea (Anthomedusae).<br>" 2. Calyptoblastea (Leptomedusae).<br>Order III. Hydrocorallinae.<br>" IV. Graptolitoidea.<br>" V. Trachylinae.<br>Sub-order 1. Trachomedusae.<br>2. Narcomedusae.<br>Order VI. Siphonophora.<br>Sub-order 1. Chondrophorida.<br>" 2. Calycophorida.<br>" 3. Physophorida.<br>" 4. Cystophorida.

## Organization and Morphology of the Hydromedusae.

As already stated, there occur in the Hydromedusae two distinct types of person, the polyp and the medusa; and either of them is capable of non-sexual reproduction by budding, a process which may lead to the formation of colonies, composed of more or fewer individuals combined and connected together. The morphology of the group thus falls naturally into four sections-(1) the hydropolyp, (2) the polyp-colony, (3) the hydromedusa, (4) the medusa-colonies. Since, however, medusa-colonies occur only in one group, the Siphonophora, and divergent views are held with regard to the morphological interpretation of the members of a siphonophore, only the first three of the above subdivisions of hydromedusa morphology will be dealt with here in a general way, and the morphology of the Siphonophora will be considered under the heading of the group itself.

1. The Hydropolyp (fig. 1)-The general characters of this organism are described above and in the articles Hydrozoa and Polyp. It is rarely free, but usually fixed and incapable of locomotion. The foot by which it is attached often sends out root-like processes-the hydrorhiza (c). The column (b) is generally long, slender and stalk-like (hydrocaulus). Just below the crown of tentacles, however, the body widens out to form a "head," termed, the hydranth (a), containing a stomach-like dilatation of the digestive cavity. On the upper face of the hydranth the crown of tentacles ( $t$ ) surrounds the peristome, from which rises the conical hypostome, bearing the mouth at its extremity. The general ectoderm covering the surface of the body has entirely lost the cilia present in the earlier larval stages (planula), and may be naked, or


Fig. 1.-Diagram of a typical Hydropolyp.
a, Hydranth;
b, Hydrocaulus;
c, Hydrorhiza;
$t$, Tentacle
ps, Perisarc, forming in the region of the hydranth a cup or hydrotheca $(h, t)$,which, however, is only found in polyps of the order Calyptoblastea. clothed in a cuticle or exoskeleton, the perisarc ( $p s$ ), which in its simplest condition is a chitinous membrane secreted by the ectoderm. The perisarc when present invests the hydrorhiza and hydrocaulus; it may stop short below the hydranth, or it may extend farther. In general there are two types of exoskeleton, characteristic of the two principal divisions of the Hydroidea. In the Gymnoblastea the perisarc either stops below the hydranth, or, if continued on to it, forms a closely-fitting investment extending as a thin cuticle as far as the bases of the tentacles (e.g. Bimeria, see G. J. Allman [1], ${ }^{2}$ pl. xii. figs, 1 and 3). In the Calyptoblastea the perisarc is always continued above the hydrocaulus, and forms a cup, the hydrangium or hydrotheca ( $h, t$ ), standing off from the body, into which the hydranth can be


From Allman's Gymnoblastic Hydroids, by permission of the Council of the Ray Society.
Fig. 2.-Stauridium productum, portion of the colony magnified; p, polyp; rh, hydrorhiza.
The architecture of the hydropolyp, simple though it be, furnishes a long series of variations affecting each part of the body. The greatest variation, however, is seen in the tentacles. As regards number, we find in the aberrant forms Protohydra and Microhydra tentacles entirely absent. In the curious hydroid Monobrachium a single tentacle is present, and the same is the case in Clathrozoon; in Amphibrachium and in Lar (fig. 11, A) the polyp bears two tentacles only. The reduction of the tentacles in all these forms may be correlated with their mode of life, and especially with living in a constant current of water, which brings food-particles always from one direction and renders a complete whorl or circle of tentacles unnecessary. Thus Microhydra lives amongst Bryozoa, and appears to utilize the currents produced by these animals. Protohydra occurs in oyster-banks and Monobrachium also grows on the shells of bivalves, and both these hydroids probably fish in the currents produced by the lamellibranchs. Amphibrachium grows in the tissues of a sponge, Euplectella, and protrudes its hydranth into the canal-system of the sponge; and Lar grows on the tubes of the worm Sabella. With the exception of these forms, reduced for the most part in correlation with a semi-parasitic mode of life, the tentacles are usually numerous. It is rare to find in the polyp a regular, symmetrical disposition of the tentacles as in the medusa. The primitive number of four in a whorl is seen, however, in Stauridium (fig. 2) and Cladonema (Allman [1], pl. xvii.), and in Clavatella each whorl consists regularly of eight (Allman, loc. cit. pl. xviii.). As a rule, however, the number in a whorl is irregular. The tentacles may form a single whorl, or more than one; thus in Corymorpha (fig. 3) and Tubularia (fig. 4) there are two circlets; in Stauridium (fig. 2) several; in Coryne and Cordylophora the tentacles are scattered irregularly over the elongated hydranth.

As regards form,


Fig. 3.-Diagram of Corymorpha. A, A hydriform person giving rise to medusiform persons by budding from the margin of the disk; B, free swimming medusa (Steenstrupia of Forbes) detached from the same, with manubrial genitalia, (Anthomedusae) and only one tentacle. (After Allman).

the tentacles show a number of types, of which the most important are (1) filiform, i.e. cylindrical or tapering from base to extremity, as in Clava (fig. 5); (2) capitate, i.e. knobbed at the extremity, as in Coryne (see Allman, loc. cit. pl. iv.); (3) branched, a rare form in the polyp, but seen in Cladocoryne (see Allman, loc. cit. p. 380, fig. 82). Sometimes more than one type of form is found in the same polyp; in Pennaria and Stauridium (fig. 2) the upper whorls are capitate, the lower filiform. Finally, as regards structure, the tentacles may retain their primitive hollow nature, or become solid by obliteration of the axial cavity.

Fig. 4.-Diagram of Tubularia indivisa. A single hydriform person a bearing a stalk carrying numerous degenerate medusiform persons or sporosacs $b$. (After Allman.)

The hypostome of the hydropolyp may be small, or, on the other hand, as in Eudendrium (Allman, loc. cit. pls. xiii., xiv.), large and trumpet-shaped. In the curious polyp Myriothela the body of the polyp is differentiated into nutritive and reproductive portions.

Histology.-The ectoderm of the hydropolyp is chiefly sensory, contractile and protective in function. It may also be glandular in places. It consists of two regions, an external epithelial layer and a more internal sub-epithelial layer.

The epithelial layer consists of (1) so-called "indifferent" cells secreting the perisarc or cuticle and modified to form glandular cells in places; for example, the adhesive cells in the foot. (2) Sensory cells, which may be fairly numerous in places, especially on the tentacles, but which occur always scattered and isolated, never aggregated to form sense-organs as in the medusa. (3) Contractile or myo-epithelial cells, with the cell prolonged at the base into a contractile muscle-fibre (fig. 6, B). In the hydropolyp the ectodermal muscle-fibres are always directed longitudinally. Belonging primarily to the epithelial layer, the muscular cells may become secondarily sub-epithelial.


From Allman's Gymnoblastic Hydroids, by permission of the Council of the Ray Society.
Fig. 5.-Colonies of Clava. A, Clava squamata, magnified. B, C. multicornis, natural size; p, polyp; gon, gonophores; rh, hydrorhiza.

The sub-epithelial layer consists primarily of the so-called interstitial cells, lodged between the narrowed basal portions of the epithelial cells. From them are developed two distinct types of histological elements; the genital cells and the cnidoblasts or mother-cells of the nematocysts. The sub-epithelial layer thus primarily constituted may be recruited by immigration from without of other elements, more especially by nervous (ganglion) cells and muscle-cells derived from the epithelial layer. In its fullest development, therefore, the subepithelial layer consists of four classes of cell-elements.


Fig. 6 A.-Portion of the body-wall of Hydra, showing ectoderm cells above, separated by "structureless lamella" from three flagellate endoderm cells below. The latter are vacuolated, and contain each a nucleus and several dark granules. In the middle ectoderm cell are seen a nucleus and three nematocysts, with trigger hairs projecting beyond the cuticle. A large nematocyst, with everted thread, is seen in the right-hand ectodermal cell. (After F. E. Schulze.)

The genital cells are simple wandering cells (archaeocytes), at first minute and without any specially distinctive features, until they begin to develop into germ-cells. According to Wulfert [60] the primitive germ-cells of Gonothyraea can be distinguished soon after the fixation of the planula, appearing amongst the interstitial cells of the ectoderm. The germ-cells are capable of extensive migrations, not only in the body of the same polyp, but also from parent to bud through many non-sexual generations of polyps in a colony (A. Weismann [58]).


Fig. 7.-Diagrams to show the structure of Nematocysts and their mode of working. (After Iwanzov.)
a, Undischarged nematocyst.
$b$, Commencing discharge.
c, Discharge complete.
cn, Cnidocil.
N, Nucleus of cnidoblast.
o.c, Outer capsule.
$x$, Plug closing the opening of the outer capsule.

The


Fig. 6 B.-Epidermo-muscular cells of Hydra. m, muscularfibre processes. (After Kleinenberg, from Gegenbaur.)
cnidoblasts are the mother-cells of the nematocysts, each cell producing one nematocyst in its interior. The complete nematocyst (fig. 7) is a spherical or oval capsule containing a hollow thread, usually barbed, coiled in its interior. The capsule has a double wall, an outer one (o.c.), tough and rigid in nature, and an inner one (i.c.) of more flexible consistence. The outer wall of the capsule is incomplete at one pole, leaving an aperture through which the thread is discharged. The inner membrane is continuous with the wall of the hollow thread at a spot immediately below the aperture in the outer wall, so that the thread itself ( $f$ ) is simply a hollow prolongation of the wall of the inner capsule inverted and pushed into its cavity. The entire nematocyst is enclosed in the cnidoblast which formed it. When the nematocyst is completely developed, the cnidoblast passes outwards so as to occupy a superficial position in the ectoderm, and a delicate protoplasmic process of sensory nature, termed the cnidocil (cn) projects from the cnidoblast like a fine hair or cilium. Many points in the development and mechanism of the nematocyst are disputed, but it is tolerably certain (1) that the cnidocil is of sensory nature, and that stimulation, by contact with prey or in other ways, causes a
i.c., Inner capsule, continuous with the wall of the filament, $f$.
$b$, Barbs.
reflex discharge of the nematocyst; (2) that the discharge is an explosive change whereby the in-turned thread is suddenly everted and turned inside out, being thus shot through the opening in the outer wall of the capsule, and forced violently into the tissues of the prey, or, it may be, of an enemy; (3) that the thread inflicts not merely a mechanical wound, but instils an irritant poison, numbing and paralysing in its action. The points most in dispute are, first, how the explosive discharge is brought about, whether by pressure exerted external to the capsule (i.e. by contraction of the cnidoblast) or by internal pressure. N. Iwanzov [27] has brought forward strong grounds for the latter view, pointing out that the cnidoblast has no contractile mechanism and that measurements show discharged capsules to be on the average slightly larger than undischarged ones. He believes that the capsule contains a substance which swells very rapidly when brought into contact with water, and that in the undischarged condition the capsule has its opening closed by a plug of protoplasm ( $x$, fig. 7) which prevents access of water to the contents; when the cnidocil is stimulated it sets in action a mechanism or perhaps a series of chemical changes by which the plug is dissolved or removed; as a result water penetrates into the capsule and causes its contents to swell, with the result that the thread is everted violently. A second point of dispute concerns the spot at which the poison is lodged. Iwanzov believes it to be contained within the thread itself before discharge, and to be introduced into the tissues of the prey by the eversion of the thread. A third point of dispute is whether the nematocysts are formed in situ, or whether the cnidoblasts migrate with them to the region where they are most needed; the fact that in Hydra, for example, there are no interstitial cells in the tentacles, where nematocysts are very abundant, is certainly in favour of the view that the cnidoblasts migrate on to the tentacles from the body, and that like the genital cells the cnidoblasts are wandering cells.

The muscular tissue consists primarily of processes from the bases of the epithelial cells, processes which are contractile in nature and may be distinctly striated. A further stage in evolution is that the muscle-cells lose their connexion with the epithelium and come to lie entirely beneath it, forming a sub-epithelial contractile layer, developed chiefly in the tentacles of the polyp. The evolution of the ganglion-cells, is probably similar; an epithelial cell develops processes of nervous nature from the base, which come into connexion with the bases of the sensory cells, with the muscular cells, and with the similar processes of other nerve-cells; next the nerve-cell loses its connexion with the outer epithelium and becomes a sub-epithelial ganglion-cell which is closely connected with the muscular layer, conveying stimuli from the sensory cells to the contractile elements. The ganglion-cells of Hydromedusae are generally very small. In the polyp the nervous tissue is always in the form of a scattered plexus, never concentrated to form a definite nervous system as in the medusa.

The endoderm of the polyp is typically a flagellated epithelium of large cells (fig. 6), from the bases of which arise contractile muscular processes lying in the plane of the transverse section of the body. In different parts of the coelenteron the endoderm may be of three principal types-(1) digestive endoderm, the primitive type, with cells of large size and considerably vacuolated, found in the hydranth; some of these cells may become special glandular cells, without flagella or contractile processes; (2) circulatory endoderm, without vacuoles and without basal contractile processes, found in the hydrorhiza and hydrocaulus; (3) supporting endoderm (fig. 8), seen in solid tentacles as a row of cubical vacuolated cells, occupying the axis of the tentacle, greatly resembling notochordal tissue, particularly that of Amphioxus at a certain stage of development; as a fourth variety of endodermal cells excretory cells should perhaps be reckoned, as seen in the pores in the foot of Hydra and elsewhere (cf. C. Chun, Hydrozoa [1], pp. 314, 315).

The mesogloea in the hydropolyp is a thin elastic layer, in which may be lodged the muscular fibres and ganglion cells mentioned above, but which never contains any connective tissue or skeletogenous cells or any other kind of special mesogloeal corpuscles.


From Allman's Gymnoblastic Hydroids, by permission of the Council of the Ray Society.

Fig. 10.-Polyps from a Colony of Hydractinia, magnified. $d z$, dactylozoid; $g z$, gastrozoid: $b$, blastostyle; gon, gonophores; rh, hydrorhiza.


From Allman's Gymnoblastic Hydroids, by permission of the Council of the Ray Society.

Fig. 9.-Colony of Hydractinia echinata, growing on the Shell of a Whelk. Natural size.
2. The Polyp-colony.-All known hydropolyps possess the power of reproduction by budding, and the buds produced may become either polyps or medusae. The buds may all become detached after a time and give rise to separate and independent individuals, as in the common Hydra, in which only polyp-individuals are produced and sexual elements are developed upon the polyps themselves; or, on the other hand, the polypindividuals produced by budding may remain permanently in connexion with the parent polyp, in which case sexual elements are never developed on polyp-individuals but only on medusa-individuals, and a true colony is formed. Thus the typical hydroid colony starts from a "founder" polyp, which in the vast majority of cases is fixed, but which may be floating, as in Nemopsis, Pelagohydra, \&c. The founder-polyp usually produces by budding polyp-individuals, and these in their turn produce other buds. The polyps are all non-sexual individuals whose function is purely nutritive. After a time the polyps, or certain of them, produce by budding medusa-individuals, which sooner or later develop sexual elements; in some cases, however, the founder-polyp remains solitary, that is to say, does not produce polyp-buds, but only medusa-buds, from the first (Corymorpha, fig. 3, Myriothela, \&c.). In primitive forms the medusa-individuals are set free before reaching sexual maturity and do not contribute anything to the colony. In other cases, however, the medusa-individuals become sexually mature while still attached to the parent polyp, and are then not set free at all, but become appanages of the hydroid colony and undergo degenerative changes leading to reduction and even to complete obliteration of their original medusan structure. In this way the hydroid colony becomes composed of two portions of different function, the nutritive "trophosome," composed of nonsexual polyps, and the reproductive "gonosome," composed of sexual medusa-individuals, which never exercise a nutritive function while attached to the colony. As a general rule polyp-buds are produced from the hydrorhiza and hydrocaulus, while medusa-buds are formed on the hydranth. In some cases, however, medusa-buds are formed on the hydrorhiza, as in Hydrocorallines.
In such a colony of connected individuals, the exact limits of the separate "persons" are not always clearly marked out. Hence it is necessary to distinguish between, first, the "zooids," indicated in the case of the polyps by the hydranths, each with mouth and tentacles; and, secondly, the "coenosarc," or common flesh, which cannot be assigned more to one individual than another, but consists of a more or less complicated network of tubes, corresponding to the hydrocaulus and hydrorhiza of the primitive independent polyp-individual. The coenosarc constitutes a system by which the digestive cavity of any one polyp is put into communication with that of any other individual either of the trophosome or gonosome. In this manner the food absorbed by one individual contributes to the welfare of the whole colony, and the coenosarc has the function of circulating and distributing nutriment through the colony.

The hydroid colony shows many variations in form and architecture which depend simply upon differences in the methods in which polyps are budded.


After Hincks, Forbes, and Browne. A and B modified from Hincks; C modified from Forbes's Brit. Naked-eyed Medusae.

Fig. 11.-Lar sabellarum and two stages of its Medusa, Willia stellata. A, colony of Lar, B and C, young and adult medusae.

In the first place, buds may be produced only from the hydrorhiza, which grows out and branches to form a basal stolon, typically net-like, spreading over the substratum to which the founder-polyp attached itself. From the stolon the daughter-polyps grow up vertically. The result is a spreading or creeping colony, with the coenosarc in the form of a root-like horizontal network (fig. $5, \mathrm{~B} ; 11, \mathrm{~A})$. Such a colony may undergo two principal modifications. The meshes of the basal network may become very small or virtually obliterated, so that the coenosarc becomes a crust of tubes tending to fuse together, and covered over by a common perisarc. Encrusting colonies of this kind are seen in Clava squamata (fig. 5, A) and Hydractinia (figs. 9, 10), the latter having the perisarc calcified. A further very important modification is seen when the tubes of the basal perisarc do not remain spread out in one plane, but grow in all planes forming a feltwork; the result is a massive colony, such as is seen in the so-called Hydrocorallines (fig. 60), where the interspaces between the coenosarcal tubes are filled up with calcareous matter, or coenosteum, replacing the chitinous perisarc. The result is a stony, solid mass, which contributes to the building up of coral reefs. In massive colonies of this kind no sharp distinction can be drawn between hydrorhiza and hydrocaulus in the coenosarc; it is practically all hydrorhiza. Massive colonies may assume various forms and are often branching or tree-like. A further peculiarity of this type of colony is that the entire coenosarcal complex is covered externally by a common layer of ectoderm; it is not clear how this covering layer is developed.
In the second place, the buds may be produced from the hydrocaulus, growing out laterally from it; the result is an arborescent, tree-like colony (figs. 12, 13). Budding from the hydrocaulus may be combined with budding from the hydrorhiza, so that numerous branching colonies arise from a common basal stolon. In the formation of arborescent colonies, two
sharply distinct types of budding are found, which are best described in botanical terminology as the monopodial or racemose, and the sympodial or cymose types respectively; each is characteristic of one of the two sub-orders of the Hydroidea, the Gymnoblastea and Calyptoblastea.

In the monopodial method (figs. 12, 14) the founder-polyp is, theoretically, of unlimited growth in a vertical direction, and as it grows up it throws out buds right and left alternately, so that the first bud produced by it is the lowest down, the second bud is above the first, the third above this again, and so on. Each bud produced by the founder proceeds to grow and to bud in the same way as the founder did, producing a side branch of the main stem. Hence, in a colony of gymnoblastic hydroids, the oldest polyp of each system, that is to say, of the main stem or of a branch, is the topmost polyp; the youngest polyp of the system is the one nearest to the topmost polyp; and the axis of the system is a true axis.


Fig. 13.-Portion of colony of Bougainvillea fruticosa (Anthomedusae-Gymnoblastea) more magnified. (From Lubbock, after Allman.)


Fig. 14.-Diagrams of the monopodial method of budding, shown in five stages (1-5). F, the founder-polyp; 1, 2, 3, 4, the succession of polyps budded from the founder-polyp; $a^{\prime}, b^{\prime}, c^{\prime}$, the

In the sympodial method of budding, on the other hand, the founder-polyp is of limited growth, and forms a bud from its side, which is also of limited growth, and forms a bud in its turn, and so on (figs. 15, 16). Hence, in a colony of calyptoblastic hydroids, the oldest polyp of a system is the lowest; the youngest polyp is the topmost one; and the axis of the system is a false axis composed of portions of each of the consecutive polyps. In this method of budding there are two types. In one, the biserial type (fig. 15), the polyps produce buds right and left alternately, so that the hydranths are arranged in a zigzag fashion, forming a "scorpioid cyme," as in Obelia and Sertularia. In the other, the uniserial type (fig. 16), the buds are formed always on the same side, forming a "helicoid cyme," as in Hydrallmania, according to $H$. Driesch, in which, however, the primitively uniserial arrangement becomes masked later by secondary torsions of the hydranths.

In a colony formed by sympodial budding, a polyp always produces first a bud, which contributes to the system to which it belongs, i.e. continues the stem or branch of which its parent forms a part. The polyp may then form a second bud, which becomes the starting point of a new system, the beginning, that is, of a new branch; and even a third bud, starting yet another system, may be produced from the same polyp. Hence the colonies of Calyptoblastea may be complexly branched, and the budding may be biserial throughout, uniserial throughout, or partly one, partly the other. Thus in Plumularidae (figs. 17, 18) there is formed a main stem by biserial budding; each polyp on the main stem forms a second bud, which usually forms a side branch or pinnule by uniserial budding. In this way are formed the familiar feathery colonies of Plumularia, in which the pinnules are all in one plane, while in the allied Antennularia the pinnules are arranged in whorls round the main biserial stem. The pinnules never branch again, since in the uniserial mode of budding a polyp never forms a second polyp-bud. On the other hand, a polyp on the main stem may form a second bud which, instead of forming a pinnule by uniserial budding, produces by biserial budding a branch, from which pinnules arise as from the main stem (fig. 18-3, 6). Or a polyp on the main stem, after having budded a second time to form a pinnule, may give rise to a third bud, which starts a new biserial system, from which uniserial pinnules arise as from the main stem-type of Aglaophenia (fig. 19). The laws of budding in hydroids have been worked out in an interesting manner by H. Driesch [13], to whose memoirs the reader must be referred for further details.

Individualization of Polyp-Colonies.-As in other cases where animal colonies are formed by organic union of separate individuals, there is ever a tendency for the polyp-colony as a whole to act as a single individual, and for the members to become subordinated to the needs of the colony and to undergo specialization for particular functions, with the result that they simulate organs and their individuality becomes masked to a greater or less degree. Perhaps the earliest of such specializations is connected with the reproductive function. Whereas primitively any polyp in a colony may produce medusa-buds, in many hydroid colonies medusae are budded only by certain polyps termed blastostyles (fig. 10, b). At first not differing in any way from other polyps (fig. 5), the blastostyles gradually lose their nutritive function and the organs connected with it; the mouth and tentacles disappear, and the blastostyle obtains the nutriment necessary for its activity by way of the coenosarc. In the Calyptoblastea, where the polyps are protected by special capsules of the perisarc, the gonothecae enclosing the blastostyles differ from the hydrothecae protecting the hydranth (fig. 54).


Fig. 18.-Diagram showing method of branching in the Plumularia-type; compare with fig. 17. Polyps 3 and 6, instead of producing uniserial pinnules, have produced biserial branches $\left(3^{1}, 3^{2}\right.$, $3^{3}, 3^{4} ; 6^{1}-6^{3}$ ), which give off uniserial branches in their turn.


Fig. 19.-Diagram showing method of branching in the Aglaophenia-type. Polyp 7 has produced as its first bud, 8; as its second bud, $\mathrm{a}^{7}$, which starts a uniserial pinnule; and as a third bud $\mathrm{I}^{7}$, which starts a biserial branch $\left(\mathrm{II}^{7}-\mathrm{VI}^{7}\right)$ that repeats the structure of the main stem and gives off pinnules. The main stem is indicated by-••-- , the new stem by $\cdots \cdots$.

In other colonies the two functions of the nutritive polyp, namely, capture and digestion of food, may be shared between different polyps (fig. 10). One class of polyps, the dactylozoids $(d z)$, lose their mouth and stomach, and become elongated and tentacle-like, showing great activity of movement. Another class, the gastrozoids ( $g z$ ), have the tentacles reduced or absent, but have the mouth and stomach enlarged. The dactylozoids capture food, and pass it on to the gastrozoids, which swallow and digest it.

Besides the three types of individual above mentioned, there are other appendages of hydroid colonies, of which the individuality is doubtful. Such are the "guard-polyps" (machopolyps) of Plumularidae, which are often regarded as individuals of the nature of dactylozoids, but from a study of the mode of budding in this hydroid family Driesch concluded that the guard-polyps were not true polyp-individuals, although each is enclosed in a small protecting cup of the perisarc, known as a nematophore. Again, the spines arising from the basal crust of Podocoryne have been interpreted by some authors as reduced polyps.
3. The Medusa.-In the Hydromedusae the medusa-individual occurs, as already stated, in one of two conditions, either as an independent organism leading a true life in the open seas, or as a subordinate individuality in the hydroid colony, from which it is never set free; it then becomes a mere reproductive appendage or gonophore, losing successively its organs of sense, locomotion and nutrition, until its medusoid nature and organization become scarcely recognizable. Hence it is convenient to consider the morphology of the medusa from these two aspects.
(a) The Medusa as an Independent Organism.-The general structure and characteristics of the medusa are described elsewhere (see articles Hydrozoan and Medusa), and it is only necessary here to deal with the peculiarities of the Hydromedusa.


From Allman's Gymnoblastic Hydroids, by permission of the Council of the Ray Society.

Fig. 20.-Cladonema radiatum, the medusa walking on the basal branches of its tentacles ( $t$ ), which are turned up over the body.


From Allman's Gymnoblastic Hydroids, by permission of the Council of the Ray Society.

Fig. 21.-Clavatella prolifera, ambulatory medusa. $t$, tentacles; $o c$, ocelli.

As regards habit of life the vast majority of Hydromedusae are pelagic organisms, floating on the surface of the open sea, propelling themselves feebly by the pumping movements of the umbrella produced by contraction of the sub-umbral musculature, and capturing their prey with their tentacles. The genera Cladonema (fig. 20) and Clavatella (fig. 21), however, are ambulatory, creeping forms, living in rock-pools and walking, as it were, on the tips of the proximal branches of each of the tentacles, while the remaining branches serve for capture of food. Cladonema still has the typical medusan structure, and is able to swim about, but in Clavatella the umbrella is so much reduced, that swimming is no longer possible. The remarkable medusa Mnestra parasites is ecto-parasitic throughout life on the pelagic mollusc Phyllirrhoe, attached to it by the sub-umbral surface, and its tentacles have become rudimentary or absent. It is interesting to note that Mnestra has been shown by J. W. Fewkes [15] and R. T. Günther [19] to belong to the same family (Cladonemidae) as Cladonema and Clavatella, and it is reasonable to suppose that the non-parasitic ancestor of Mnestra was, like the other two genera, an ambulatory medusa which acquired louse-like habits. In some species of the genus Cunina (Narcomedusae) the youngest individuals (actinulae) are parasitic on other medusae (see below), but in later life the parasitic habit is abandoned. No other instances are known of sessile habit in Hydromedusae.

The external form of the Hydromedusae varies from that of a deep bell or thimble, characteristic of the Anthomedusae, to the shallow saucer-like form characteristic of the Leptomedusae. It is usual for the umbrella to have an even, circular, uninterrupted margin; but in the order Narcomedusae secondary down-growths between the tentacles produce a lobed, indented margin to the umbrella. The marginal tentacles are rarely absent in non-parasitic forms, and are typically four in number, corresponding to the four perradii marked by the radial canals. Interradial tentacles may be also developed, so that the total number present may be increased to eight or to an indefinitely large number. In Willia, Geryonia, \&c., however, the tentacles and radial canals are on the plan of six instead of four (figs. 11 and 26). On the other hand, in some cases the tentacles are less in number than the perradii; in Corymorpha (figs. 3 and 22) there is but a single tentacle, while two are found in Amphinema and Gemmaria (Anthomedusae), and in Solmundella bitentaculata (fig. 67) and Aeginopsis hensenii (fig. 23) (Narcomedusae). The tentacles also vary considerably in other ways than in number: first, in form, being usually simple, with a basal bulb, but in Cladonemidae they are branched, often in complicated fashion; secondly, in grouping, being usually given off singly, and at regular intervals from the margin of the umbrella, but in Margelidae and in some Trachomedusae they are given off in tufts or bunches (fig. 24); thirdly, in position and origin, being usually implanted on the extreme edge of the umbrella, but in


After E. T. Browne, from Proc. Zool. Soc. of London.

Fig. 22.

- Corymorpha nutans, adult female Medusa. Magnified 10 diameters. Narcomedusae they become secondarily shifted and are given off high up on the ex-umbrella (figs. 23 and 25); and, fourthly, in structure, being hollow or solid, as in the polyp. In some
medusae, for instance, the remarkable deep-sea family Pectyllidae, the tentacles may bear suckers, by which the animal may attach itself temporarily. It should be mentioned finally that the tentacles are very contractile and extensible, and may therefore present themselves, in one and the same individual, as long, drawn-out threads, or in the form of short corkscrew-like ringlets; they may stream downwards from the sub-umbrella, or be held out horizontally, or be directed upwards over the ex-umbrella (fig. 23). Each species of medusa usually has a characteristic method of carrying its tentacles.


After O. Maas, Die craspedoten Medusen der Plankton Expedition, by permission of Lipsius and Tischer.

Fig. 23.-Aeginopsis hensenii, slightly magnified, showing the manner in which the tentacles are carried in life.

The sub-umbrella invariably shows a velum as an inwardly projecting ridge or rim at its margin, within the circle of tentacles; hence the medusae of this sub-class are termed craspedote. The manubrium is absent altogether in the fresh-water medusa Limnocnida, in which the diameter of the mouth exceeds half that of the umbrella; on the other hand, the manubrium may attain a great length, owing to the centre of the sub-umbrella with the stomach being drawn into it, as it were, to form a long proboscis, as in Geryonia. The mouth may be a simple, circular pore at the extremity of the manubrium, or by folding of the edges it may become square or shaped like a Maltese cross, with four corners and four lips. The corners of the mouth may then be drawn out into lobes or lappets, which may have a branched or fringed outline (fig. 27), and in Margelidae the subdivisions of the fringe simulate tentacles (fig. 24).


After O. Maas, Craspedoten Medusen der SibogaExpedition, by permission of E. S. Brill \& Co.

Fig. 24.-Rathkea octonemalis.


After O. Maas, Medusae, in Prince of Monaco's series.

Fig. 25.-Aeginura grimaldii.

The internal anatomy of the Hydromedusae shows numerous variations. The stomach may be altogether lodged in the manubrium, from which the radial canals then take origin directly as in Geryonia (Trachomedusae); it may be with or without gastric pouches. The radial canals may be simple or branched, primarily four, rarely six in number. The ring-canal is drawn out in Narcomedusae into festoons corresponding with the lobes of the margin, and may be obliterated altogether (Solmaris). In this order the radial canals are represented only by wide gastric pouches, and in the family Solmaridae are suppressed altogether, so that the tentacles and the festoons of the ring-canal arise directly from the stomach. In Geryonia, centripetal canals, ending blindly, arise from the ring-canal and run in a radial direction towards the centre of the umbrella (fig. 26).

Histology of the Hydromedusa.-The histology described above for the polyp may be taken as the primitive type, from which that of the medusa differs only in greater elaboration and differentiation of the cell-elements, which are also more concentrated to form distinct tissues.


Fig. 26.-Carmarina (Geryonia) hastata, one of the Trachomedusae. (After Haeckel.)
a, Nerve ring.
$a^{\prime}$, Radial nerve.
$b$, Tentaculocyst.
c, Circular canal.
$e$, Radiating canal.
$g^{\prime \prime}$. Ovary.
h, Peronia or cartilaginous process ascending from the cartilaginous margin of the disk centripetally in the outer surface of the jelly-like disk; six of these are perradial, six interradial, corresponding to the twelve solid larval tentacles, resembling those of Cunina.
$k$, Dilatation (stomach) of the manubrium.
1 , Jelly of the disk.
p, Manubrium.
$t$, Tentacle (hollow and tertiary, i.e. preceded by six perradial and six interradial solid larval tentacles).
$u$, Cartilaginous margin of the disk covered by thread-cells.
v. Velum.

The ectoderm furnishes the general epithelial covering of the body, and the muscular tissue, nervous system and sense-organs. The external epithelium is flat on the ex-umbral surface, more columnar on the sub-umbral surface, where it forms the muscular tissue of the sub-umbrella and the velum. The nematocysts of the ectoderm may be grouped to form batteries on the tentacles, umbrellar margin and oral lappets. In places the nematocysts may be crowded so thickly as to form a tough, supporting, "chondral" tissue, resembling cartilage, chiefly developed at the margin of the umbrella and forming streaks or bars supporting the tentacles ("Tentakelspangen," peronia) or the tentaculocysts ("Gehörspangen," otoporpae).
The muscular tissue of the Hydromedusae is entirely ectodermal. The muscle-fibres arise as processes from the bases of the epithelial cells; such cells may individually become subepithelial in position, as in the polyp; or, in places where muscular tissue is greatly developed, as in the velum or subumbrella, the entire muscular epithelium may be thrown into folds in order to increase its surface, so that a deeper subepithelial muscular layer becomes separated completely from a more superficial body-epithelium.


After O. Maas in Results of the "Albatross" Expedition, Museum of Comparative Zoology, Cambridge, Mass., U.S.A.

Fig. 27.-Stomotoca divisa, one of the Tiaridae
(Anthomedusae).
fibres arranged circularly, that is to say, concentrically round the central axis of the umbrella; the other of non-striated fibres running longitudinally, that is to say, in a radial direction from, or (in the manubrium) parallel to, the same ideal axis. The circular system is developed continuously over the entire sub-umbral surface, and the velum represents a special local development of this system, at a region where it is able to act at the greatest mechanical advantage in producing the contractions of the umbrella by which the animal progresses. The longitudinal system is discontinuous, and is subdivided into proximal, medial and distal portions. The proximal portion forms the retractor muscles of the manubrium, or proboscis, well developed, for example, in Geryonia. The medial portion forms radiating tracts of fibres, the so-called "bell-muscles" running underneath, and parallel to, the radial canals; when greatly developed, as in Tiaridae, they form ridges, so-called mesenteries, projecting into the subumbral cavity. The distal portions form the muscles of the tentacles. In contrast with the polyp, the longitudinal muscle-system is entirely ectodermal, there being no endodermal muscles in craspedote medusae.


Fig. 28.-Muscular Cells of Medusae (Lizzia). The uppermost is a purely muscular cell from the subumbrella; the two lower are epidermo-muscular cells from the base of a tentacle; the upstanding nucleated portion forms part of the epidermal mosaic on the free surface of the body. (After Hertwig.)

The nervous system of the medusa consists of sub-epithelial ganglion-cells, which form, in the first place, a diffuse plexus of nervous tissue, as in the polyp, but developed chiefly on the subumbral surface; and which are concentrated, in the second place, to form a definite central nervous system, never found in the polyp. In Hydromedusae the central nervous system forms two concentric nerve-rings at the margin of the umbrella, near the base of the velum. One, the "upper" or ex-umbral nerve-ring, is derived from the ectoderm on the ex-umbral side of the velum; it is the larger of the two rings, containing more numerous but smaller ganglion-cells, and innervates the tentacles. The other, the "lower" or sub-umbral nerve-ring, is derived from the ectoderm on the sub-umbral side of the velum; it contains fewer but larger ganglion-cells and innervates the muscles of the velum (see diagram in article Medusae). The two nerve-rings are connected by fibres passing from one to the other.

The sensory cells are slender epithelial cells, often with a cilium or stiff protoplasmic process, and should perhaps be regarded as the only ectoderm-cells which retain the primitive ciliation of the larval ectoderm, otherwise lost in all Hydrozoa. The sense-cells form, in the first place, a diffuse system of scattered sensory cells, as in the polyp, developed chiefly on the manubrium, the tentacles and the margin of the umbrella, where they form a sensory ciliated epithelium covering the nerve-centres; in the second place, the sense-cells are concentrated to form definite sense-organs, situated always at the margin of the umbrella, hence often termed "marginal bodies." The possession of definite sense-organs at once distinguishes the medusa from the polyp, in which they are never found.

The sense-organs of medusae are of two kinds-first, organs sensitive to light, usually termed ocelli (fig. 29); secondly, organs commonly termed otocysts, on account of their resemblance to the auditory vesicles of higher animals, but serving for the sense of balance and orientation, and therefore given the special name of statocysts (fig. 30). The sense-organs may be tentaculocysts, i.e. modifications of a tentacle, as in Trachylinae, or developed from the margin of the umbrella, in no connexion with a tentacle (or, if so connected, not producing any modification in the tentacle), as in Leptolinae. In Hydromedusae the sense-organs are always
exposed at the umbrellar margin (hence Gymnophthalmata), while in Scyphomedusae they are covered over by flaps of the umbrellar margin (hence Steganophthalmata).


Modified after Linko, Traveaux Soc. Imp. Nat., St. Petersbourg, xxix.

Fig. 30.-Section of a Statocyst and Ocellus of Tiaropsis diademata; cf. fig. 29.
$e x$, Ex-umbral ectoderm.
$s u b$, Sub-umbral ectoderm.
c.c, Circular canal.
$v$, Velum.
st.e, Cavity of statocyst.
con, Concrement-cell with otolith.


Modified after O. and R, Hertwig, Nervensystem und Sinnesorgane der Medusen, by permission of F. C. W. Vogel.

Fig. 31.-Section of a Statocyst of Mitrocoma annae.
sub, Sub-umbral ectoderm.
c.c, Circular canal.
v, Velum.
st.c, Cavity of statocyst.
con, Concrement-cell with otolith.

The statocysts present in general the structure of either a knob or a closed vesicle, composed of (1) indifferent supporting epithelium: (2) sensory, so-called auditory epithelium of slender cells, each bearing at its free upper end a stiff bristle and running out at its base into a nervefibre; (3) concrement-cells, which produce intercellular concretions, so-called otoliths. By means of vibrations or shocks transmitted through the water, or by displacements in the balance or position of the animal, the otoliths are caused to impinge against the bristles of the sensory cells, now on one side, now on the other, causing shocks or stimuli which are transmitted by the basal nerve-fibre to the central nervous system. Two stages in the development of the otocyst can be recognized, the first that of an open pit on a freely-projecting knob, in which the otoliths are exposed, the second that of a closed vesicle, in which the otoliths are covered over. Further, two distinct types of otocyst can be recognized in the Hydromedusae: that of the Leptolinae, in which the entire organ is ectodermal, concrementcells and all, and the organ is not a tentaculocyst; and that of the Trachylinae, in which the organ is a tentaculocyst, and the concrement-cells are endodermal, derived from the endoderm of the modified tentacle, while the rest of the organ is ectodermal.


Modified after O. and R, Hertwig, Nervensystem und Sinnesorgane der Medusen, by permission of F. C. W. Vogel.

> Fig. 32.-Section of a Statocyst of Phialidium.
$e x$, Ex-umbral ectoderm. $s u b$, Sub-umbral ectoderm. $v$, Velum.
st.c, Cavity of statocyst. con, Concrement-cell with otolith.


Modified after O. and R, Hertwig, Nervensystem und Sinnesorgane der Medusen, by permission of F. C. W. Vogel.

Fig. 33.-Optical Section of a Statocyst of Octorchis.
con, Concrement-cell with otolith.
st.c, Cavity of statocyst.

In the Leptolinae the otocysts are seen in their first stage in Mitrocoma annae (fig. 31) and Tiaropsis (figs. 29, 30) as an open pit at the base of the velum, on its sub-umbral side. The pit
has its opening turned towards the sub-umbral cavity, while its base or fundus forms a bulge, more or less pronounced, on the ex-umbral side of the velum. At the fundus are placed the concrement-cells with their conspicuous otoliths (con) and the inconspicuous auditory cells, which are connected with. the sub-umbral nerve-ring. From the open condition arises the closed condition very simply by closing up of the aperture of the pit. We then find the typical otocyst of the Leptomedusae, a vesicle bulging on the ex-umbral side of the velum (figs. 32, 33). The otocysts are placed on the outer wall of the vesicle (the fundus of the original pit) or on its sides; their arrangement and number vary greatly and furnish useful characters for distinguishing genera. The sense-cells are innervated, as before, from the sub-umbral nervering. The inner wall of the vesicle (region of closure) is frequently thickened to form a so-called "sense-cushion," apparently a ganglionic offshoot from the sub-umbral nerve-ring. In many Leptomedusae the otocysts are very small, inconspicuous and embedded completely in the tissues; hence they may be easily overlooked in badly-preserved material, and perhaps are present in many cases where they have been said to have been wanting.


After O. and R, Hertwig, Nervensystem und Sinnesorgane der Medusen, by permission of F. C. W. Vogel.

Fig. 34.-Tentaculocyst (statorhabd) of Cunina solmaris. n.c, Nerve-cushion; end, endodermal concrement-cells; con, otolith.


After O. and R, Hertwig, Nervensystem und Sinnesorgane der Medusen, by permission of F. C. W. Vogel.

Fig. 35.-Tentaculocyst of Cunina lativentris.

## ect, Ectoderm.

n. $c$, Nerve-cushion.
end, Endodermal concrement-cells.
con, Otolith.

In the Trachylinae the simplest condition of the otocyst is a freely projecting club, a so-called statorhabd (figs. 34, 35), representing a tentacle greatly reduced in size, covered with sensory ectodermal epithelium (ect.), and containing an endodermal core (end.), which is at first continuous with the endoderm of the ring-canal, but later becomes separated from it. In the endoderm large concretions are formed (con.). Other sensory cells with long cilia cover a sort of cushion (n.c.) at the base of the club; the club may be long and the cushion small, or the cushion large and the club small. The whole structure is innervated, like the tentacles, from the ex-umbral nerve-ring. An advance towards the second stage is seen in such a form as Rhopalonema (fig. 36), where the ectoderm of the cushion rises up in a double fold to enclose the club in a protective covering forming a cup or vesicle, at first open distally; finally the opening closes and the closed vesicle may sink inwards and be found far removed from the surface, as in Geryonia (fig. 37).


Fig. 36.-Simple tentaculocyst of Rhopalonema

velatum. The process carrying the otolith or concretion $h k$, formed by endoderm cells, is enclosed by an upgrowth forming the "vesicle," which is not yet quite closed in at the top. (After Hertwig.)

Medusen, by permission of F. C. W. Vogel.
Fig. 37.-Section of statocyst of Geryonia ( Carmarina hastata).
st.c, Statocyst containing the minute tentaculocyst. $n r_{1}$, Ex-umbral nerve-ring.
$n r_{2}$, Sub-umbral nerve-ring.
$e x$, Ex-umbral ectoderm.
sub, Sub-umbral ectoderm.
c.c, Circular canal.
$v$, Velum.

The ocelli are seen in their simplest form as a pigmented patch of ectoderm, which consists of two kinds of cells-(1) pigment-cells, which are ordinary indifferent cells of the epithelium containing pigment-granules, and (2) visual cells, slender sensory epithelial cells of the usual type, which may develop visual cones or rods at their free extremity. The ocelli occur usually either on the inner or outer sides of the tentacles; if on the inner side, the tentacle is turned upwards and carried over the ex-umbrella, so as to expose the ocellus to the light; if the ocellus be on the outer side of a tentacle, two nerves run round the base of the tentacle to it. In other cases ocelli may occur between tentacles, as in Tiaropsis (fig. 29).

The simple form of ocellus described in the foregoing paragraph may become folded into a pit or cup, the interior of which becomes filled with a clear gelatinous secretion forming a sort of vitreous body. The distal portion of the vitreous body may project from the cavity of the cup, forming a non-cellular lens as in Lizzia (fig. 28). Beyond this simple condition the visual organs of the Hydromedusae do not advance, and are far from reaching the wonderful development of the eyes of Scyphomedusae (Charybdaea).

Besides the ordinary type of ocellus just described, there is found in one genus (Tiaropsis) a type of ocellus in which the visual elements are inverted, and have their cones turned away from the light, as in the human retina (fig. 30). In this case the pigment-cells are endodermal, forming a cup of pigment in which the visual cones are embedded. A similar ocellus is formed in Aurelia among the Scyphomedusae (q.v.).

Other sense organs of Hydromedusae are the so-called sense-clubs or cordyli found in a few Leptomedusae, especially in those genera in which otocysts are inconspicuous or absent (fig. 39). Each cordylus is a tentacle-like structure with an endodermal axis containing an axial cavity which may be continuous with the ring-canal, or may be partially occluded. Externally the cordylus is covered, by very flattened ectoderm, and bears no otoliths or sense-cells, but the base of the club rests upon the ex-umbral nerve-ring. Brooks regards these organs as sensory, serving for the sense of balance, and representing a primitive stage of the tentaculocysts of Trachylinae; Linko, on the other hand, finding no nerve-elements connected with them, regards them as digestive (?) in function.

The sense-organs of the two fresh-water medusae Limnocodium and Limnocnida are peculiar and of rather doubtful nature (see E. T. Browne [10]).

The endoderm of the medusa shows the same general types of structure as in the polyp, described above. We can distinguish (1) digestive endoderm, in the stomach, often with special glandular elements; (2) circulatory endoderm, in the radial and ring-canals; (3) supporting endoderm in the axes of the tentacles and in the endoderm-lamella; the latter is primitively a double layer of cells, produced by concrescence of the ex-umbral and sub-umbral layers of the coelenteron, but it is usually found as a single layer of flattened cells (fig. 40); in Geryonia, however, it remains double, and the centripetal canals arise by parting of the two layers; (4) excretory endoderm, lining pores at the margin of the umbrella, occurring in certain Leptomedusae as so-called "marginal tubercles," opening, on the one hand, into the ring-canal and, on the other hand, to the exterior by "marginal funnels," which debouch into the sub-umbral cavity above the velum. As has been described above, the endoderm may also contribute to the senseorgans, but such contributions are always of an accessory nature, for instance, concrement-cells in the otocysts, pigment in the ocelli, and never of sensory nature, sense-cells being in all cases ectodermal.

The reproductive cells may be regarded as belonging primarily to neither ectoderm nor endoderm, though lodged in the ectoderm in


Fig. 38.-Ocellus of Lizzia koellikeri. oc, Pigmented ectodermal cells; l, lens. (After
Hertwig.) all Hydromedusae. As described for the polyp, they are wandering cells capable of extensive migrations before reaching the particular spot at which they ripen. In the Hydromedusae they
usually, if not invariably, ripen in the ectoderm, but in the neighbourhood of the main sources of nutriment, that is to say, not far from the stomach. Hence the gonads are found on the manubrium in Anthomedusae generally; on the base of the manubrium, or under the gastral pouches, or in both these situations (Octorchidae), or under the radial canals, in Trachomedusae; under the gastral pouches or radial canals, in Narcomedusae. When ripe, the germ-cells are dehisced directly to the exterior.


After W. K. Brooks, Journal of Morphology, x., by permission of Ginn \& Co.

Fig. 39.-Section of a Cordylus of Laodice.
c.c, Circular canal.
$v$, Velum.
$t$, Tentacle.
c, Cordylus, composed of flattened ectoderm $e c$ covering a large-celled endodermal axis en.


Fig. 40.-Portions of Sections through the Disk of Medusae-the upper one of Lizzia, the lower of Aurelia. (After Hertwig.)
el, Endoderm lamella.
$m$, Muscular processes of the ectoderm-cells in cross section.
d, Ectoderm.
en, Endoderm lining the enteric cavity.
$e$, Wandering endoderm cells of the gelatinous substance.

Hydromedusae are of separate sexes, the only known exception being Amphogona apsteini, one of the Trachomedusae (Browne [9]). Moreover, all the medusae budded from a given hydroid colony are either male or female, so that even the non-sexual polyp must be considered to have a latent sex. (In Hydra, on the other hand, the individual is usually hermaphrodite.) The medusa always reproduces itself sexually, and in some cases non-sexually also. The non-sexual reproduction takes the form of fission, budding or sporogony, the details of which are described below. Buds may be produced from the manubrium, radial canals, ring-canal, or tentacle-bases, or from an aboral stolon (Narcomedusae). In all cases only medusa-buds are produced, never polyp-buds.

The mesogloea of the medusa is largely developed and of great thickness in the umbrella. The sub-epithelial tissues, i.e. the nervous and muscular cells, are lodged in the mesogloea, but in Hydromedusae it never contains tissue-cells or mesogloeal corpuscles.
(b) The Medusae as a Subordinate Individuality.-It has been shown above that polyps are budded only from polyps and that the medusae may be budded either from polyps or from medusae. In any case the daughter-individuals produced from the buds may be imagined as remaining attached to the parent and forming a colony of individuals in organic connexion with one another, and thus three possible cases arise. The first case gives a colony entirely composed of polyps, as in many Hydroidea. The second case gives a colony partly composed of polyp-individuals, partly of medusa-individuals, a possibility also realized in many colonies of Hydroidea. The third case gives a colony entirely composed of medusa-individuals, a possibility perhaps realized in the Siphonophora, which will be discussed in dealing with this group.

The first step towards the formation of a mixed hydroid colony is undoubtedly a hastening of the sexual maturity of the medusa-individual. Normally the medusae are liberated in quite an immature state; they swim away, feed, grow and become adult mature individuals. From the bionomical point of view, the medusa is to be considered as a means of spreading the species, supplementing the deficiencies of the sessile polyp. It may be, however, that increased reproductiveness becomes of greater importance to the species than wide diffusion; such a condition will be brought about if the medusae mature quickly and are either set free in a mature condition or remain in the shelter of the polyp-colony, protected from risks of a free life in the open sea. In this way the medusa sinks from an independent personality to an organ of the polyp-colony, becoming a so-called medusoid gonophore, or bearer of the reproductive
organs, and losing gradually all organs necessary for an independent existence, namely those of sense, locomotion and nutrition.

In some cases both free medusae and gonophores may be produced from the same hydroid colony. This is the case in Syncoryne mirabilis (Allman [1], p. 278) and in Campanularia volubilis; in the latter, free medusae are produced in summer, gonophores in winter (Duplessis [14]). Again in Pennaria, the male medusae are set free in a state of maturity, and have ocelli; the female medusae remain attached and have no sense organs.


Modified from Weismann, Entstehung der Sexualzellen bei den Hydromedusen.

Fig. 41.-Diagrams of the Structure of the Gonophores of various Hydromedusae, based on the figures of G. J.

Allman and A. Weismann.

A, "Meconidium" of
Gonothyraea.
B, Type of Tubularia.
C, Type of Garveia, \&c.
D, Type of Plumularia,
Agalma, \&c.
E, Type of Coryne, Forskalia, \&c.
F, G, H, Sporosacs.
$F$, With simple spadix.
G, With spadix prolonged (Eudendrium).
H, With spadix branched (Cordylophora).
s.c, Sub-umbral cavity. $t$, Tentacles.
c.c, Circular canal,
$g$, Gonads.
$s p$, Spadix.
e.l, Endoderm-lamella.
$e x$, Ex-umbral ectoderm.
ect, Ectotheca.

The gonophores of different hydroids differ greatly in structure from one another, and form a series showing degeneration of the medusa-individual, which is gradually stripped, as it were, of its characteristic features of medusan organization and finally reduced to the simplest structure. A very early stage in the degeneration is well exemplified by the so-called "meconidium" of

Gonothyraea (fig. 41, A). Here the medusoid, attached by the centre of its exumbral surface, has lost its velum and sub-umbral muscles, its sense organs and mouth, though still retaining rudimentary tentacles. The gonads ( $g$ ) are produced on the manubrium, which has a hollow endodermal axis, termed the spadix ( $s p$.), in open communication with the coenosarc of the polyp-colony and serving for the nutrition of the generative cells. A very similar condition is seen in Tubularia (fig. 41, B), where, however, the tentacles have quite disappeared, and the circular rim formed by the margin of the umbrella has nearly closed over the manubrium leaving only a small aperture through which the embryos emerge. The next step is illustrated by the female gonophores of Cladocoryne, where the radial and ringcanals have become obliterated by coalescence of their walls, so that the entire endoderm of the umbrella is in the condition of the endoderm-lamella. Next the opening of the umbrella closes up completely and disappears, so that the sub-umbral cavity forms a closed space surrounding the manubrium, on which the gonads are developed; such a condition is seen in the male gonophore of Cladocoryne and in Garveia (fig. 41, C), where, however, there is a further complication in the form of an adventitious envelope or ectotheca (ect.) split off from the gonophore as a protective covering, and not present in Cladocoryne. The sub-umbral cavity (s.c.) functions as a brood-space for the developing embryos, which are set free by rupture of the wall. It is evident that the outer envelope of the gonophore represents the ex-umbral ectoderm (ex.), and that the inner ectoderm lining the cavity represents the sub-umbral ectoderm of the free medusa. The next step is the gradual obliteration of the sub-umbral cavity (s.c.) by disappearance of which the sub-umbral ectoderm comes into contact with the ectoderm of the manubrium. Such a type is found in Plumularia and also in Agalma (fig. 41, D); centrally is seen the spadix ( $s p$.), bearing the generative cells ( $g$ ), and external to these (1) a layer of ectoderm representing the epithelium of the manubrium; (2) the layer of sub-umbral ectoderm; (3) the endoderm-lamella (e.l.); (4) the ex-umbral ectoderm (ex.); and (5) there may or may not be present also an ectotheca. Thus the gonads are covered over by at least four layers of epithelium, and since these are unnecessary, presenting merely obstacles to the dehiscence of the gonads, they gradually undergo reduction. The sub-umbral ectoderm and that covering the manubrium undergo concrescence to form a single layer (fig. 41, E), which finally disappears altogether, and the endoderm-lamella disappears. The gonophore is now reduced to its simplest condition, known as the sporosac (fig. 41, F, G, H), and consists of the spadix bearing the gonads covered by a single layer of ectoderm (ex.), with or without the addition of an ectotheca. It cannot be too strongly emphasized, however, that the sporosac should not be compared simply with the manubrium of the medusa, as is sometimes done. The endodermal spadix ( $s p$.) of the sporosac represents the endoderm of the manubrium; the ectodermal lining of the sporosac (ex.) represents the ex-umbral ectoderm of the medusa; and the intervening layers, together with the sub-umbral cavity, have disappeared. The spadix, as the organ of nutrition for the gonads, may be developed in various ways, being simple (fig. 41, F) or branched (fig. 41, H); in Eudendrium (fig. $41, \mathrm{G}$ ) it curls round the single large ovum.

The hydroid Dicoryne is remarkable for the possession of gonophores, which are ciliate and become detached and swim away by means of their cilia. Each such sporosac has two long tentacle-like processes thickly ciliated.

It has been maintained that the gonads of Hydra represent sporosacs or gonophores greatly reduced, with the last traces of medusoid structure completely obliterated. There is, however, no evidence whatever for this, the gonads of Hydra being purely ectodermal structures, while all medusoid gonophores have an endodermal portion. Hydra is, moreover, bisexual, in contrast
with what is known of hydroid colonies.
In some Leptomedusae the gonads are formed on the radial canals and form protruding masses resembling sporosacs superficially, but not in structure. Allman, however, regarded this type of gonad as equivalent to a sporosac, and considered the medusa bearing them as a nonsexual organism, a "blastocheme" as he termed it, producing by budding medusoid gonophores. As medusae are known to bud medusae from the radial canals there is nothing impossible in Allman's theory, but it cannot be said to have received satisfactory proof.

## Reproduction and Ontogeny of the Hydromedusae.

Nearly every possible method of reproduction occurs amongst the Hydromedusae. In classifying methods of generation it is usual to make use of the sexual or non-sexual nature of the reproduction as a primary difference, but a more scientific classification is afforded by the distinction between tissue-cells (histocytes) and germinal cells, actual or potential (archaeocytes), amongst the constituent cells of the animal body. In this way we may distinguish, first, vegetative reproduction, the result of discontinuous growth of the tissues and cell-layers of the body as a whole, leading to (1) fission, (2) autotomy, or (3) vegetative budding; secondly, germinal reproduction, the result of the reproductive activity of the archaeocytes or germinal tissue. In germinal reproduction the proliferating cells may be undifferentiated, socalled primitive germ-cells, or they may be differentiated as sexual cells, male or female, i.e. spermatozoa and ova. If the germ-cells are undifferentiated, the offspring may arise from many cells or from a single cell; the first type is (4) germinal budding, the second is (5) sporogony. If the germ-cells are differentiated, the offspring arises by syngamy or sexual union of the ordinary type between an ovum and spermatozoon, so-called fertilization, of the ovum, or by parthenogenesis, i.e. development of an ovum without fertilization. The only one of these possible modes of reproduction not known to occur in Hydromedusae is parthenogenesis.
(1) True fission or longitudinal division of an individual into two equal and similar daughterindividuals is not common but occurs in Gastroblasta, where it has been described in detail by Arnold Lang [30].
(2) Autotomy, sometimes termed transverse fission, is the name given to a process of unequal fission in which a portion of the body separates off with subsequent regeneration. In Tubularia by a process of decapitation the hydranths may separate off and give rise to a separate individual, while the remainder of the body grows a new hydranth. Similarly in Schizocladium portions of the hydrocaulus are cut off to form so-called "spores," which grow into new individuals (see Allman [1]).


Much modified from C. Chun, "Coelenterata," in Bronn's Tierreich.

Fig. 43.-Direct Budding of Cunina.
A, B, C, E, F, In vertical section.
D, Sketch of external view.
st, Stomach.
$m$, Manubrium.
$t$. Tentacle.
s.o, Sense organ.
$v$, Velum.
s.c, Sub-umbral cavity.
n.s, Nervous system.


Fig. 44.-Diagrams of Medusa budding with the formation of an entocodon. The endoderm is shaded, the ectoderm left clear.

A, B, C, D, F, Successive stages in vertical section. E , Transverse section of a stage similar to D .
Gc, Entocodon.
s.c, Cavity of entocodon, forming the future subumbral cavity.
st, Stomach.
r.c, Radial canal.
c.c, Circular canal.
e.l, Endoderm lamella.
$m$, Manubrium.
$v$, Velum.
$t$, Tentacle.
(3) Vegetative budding is almost universal in the Hydromedusae. By budding is understood the formation of a new individual from a fresh growth of undifferentiated material. It is convenient to distinguish buds that give rise to polyps from those that form medusae.
(a) The Polyp.-The buds that form polyps are very simple in mode of formation. Four stages may be distinguished; the first is a simple outgrowth of both layers, ectoderm and endoderm, containing a prolongation of the coelenteric cavity; in the second stage the tentacles grow out as secondary diverticula from the side of the first outgrowth; in the third stage the mouth is formed as a perforation of the two layers; and, lastly, if the bud is to be separated, it becomes nipped off from the parent polyp and begins a free existence.
(b) The Medusae.-Two types of budding must be distinguished-the direct, so-called, palingenetic type, and the indirect, so-called coenogenetic type.

The direct type of budding is rare, but is seen in Cunina and Millepora. In Cunina there
arises, first, a simple outgrowth of both layers, as in a polyp-bud (fig. 43, A); in this the mouth is formed distally as a perforation (B); next the sides of the tube so formed bulge out laterally near the attachment to form the umbrella, while the distal undilated portion of the tube represents the manubrium (C); the umbrella now grows out into a number of lobes or lappets, and the tentacles and tentaculocysts grow out, the former in a notch between two lappets, the latter on the apex of each lappet ( $\mathrm{D}, \mathrm{E}$ ); finally, the velum arises as a growth of the ectoderm alone, the whole bud shapes itself, so to speak, and the little medusa is separated off by rupture of the thin stalk connecting it with the parent ( F ). The direct method of medusa-budding only differs from the polyp-bud by its greater complexity of parts and organs.

The indirect mode of budding (figs. 44, 45) is the commonest method by which medusa-buds are formed. It is marked by the formation in the bud of a characteristic structure termed the entocodon (Knospenkern, Glockenkern).

The first stage is a simple hollow outgrowth of both body-layers (fig. 44, A); at the tip of this is formed a thickening of the ectoderm, arising primitively as a hollow ingrowth (fig. 44, B), but more usually as a solid mass of ectoderm-cells (fig. 45, A). The ectodermal ingrowth is the entocodon ( $G c$.); it bulges into, and pushes down, the endoderm at the apex of the bud, and if solid it soon acquires a cavity (fig. 44, C, s.c.). The cavity of the entocodon increases continually in size, while the endoderm pushes up at the sides of it to form a cup with hollow walls,


Fig. 45.-Modifications of the method of budding shown in fig. 44, with solid Entocodon (Gc.) and formation of an ectotheca (ect.). enclosing but not quite surrounding the entocodon, which remains in contact at its outer side with the ectoderm covering the bud (fig. $44, \mathrm{D}, \mathrm{v}$ ). The next changes that take place are chiefly in the endoderm-cup (fig. 44, D, E); the cavity between the two walls of the cup becomes reduced by concrescence to form the radial canals (r.c.), ring-canal (c.c.), and endoderm-lamella (e.l., fig. 44, E), and at the same time the base of the cup is thrust upwards to form the manubrium ( m ), converting the cavity of the entocodon into a space which is crescentic or horse-shoe-like in section. Next tentacles ( $t$, fig. $44, \mathrm{~F}$ ) grow out from the ring-canal, and the double plate of ectoderm on the distal side of the entocodon becomes perforated, leaving a circular rim composed of two layers of ectoderm, the velum ( $v$ ) of the medusa. Finally, a mouth is formed by breaking through at the apex of the manubrium, and the now fully-formed medusa becomes separated by rupture of the stalk of the bud and swims away.

If the bud, however, is destined to give rise not to a free medusa, but to a gonophore, the development is similar but becomes arrested at various points, according to the degree to which the gonophore is degenerate. The entocodon is usually formed, proving the medusoid nature of the bud, but in sporosacs the entocodon may be rudimentary or absent altogether. The process of budding as above described may be varied or complicated in various ways; thus a secondary, amnion-like, ectodermal covering or ectotheca (fig. 45, C, ect.) may be formed over all, as in Garveia, \&c.; or the entocodon may remain solid and without cavity until after the formation of the manubrium, or may never acquire a cavity at all, as described above for the gonophores.

Phylogenetic Significance of the Entocodon.-It is seen from the foregoing account of medusa-budding that the entocodon is a very important constituent of the bud, furnishing some of the most essential portions of the medusa; its cavity becomes the sub-umbral cavity, and its lining furnishes the ectodermal epithelium of the manubrium and of the sub-umbral
cavity as far as the edge of the velum. Hence the entocodon represents a precocious formation of the sub-umbral surface, equivalent to the peristome of the polyp, differentiated in the bud prior to other portions of the organism which must be regarded as antecedent to it in phylogeny.

If the three principal organ-systems of the medusa, namely mouth, tentacles and umbrella, be considered in the light of phylogeny, it is evident that the manubrium bearing the mouth must be the oldest, as representing a common property of all the Coelentera, even of the gastrula embryo of all Enterozoa. Next in order come the tentacles, common to all Cnidaria. The special property of the medusa is the umbrella, distinguishing the medusa at once from other morphological types among the Coelentera. If, therefore, the formation of these three systems of organs took place according to a strictly phylogenetic sequence, we should expect them to appear in the order set forth above (fig. 46, $\mathrm{I} a, b, c$ ). The nearest approach to the phylogenetic sequence is seen in the budding of Cunina, where the manubrium and mouth appear first, but the umbrella is formed before the tentacles (fig. 46, II $a, b, c$ ). In the indirect or coenogenetic method of budding, the first two members of the sequence exhibited by Cunina change places, and the umbrella is formed first, the manubrium next, and then the tentacles; the actual mouth-perforation being delayed to the very last (fig. 46, IVa, b, c). Hence the budding of medusae exemplifies very clearly a common phenomenon in development, a phylogenetic series of events completely dislocated in the ontogenetic timesequence.

The entocodon is to be regarded, therefore, not as primarily an ingrowth of ectoderm, but rather as an upgrowth of both body-layers, in the form of a circular rim (IVa), representing the umbrellar margin; it is comparable to the bulging that forms the umbrella in the direct method of budding, but takes place before a manubrium is formed, and is greatly reduced in size, so as to become a little pit. By a simple modification, the open pit becomes a solid ectodermal ingrowth, just as in Teleostean fishes the hollow medullary tube, or the auditory pit of other vertebrate embryos, is formed at first as a solid cord of cells, which acquires a cavity secondarily. Moreover, the entocodon, however developed, gives rise at first to a closed cavity, representing a closing over of the umbrella, temporary in the bud destined to be a free medusa, but usually permanent in the sessile gonophore. As has been shown above, the closing up of the sub-umbral cavity is one of the earliest degenerative changes in the evolution of the gonophore, and we may regard it as the umbrellar fold taking on a protective function, either temporarily for the bud or permanently for the gonophore.

To sum up, the entocodon is a precocious formation of the umbrella, closing over to protect the organs in the umbrellar cavity. The possession of an entocodon proves the medusa-nature of the bud, and can only be explained on the theory that gonophores are degenerate medusae, and is inexplicable on the opposed view that medusae are derived from gonophores secondarily set free. In the sporosac, however, the medusa-individual has become so degenerate that even the
documentary proof, so to speak, of its medusoid nature may have been destroyed, and only circumstantial evidence of its nature can be produced.
4. Germinal Budding.-This method of budding is commonly described as budding from a single body-layer, instead of from both layers. The layer that produces the bud is invariably the ectoderm, i.e. the layer in which, in Hydromedusae, the generative cells are lodged; and in some cases the buds are produced in the exact spot in which later the gonads appear. From these facts, and from those of the sporogony, to be described below, we may regard budding to this type as taking place from the germinal epithelium rather than from ordinary ectoderm.
(a) The Polyp.-Budding from the ectoderm alone has been described by A. Lang [29] in Hydra and other polyps. The tissues of the bud become differentiated into ectoderm and endoderm, and the endoderm of the bud becomes secondarily continuous with that of the parent, but no part of the parental endoderm contributes to the building up of the daughterpolyp. Lang regarded this method of budding as universal in polyps, a notion disproved by O . Seeliger [52] who went to the opposite extreme and regarded the type of budding described by Lang as non-existent. In view, however, both of the statements and figures of Lang and of the facts to be described presently for medusae (Margellium), it is at least theoretically possible that both germinal and vegetative budding may occur in polyps as well as in medusae.
(b) The Medusa.-The clearest instance of germinal budding is furnished by Margellium (Rathkea) octopunctatum, one of the Margelidae. The budding of this medusa has been worked out in detail by Chun (Hydrozoa, [1]), to whom the reader must be referred for the interesting laws of budding regulating the sequence and order of formation of the buds.

The buds of Margellium are produced on the manubrium in each of the four interradii, and they arise from the ectoderm, that is to say, the germinal epithelium, which later gives rise to the gonads. The buds do not appear simultaneously but successively on each of the four sides of 1
the manubrium, thus: $3{ }_{2}^{1} 4$ and secondary buds may be produced on the medusa-buds
before the latter are set free as medusae. Each bud arises as a thickening of the epithelium, which first forms two or three layers (fig. 47, A), and becomes separated into a superficial layer, future ectoderm, surrounding a central mass, future endoderm (fig. 47, B). The ectodermal epithelium on the distal side of the bud becomes thickened, grows inwards, and forms a typical entocodon (fig. 37, D, E, F). The remaining development of the bud is just as described above for the indirect method of medusa-budding (fig. 47, G, H). When the bud is nearly complete, the body-wall of the parent immediately below it becomes perforated, placing the coelenteric cavity of the parent in secondary communication with that of the bud $(\mathrm{H})$, doubtless for the better nutrition of the latter.

Especially noteworthy in the germinal budding of Margellium is the formation of the entocodon, as in the vegetative budding of the indirect type.
5. Sporogony.-This method of reproduction has been described by E. Metchnikoff in Cunina and allied genera. In individuals either of the male or female sex, germ-cells which are quite undifferentiated and neutral in character, become amoeboid, and wander into the endoderm. They divide each into two sister-cells, one of which-the spore-becomes enveloped by the other. The spore-cell multiplies by division, while the enveloping cell is nutrient and protective. The spore cell gives rise to a "spore-larva," which is set free in the coelenteron and grows into a medusa. Whether sporogony occurs also in the polyp or not remains to be proved.
6. Sexual Reproduction and Embryology.-The ovum of Hydromedusae is usually one of a large number of oögonia, and grows at the expense of its sister-cells. No regular follicle is formed, but the oöcyte absorbs nutriment from the remaining oögonia. In Hydra the oöcyte is a large amoeboid cell, which sends out pseudopodia amongst the oögonia and absorbs nutriment from them. When the oöcyte is full grown, the residual oögonia die off and disintegrate.


Fig. 47.-Budding from the Ectoderm (germinal epithelium) in Margellium. (After C. Chun.)

A, The epithelium becomes twolayered.
B, The lower layer forms a solid mass of cells, which (C) becomes a vesicle, the future endoderm, containing the coelenteric cavity (coel), while the outer layer furnishes the future ectoderm.
D, E, F, a thickening of the ectoderm on the distal side of the bud forms an entocodon ( $G c$ ).

G,H, Formation of the medusae. s.c, Sub-umbral cavity.
r.c, Radial canal.
st, Stomach, which in H acquires a secondary communication with the digestive cavity of the mother.
CC, Circular canal.
$v$, Velum.
$t$, Tentacle.

The spermatogenesis and maturation and fertilization of the germ-cells present nothing out of the common and need not be described here. These processes have been studied in detail by A. Brauer [2] for Hydra.

The general course of the development is described in the article Hydrozoa. We may distinguish the following series of stages: (1) ovum; (2) cleavage, leading to formation of a blastula; (3) formation of an inner mass or parenchyma, the future endoderm, by immigration or delamination, leading to the so-called parenchymula-stage; (4) formation of an archenteric cavity, the future coelenteron, by a splitting of the internal parenchyma, and of a blastopore, the future mouth, by perforation at one pole, leading to the gastrula-stage; (5) the outgrowth of tentacles round the mouth (blastopore), leading to the actinula-stage; and (6) the actinula becomes the polyp or medusa in the manner described elsewhere (see articles Hydrozoa, Polyp and Medusa). This is the full, ideal development, which is always contracted or shortened to a greater or less extent. If the embryo is set free as a free-swimming, so-called planula-larva, in the blastula, parenchymula, or gastrula stage, then a free actinula stage is not found; if, on the other hand, a free actinula occurs, then there is no free planula stage.

The cleavage of the ovum follows two types, both seen in Tubularia (Brauer [3]). In the first, a cleavage follows each nuclear division; in the second, the nuclei multiply by division a number of times, and then the ovum divides into as many blastomeres as there are nuclei present. The result of cleavage in all cases is a typical blastula, which when set free becomes oval and develops a flagellum to each cell, but when not set free, it remains spherical in form and has no flagella.

The germ-layer formation is always by immigration or delamination, never by invagination. When the blastula is oval and free-swimming the inner mass is formed by unipolar immigration from the hinder pole. When the blastula is spherical and not set free, the germ-layer formation is always multipolar, either by immigration or by delamination, i.e. by tangential division of the cells of the blastoderm, as in Geryonia, or by a mixture of immigration and delamination, as in Hydra, Tubularia, \&c. The blastopore is formed as a secondary perforation at one spot, in freeswimming forms at the hinder pole. Formation of archenteron and blastopore may, however, be deferred till a later stage (actinula or after).

The actinula stage is usually suppressed or not set free, but it is seen in Tubularia (fig. 48), where it is ambulatory, in Gonionemus (Trachomedusae), and in Cunina (Narcomedusae),

In Leptolinae the embryonic development culminates in a polyp, which is usually formed by fixation of a planula (parenchymula), rarely by fixation of an actinula. The planula may fix itself (1) by one end, and then becomes the hydrocaulus and hydranth, while the hydrorhiza grows out from the base; or (2) partly by one side and then gives rise to the hydrorhiza as well as to the other parts of the polyp; or (3) entirely by its side, and then forms a recumbent hydrorhiza from which a polyp appears to be budded as an upgrowth.

In Trachylinae the development produces always a medusa, and there is no polyp-stage. The medusa arises direct from the actinula-stage and there is no entocodon formed, as in the budding described above.


Modified from a plate by L. Agassiz, Contributions to Nat. Hist. U.S., iv.

Fig. 48.-Free Actinula of Tubularia.

Life-cycles of the Hydromedusae.-The life-cycle of the Leptolinae consists of an alternation of generations in which non-sexual individuals, polyps, produce by budding sexual individuals, medusae, which give rise by the sexual process to the non-sexual polyps again, so completing the cycle. Hence the alternation is of the type termed metagenesis. The Leptolinae are chiefly forms belonging to the inshore fauna. The Trachylinae, on the other hand, are above all oceanic forms, and have no polyp-stage, and hence there is typically no alternation in their life-cycle. It is commonly assumed that the Trachylinae are forms which have lost the alternation of generations possessed by them ancestrally, through secondary simplification of the life-cycle. Hence the Trachylinae are termed "hypogenetic" medusae to contrast them with the metagenetic Leptolinae. The whole question has, however, been argued at length by W. K. Brooks [4], who adduces strong evidence for a contrary view, that is to say, for regarding the direct type of development seen in Trachylinae as more primitive, and the metagenesis seen in Leptolinae as a secondary complication introduced into the life-cycle by the acquisition of larval budding. The polyp is regarded, on this view, as a form phylogenetically older than the medusa, in short, as nothing more than a sessile actinula. In Trachylinae the polyp-stage is passed over, and is represented only by the actinula as a transitory embryonic stage. In Leptolinae the actinula becomes the sessile polyp which has acquired the power of budding and producing individuals either of its own or of a higher rank; it represents a persistent larval stage and remains in a sexually immature condition as a neutral individual, sex being an attribute only of the final stage in the development, namely the medusa. The polyp of the Leptolinae has reached the limit of its individual development and is incapable of becoming itself a medusa, but only produces medusa-buds; hence a true alternation of generations is produced. In Trachylinae also the beginnings of a similar metagenesis can be found. Thus in Cunina octonaria, the ovum develops into an actinula which buds daughter-actinulae; all of them, both parent and offspring, develop into medusae, so that there is no alternation of generations, but only larval multiplication. In Cunina parasitica, however, the ovum develops into an actinula, which buds actinulae as before, but only the daughter-actinulae develop into medusae, while the original, parent-actinula dies off; here, therefore, larval budding has led to a true alternation of generations. In Gonionemus the actinula becomes fixed and polyp-like, and reproduces by budding, so that here also an alternation of generations may occur. In the Leptolinae we must first substitute polyp for actinula, and then a condition is found which can be compared to the case of Cunina parasitica or Gonionemus, if we suppose that neither the parent-actinula (i.e. founder-polyp) nor its offspring by budding (polyps of the colony) have the power of becoming medusae, but only of producing medusae by budding. For further arguments and illustrations the reader must be referred to Brooks's most interesting memoir. The whole theory is one most intimately connected with the question of the relation between polyp and medusa, to be discussed presently. It will be seen elsewhere, however, that whatever view may be held as to the origin of metagenesis in Hydromedusae, in the case of Scyphomedusae (q.v.) no other view is possible than that the alternation of generations is the direct result of larval proliferation.

To complete our survey of life-cycles in the Hydromedusae it is necessary to add a few words about the position of Hydra and its allies. If we accept the view that Hydra is a true sexual polyp, and that its gonads are not gonophores (i.e. medusa-buds) in the extreme of degeneration, then it follows from Brooks's theory that Hydra must be descended from an archaic form in which the medusan type of organization had not yet been evolved. Hydra must, in short, be a living representative of the ancestor of which the actinula-stage is a transient reminiscence in the development of higher forms. It may be pointed out in this connexion that the fixation of Hydra is only temporary, and that the animal is able at all times to detach itself, to move to a new situation, and to fix itself again. There is no difficulty whatever in regarding Hydra as bearing the same relation to the actinula-stage of other Hydromedusae that a Rotifer bears to a trochophore-larva or a fish to a tadpole.

The Relation of Polyp and Medusa.-Many views have been put forward as to the morphological relationship between the two types of person in the Hydromedusae. For the most part, polyp and medusa have been regarded as modifications of a common type, a view
supported by the existence, among Scyphomedusae (q.v.), of sessile polyp-like medusae (Lucernaria, \&c.). R. Leuckart in 1848 compared medusae in general terms to flattened polyps. G. J. Allman [1] put forward a more detailed view, which was as follows. In some polyps the tentacles are webbed at the base, and it was supposed that a medusa was a polyp of this kind set free, the umbrella being a greatly developed web or membrane extending between the tentacles. A very different theory was enunciated by E. Metchnikoff. In some hydroids the founder-polyp, developed from a planula after fixation, throws out numerous outgrowths from the base to form the hydrorhiza; these outgrowths may be radially arranged so as to form by contact or coalescence a flat plate. Mechnikov considered the plate thus formed at the base of the polyp as equivalent to the umbrella, and the body of the polyp as equivalent to the manubrium, of the medusa; on this view the marginal tentacles almost invariably present in medusae are new formations, and the tentacles of the polyp are represented in the medusa by the oral arms which may occur round the mouth, and which sometimes, e.g. in Margelidae, have the appearance and structure of tentacles. Apart from the weighty arguments which the development furnishes against the theories of Allman and Mechnikov, it may be pointed out that neither hypothesis gives a satisfactory explanation of a structure universally present in medusae of whatever class, namely the endoderm-lamella, discovered by the brothers O. and R. Hertwig. It would be necessary to regard this structure as a secondary extension of the endoderm in the tentacle-web, on Allman's theory, or between the outgrowths of the hydrorhiza, on Mechnikov's hypothesis. The development, on the contrary, shows unequivocally that the endoderm-lamella arises as a local coalescence of the endodermal linings of a primitively extensive gastral space.

The question is one intimately connected with the view taken as to the nature and individuality of polyp, medusa and gonophore respectively. On this point the following theories have been put forward.

1. The theory that the medusa is simply an organ, which has become detached and has acquired a certain degree of independence, like the well-known instance of the hectocotyle of the cuttle-fish. On this view, put forward by E. van Beneden and T. H. Huxley, the sporosac is the starting-point of an evolution leading up through the various types of gonophores to the free medusa as the culminating point of a phyletic series. The evidence against this view may be classed under two heads: first, comparative evidence; hydroids very different in their structural characters and widely separate in the systematic classification of these organisms may produce medusae very similar, at least so far as the essential features of medusan organization are concerned; on the other hydroids closely allied, perhaps almost indistinguishable, may produce gonophores in the one case, medusae in the other; for example, Hydractinia (gonophores) and Podocoryne (medusae), Tubularia (gonophores) and Ectopleura (medusae), Coryne (gonophores) and Syncoryne (medusae), and so on. If it is assumed that all these genera bore gonophores ancestrally, then medusa of similar type must have been evolved quite independently in a great number of cases. Secondly, there is the evidence from the development, namely, the presence of the entocodon in the medusa-bud, a structure which, as explained above, can only be accounted for satisfactorily by derivation from a medusan type of organization. Hence it may be concluded that the gonophores are degenerate medusae, and not that the medusae are highly elaborated gonophores, as the organ-theory requires.
2. The theory that the medusa is an independent individual, fully equivalent to the polyp in this respect, is now universally accepted as being supported by all the facts of comparative morphology and development. The question still remains open, however, which of the two types of person may be regarded as the most primitive, the most ancient in the race-history of the Hydromedusae. F. M. Balfour put forward the view that the polyp was the more primitive type, and that the medusa is a special modification of the polyp for reproductive purposes, the result of division of labour in a polyp-colony, whereby special reproductive persons become detached and acquire organs of locomotion for spreading the species. W. K. Brooks, on the other hand, as stated above, regards the medusa as the older type and looks upon both polyp and medusa, in the Hydromedusae, as derived from a free-swimming or floating actinula, the polyp being thus merely a fixed nutritive stage, possessing secondarily acquired powers of multiplication by budding.

The Hertwigs when they discovered the endoderm-lamella showed on morphological grounds that polyp and medusa are independent types, each produced by modification in different directions of a more primitive type represented in development by the actinula-stage. If a polyp, such as Hydra, be regarded simply as a sessile actinula, we must certainly consider the polyp to be the older type, and it may be pointed out that in the Anthozoa only polyp-individuals occur. This must not be taken to mean, however, that the medusa is derived from a sessile polyp; it must be regarded as a direct modification of the more ancient free actinula form, without primitively any intervening polyp-stage, such as has been introduced secondarily into the development of the Leptolinae and represents a revival, so to speak, of an ancestral form or larval stage, which has taken on a special role in the economy of the species.

Order I. Eleutheroblastea.-Simple polyps which become sexually mature and which also reproduce non-sexually, but without any medusoid stage in the life-cycle.

The sub-order includes the family Hydridae, containing the common fresh-water polyps of the genus Hydra. Certain other forms of doubtful affinities have also been referred provisionally to this section.

Hydra.-This genus comprises fresh-water polyps of simple structure. The body bears tentacles, but shows no division into hydrorhiza, hydrocaulus or hydranth; it is temporarily fixed and has no perisarc. The polyp is usually hermaphrodite, developing both ovaries and testes in the same individual. There is no free-swimming planula larva, but the stage corresponding to it is passed over in an enveloping cyst, which is secreted round the embryo by its own ectodermal layer, shortly after the germ-layer formation is complete, i.e. in the parenchymula-stage. The envelope is double, consisting of an external chitinous stratified shell, and an internal thin elastic membrane. Protected by the double envelope, the embryo is set free as a so-called "egg," and in Europe it passes the winter in this condition. In the spring the embryo bursts its shell and is set free as a minute actinula which becomes a Hydra.
Many species are known, of which three are common in European waters. It has been shown by C. F. Jickeli (28) that the species are distinguishable by the characters of their nematocysts. They also show characteristic differences in the egg (Brauer [2]). In Hydra viridis the polyp is of a green colour and produces a spherical egg with a smooth shell which is dropped into the mud. H. grisea is greyish in tint and produces a spherical egg with a spiky shell, which also is dropped into the mud. H. fusca ( $=H$. vulgaris) is brown in colour, and produces a bun-shaped egg, spiky on the convex surface, and attached to a water-weed or some object by its flattened side. Brauer found a fourth species, similar in appearance to H. fusca, but differing from the three other species in being of separate sexes, and in producing a spherical egg with a knobby shell, which is attached like that of $H$. fusca.

The fact already noted that the species of Hydra can be distinguished by the characters of their nematocysts is a point of great interest. In each species, two or three kinds of nematocysts occur, some large, some small, and for specific identification the nematocysts must be studied collectively in each species. It is very remarkable that this method of characterizing and diagnozing species has never been extended to the marine hydroids. It is quite possible that the characters of the nematocysts might afford data as useful to the systematist in this group as do the spicules of sponges, for instance. It would be particularly interesting to ascertain how the nematocysts of a polyp are related to those possessed by the medusa budded from it, and it is possible that in this manner obscure questions of relationship might be cleared up.

Protohydra is a marine genus characterized by the absence of tentacles, by a great similarity to Hydra in histological structure, and by reproduction by transverse fission. It was found originally in an oyster-farm at Ostend. The sexual reproduction is unknown. For further information see C. Chun (HydrozoA [1]. Pl. I.).

Polypodium hydriforme Ussow is a freshwater form parasitic on the eggs of the sterlet. A "stolon" of unknown origin produces thirtytwo buds, which become as many Polypodia; each has twenty-four tentacles and divides by fission repeated twice into four individuals, each with six tentacles. The daughterindividuals grow, form the full number of twenty-four tentacles and divide again. The polyps are free and walk on their tentacles. See Ussow [54].

Tetraplatia volitans Viguier is a remarkable floating marine form. See C. Viguier [56] and Delage and Hérouard (Hydrozoa [2]).

Haleremita Schaudinn. See F. Schaudinn [50] and Delage and Hérouard (Hydrozoa [2]).

In all the above-mentioned genera, with the exception of Hydra, the life-cycle is so imperfectly known that their true position cannot be determined in the present state of


Fig. 49.-Diagram showing possible modifications of persons of a gymnoblastic Hydromedusa. (After Allman.)
a, Hydrocaulus (stem).
our knowledge. They may prove eventually to belong to other orders. Hence only the genus Hydra can be considered as truly representing the order Eleutheroblastea. The phylogenetic position of this genus has been discussed above.

Order II. Hydroidea seu Leptolinae.Hydromedusae with alternation of generations (metagenesis) in which a non-sexual polypgeneration (trophosome) produces by budding a sexual medusa-generation (gonosome). The polyp may be solitary, but more usually produces polyps by budding and forms a polyp-colony. The polyp usually has the body distinctly divisible into hydranth, hydrocaulus and hydrorhiza, and is usually clothed in a perisarc. The medusae may be set free or may remain attached to the polyp-colony and degenerate into a gonophore. When fully developed the medusa is characterized by the sense organs being composed entirely of ectoderm, developed independently of the tentacles, and innervated from the sub-umbral nerve-ring.

The two kinds of persons present in the typical Hydroidea make the classification of the group extremely difficult, for reasons explained above. Hence the systematic arrangement that follows must be considered purely provisional. A natural classification of the Hydroidea has yet to be put forward. Many genera and families are separated by purely artificial characters, mere shelf-and-bottle groupings devised, for the convenience of the museum curator and the collector. Thus many subdivisions are diagnosed by setting free medusae in one case, or producing gonophores in another, although it is very obvious, as pointed out above, that a genus producing medusae may be far more closely allied to one producing gonophores than to another producing medusae, or vice versa, and that in some cases the production of medusae or gonophores varies with the season or the sex. Moreover, P. Hallez [22] has recently shown that hydroids hitherto regarded as distinct species are only forms of the same species grown under different conditions.

Sub-Order 1. Hydroidea Gymnoblastea (Anthomedusae).-Trophosome without hydrothecae or gonothecae, with monopodial type of budding. Gonosome with free medusae or gonophores; medusae usually with ocelli, never with otocysts. The gymnoblastic polyp usually has a distinct perisarc investing the hydrorhiza and the hydrocaulus, sometimes also the hydranth as far as the bases of the tentacles (Bimeria); but in such cases the perisarc forms a closely-fitting investment or cuticule on the hydranth, never a hydrotheca standing off from it, as in the next sub-order. The polyps may be solitary, or form colonies, which may be of the spreading or encrusting type, or arborescent, and then always of monopodial growth and budding. In some cases, any polyp of the colony may bud medusae; in other cases, only certain polyps, the blastostyles, have this power. When blastostyles are present, however, they are never enclosed in special gonothecae as in the next sub-order. In this sub-order the characters of the hydranth are very variable, probably owing to the fact that it is exposed and not protected by a hydrotheca, as in Calyptoblastea.


Fig. 50.-Sarsia (Dipurena) gemnifera. $b$, The long manubrium, bearing medusiform buds; a, mouth.

Fig. 51.-Sarsia prolifera. Ocelli are seen at the base of the tentacles, and also (as an exception) groups of medusiform buds.

Speaking generally, three principal types of hydranth can be distinguished, each with subordinate varieties of form.

1. Club-shaped hydranths with numerous tentacles, generally scattered irregularly, sometimes with a spiral arrangement, or in whorls ("verticillate").
(a) Tentacles filiform; type of Clava (fig. 5), Cordylophora, \&c.
(b) Tentacles capitate, simple; type of Coryne and Syncoryne; Myriothela is an aberrant form with some of the tentacles modified as "claspers" to hold the ova.
(c) Tentacles capitate, branched, wholly or in part; type of Cladocoryne.
(d) Tentacles filiform or capitate, tending to be arranged in definite whorls; type of Stauridium (fig. 2), Cladonema and Pennaria.
2. Hydranth more shortened, daisy-like in form, with two whorls of tentacles, oral and aboral.
(a) Tentacles filiform, simple, radially arranged or scattered irregularly; type of Tubularia (fig. 4), Corymorpha (fig. 3), Nemopsis, Pelagohydra, \&c.
(b) Tentacles with a bilateral arrangement, branched tentacles in addition to simple filiform ones; type of Branchiocerianthus.
3. Hydranth with a single circlet of tentacles.
(a) With filiform tentacles; the commonest type, seen in Bougainvillea (fig. 13), Eudendrium, \&c.
(b) With capitate tentacles; type of Clavatella.
4. Hydranth with tentacles reduced below four; type of Lar (fig. 11), Monobrachium, \&c.

The Anthomedusa in form is generally deep, bell-shaped. The sense organs are typically ocelli, never otocysts. The gonads are borne on the manubrium, either forming a continuous ring (Codonid type), or four masses or pairs of masses (Oceanid type). The tentacles may be scattered singly round the margin of the umbrella ("monerenematous") or arranged in tufts ("lophonematous"); in form they may be simple or branched (Cladonemid type); in structure they may be hollow ("coelomerinthous"); or solid ("pycnomerinthous"). When sessile gonophores are produced, they may show all stages of degeneration.

Classification.-Until quite recently the hydroids (Gymnoblastea) and the medusae (Anthomedusae) have been classified separately, since the connexion between them was insufficiently known. Delage and Hérouard (Hydrozoa [2]) were the first to make an heroic attempt to unite the two classifications into one, to which Hickson (Hydrozoa [4]) has made some additions and slight modifications. The classification given here is for the most part that of Delage and Hérouard. It is certain, however, that no such classification can be considered final at present, but must undergo continual revision in the future. With this reservation we may recognize fifteen well-characterized families and others of more doubtful nature. Certain discrepancies must also be noted.

1. Margelidae (= medusa-family Margelidae + hydroid families Bougainvillidae, Dicorynidae, Bimeridae and Eudendridae). Trophosome arborescent, with hydranths of Bougainvillea-type; gonosome free medusae or gonophores, the medusae with solid tentacles in tufts (lophonematous). Common genera are the hydroid Bougainvillea (figs. 12, 13), and the medusae Hippocrene (budded from Bougainvillea), Margelis, Rathkea (fig. 24), and Margellium. Other hydroids are Garveia, Bimeria, Eudendrium and Heterocordyle, with gonophores, and Dicoryne with peculiar sporosacs.
2. Podocorynidae ( $=$ medusa-families Thamnostomidae and Cytaeidae + hydroid families Podocorynidae and Hydractiniidae). Trophosome encrusting with hydranths of Bougainvillea-type, polyps differentiated into blastostyles, gastrozoids and dactylozoids; gonosome free medusae or gonophores. The typical genus is the well-known hydroid Podocoryne, budding the medusa known as Dysmorphosa; Thamnostylus, Cytaeis, \&c., are other medusae with unknown hydroids. Hydractinia (figs. 9, 10) is a familiar hydroid genus, bearing
gonophores.
3. Cladonemidae.-Trophosome, polyps with two whorls of tentacles, the lower filiform, the upper capitate; gonosome, free medusae, with tentacles solid and branched. The type-genus Cladonema (fig. 20 ) is a common British form.
4. Clavatellidae.-Trophosome, polyps with a single whorl of capitate tentacles; gonosome, free medusae, with tentacles branched, solid. Clavatella (fig. 21), with a peculiar ambulatory medusa is a British form.
5. Pennariidae.-Trophosome, polyps with an upper circlet of numerous capitate tentacles, and a lower circlet of filiform tentacles. Pennaria, with a free medusa known as Globiceps, is a common Mediterranean form. Stauridium (fig. 2) is a British hydroid.
6. Tubulariidae.-Trophosome, polyps with two whorls of tentacles, both filiform. Tubularia (fig. 4), a well-known British hydroid, bears gonophores.
7. Corymorphidae (including the medusa-family Hybocodonidae).Trophosome solitary polyps, with two whorls of tentacles; gonosome, free medusae or gonophores. Corymorpha (fig. 3), a well-known British genus, sets free a medusa known as Steenstrupia (fig. 22).


After Haeckel, System der Medusen, by permission of Gustav Fischer.

Fig. 52.-Tiara pileata, L. Agassiz. Here belong the deep-sea genera Monocaulus and Branchiocerianthus, including the largest hydroid polyps known, both genera producing sessile gonophores.


After Haeckel, System der Medusen, by permission of Gustav Fischer.
Fig. 53.-Pteronema darwinii. The apex of the stomach is prolonged into a
8. Dendroclavidae.-Trophosome, polyp with filiform
tentacles in three or four whorls. Dendroclava, a hydroid, produces the medusa known as Turritopsis.
9. Clavidae (including the medusa-family Tiaridae (figs. 27 and 51). Trophosome, polyps with scattered filiform tentacles; gonosome, medusae or gonophores, the medusae with hollow tentacles. Clava (fig. 5), a common British hydroid, produces gonophores; so also does Cordylophora, a form inhabiting fresh or brackish water. Turris produces free medusae. Amphinema is a medusan genus of unknown hydroid.
10. Bythotiaridae.-Trophosome unknown; gonosome, free medusae, with deep, bell-shaped umbrella, with interradial gonads on the base of the stomach, with branched radial canals, and correspondingly numerous hollow tentacles. Bythotiara, Sibogita.
11. Corynidae ( $=$ hydroid families Corynidae, Syncorynidae and Cladocorynidae + medusan family Sarsiidae).-Trophosome polyps with capitate tentacles, simple or branched, scattered or verticillate; gonosome, free medusae or gonophores. Coryne, a common British hydroid, produces gonophores; Syncoryne, indistinguishable from it, produces medusae known as Sarsia (fig. 51). Cladocoryne is another hydroid genus; Codonium and Dipurena (fig. 50) are medusan genera.
12. Myriothelidae.-The genus Myriothela is a solitary polyp with scattered capitate tentacles, producing sporosacs.
13. Hydrolaridae.-Trophosome (only known in one genus), polyps with two tentacles forming a creeping colony;
brood pouch containing embryos.
gonosome, free medusae with four, six or more radial canals, giving off one or more lateral branches which run to the margin of the umbrella, with the stomach produced into four, six or more lobes, upon which the gonads are developed; the mouth with four lips or with a folded margin; the tentacles simple, arranged evenly round the margin of the umbrella. The remarkable hydroid Lar (fig. 11) grows upon the tubes of the worm Sabella and produces a medusa known as Willia. Another medusan genus is Proboscidactyla.
14. Monobrachiidae.-The genus Monobrachium is a colony-forming hydroid which grows upon the shells of bivalve molluscs, each polyp having but a single tentacle. It buds medusae, which, however, are as yet only known in an immature condition (C. Mereschkowsky [41]).
15. Ceratellidae.-Trophosome polyps forming branching colonies of which the stem and main branches are thick and composed of a network of anastomosing coenosarcal tubes covered by a common ectoderm and supported by a thick chitinous perisarc; hydranths similar to those of Coryne; gonosome, sessile gonophores. Ceratella, an exotic genus from the coast of East Africa, New South Wales and Japan. The genera Dehitella Gray and Dendrocoryne Inaba should perhaps be referred to this family; the last-named is regarded by S. Goto [16] as the type of a distinct family, Dendrocorynidae.

Doubtful families, or forms difficult to classify, are: Pteronemidae, Medusae of Cladonemid type, with hydroids for the most part unknown. The British genus Gemmaria, however, is budded from a hydroid referable to the family Corynidae. Pteronema (fig. 53).

Nemopsidae, for the floating polyp Nemopsis, very similar to Tubularia in character; the medusa, on the other hand, is very similar to Hippocrene (Margelidae). See C. Chun (Hydrozoa [1]).

Pelagohydridae, for the floating polyp Pelagohydra, Dendy, from New Zealand. The animal is a solitary polyp bearing a great number of medusa-buds. The body, representing the hydranth of an ordinary hydroid, has the aboral portion modified into a float, from which hangs down a proboscis bearing the mouth. The float is covered with long tentacles and bears the medusabuds. The proboscis bears at its extremity a circlet of smaller oral tentacles. Thus the affinities of the hydranth are clearly, as Dendy points out, with a form such as Corymorpha, which also is not fixed but only rooted in the mud. The medusae, on the other hand, have the tentacles in four tufts of (in the buds) five each, and thus resemble the medusae of the family Margelidae. See A. Dendy [12].

Perigonimus.-This common British hydroid belongs by its characters to the family Bougainvillidae; it produces, however, a medusa of the genus Tiara (fig. 52), referable to the family Clavidae; a fact sufficient to indicate the tentative character of even the most modern classifications of this order.

Sub-order II. Hydroidea (Leptomedusae).-Trophosome always differentiated into reproductive individuals

Calyptoblastea with polyps nutritive and (blastostyles) enclosed in hydrothecae and gonothecae respectively; with sympodial type of budding. Gonosome with free medusae or gonophores; the medusae typically with otocysts, sometimes with cordyli or ocelli (figs. 54, 55).


Fig. 55.-View of the Oral Surface of one of the Leptomedusae (Irene


Fig. 54.-Diagram showing possible modifications of the persons of a Calyptoblastic Hydromedusa. Letters $a$ to $h$ same as in fig. 49. $i$, The horny cup or hydrotheca of the hydriform persons; $l$, medusiform person springing from $m$, a modified, hydriform person (blastostyle); n, the horny case or gonangium enclosing the
pellucida, Haeckel), to show the numerous tentacles and the otocysts.
$g e$, Genital glands.
$M$, Manubrium.
ot, Otocysts.
$r c$, The four radiating canals.
$V e$, The velum.
blastostyle and its buds. This and the hydrotheca $i$ give origin to the name Calyptoblastea. (After Allman.)

The calyptoblastic polyp of the nutritive type is very uniform in character, its tendency to variation being limited, as it were, by the enclosing hydrotheca. The hydranth almost always has a single circlet of tentacles, like the Bougainvillea-type, in the preceding sub-order; an exception is the curious genus Clathrozoon, in which the hydranth has a single tentacle. The characteristic hydrotheca is formed by the bud at an early stage (fig. 56); when complete it is an open cup, in which the hydranth develops and can be protruded from the opening for the capture of food, or is withdrawn into it for protection. Solitary polyps are unknown in this sub-order; the colony may be creeping or arborescent in form; if the latter, the budding of the polyps, as already stated, is of the sympodial type, and either biserial, forming stems capable of further branching, or uniserial, forming pinnules not capable of further branching. In the biserial type the polyps on the two sides of the stem have primitively an alternating, zigzag arrangement; but, by a process of differential growth, quickened in the 1 st, 3 rd, 5 th, \&c., members of the stem, and retarded in the 2 nd, 4 th, 6 th, \&c., members, the polyps may assume secondarily positions opposite to one another on the two sides of the stem. Other variations in the mode of growth or budding bring about further differences in the building up of the colony, which are not in all cases properly understood and cannot be described in detail here. The stem may contain a single coenosarcal tube ("monosiphonic") or several united in a common perisarc ("polysiphonic"). An important variation is seen, in the form of the hydrotheca itself, which may come off from the main stem by a stalk, as in Obelia, or may be sessile, without a stalk, as in Sertularia.

In many Calyptoblastea there occur also reduced defensive polyps or dactylozoids, which in this sub-order have received the special name of sarcostyles. Such are the "snake-like zoids" of Ophiodes and other genera, and as such are generally interpreted the "machopolyps" of the Plumularidea. These organs are supported by cuplike structures of the perisarc, termed nematophores, regarded as modified hydrothecae supporting the specialized polyp-individuals. They are specially characteristic of the family Plumularidae.

The medusa-buds, as already stated, are always produced from blastostyles, reduced non-nutritive polyps without mouth or tentacles. An apparent, but not real, exception is Halecium halecinum, in which the blastostyle is produced from the side of a nutritive polyp, and both are enclosed in a common theca without a partition between them (Allman [1] p. 50, fig. 24). The gonotheca is formed in its early stage in the same way as the hydrotheca, but the remains of the hydranth persists as an operculum closing the capsule, to be withdrawn when the medusae or genital products are set free (fig. 56).

The blastostyles, gonophores and gonothecae furnish a series of variations which can best be considered as so many stages of evolution.

Stage 1, seen in Obelia. Numerous medusae are budded successively within the gonotheca and set free; they swim off and mature in the open sea (Allman [1], p. 48, figs. 18, 19).


After Allman, Gymnoblastic Hydroids, by permission of the council of the Ray Society.

Fig. 56.-Diagrams to show the mode of formation of the Hydrotheca and Gonotheca in Calyptoblastic Hydroids. A-D are stages common to both; from D arises the hydrotheca ( E ) or the gonotheca ( F ); th, theca; $s t$, stomach; $t$, tentacles; $m$, mouth; $m b$, medusa-buds.

Stage 2, seen in Gonothyraea. Medusae, so-called "meconidia," are budded but not liberated; each in turn, when it reaches sexual maturity, is protruded from the gonotheca by elongation of the stalk, and sets free the embryos, after which it withers and is replaced by another (Allman [1], p. 57, fig. 28).

Stage 3, seen in Sertularia.-The gonophores are reduced in varying degree, it may be to
sporosacs; they are budded successively from the blastostyle, and each in turn, when ripe, protrudes the spadix through the gonotheca (fig. 57, A, B). The spadix forms a gelatinous cyst, the so-called acrocyst ( $a c$ ), external to the gonotheca ( $g t h$ ), enclosing and protecting the embryos. Then the spadix withers, leaving the embryos in the acrocyst, which may be further protected by a so-called marsupium, a structure formed by tentacle-like processes growing out from the blastostyle to enclose the acrocyst, each such process being covered by perisarc like a glove-finger secreted by it (fig. 57, C). (Allman [1], pp. 50, 51, figs. 21-24; Weismann [58], p. 170, pl. ix., figs. 7, 8.)

Stage 4, seen in Plumularidae.-The generative elements are produced in structures termed corbulae, formed by reduction and modification of branches of the colony. Each corbula contains a central row of blastostyles enclosed and protected by lateral rows of branches representing stunted buds (Allman [1], p. 66, fig. 30).

The Leptomedusa in form is generally shallow, more or less saucer-like, with velum less developed than in Anthomedusae (fig. 55). The characteristic sense-organs are ectodermal otocysts, absent, however, in some genera, in which case cordyli may replace them. When otocysts are present, they are at least eight in number, situated adradially, but are often very numerous. The cordyli are scattered on the ring-canal. Ocelli, if present, are borne on the tentaclebulbs. The tentacles are usually hollow, rarely solid (Obelia). In number they are rarely less than four, but in Dissonema there are only two. Primitively there are four perradial tentacles, to which may be added four interradial, or they may become very numerous and are then scattered evenly round the margin, never arranged in tufts or clusters. In addition to tentacles, there may be marginal cirri (Laodice) with a solid endodermal axis, spirally coiled, very contractile, and bearing a terminal battery of nematocysts. The gonads are developed typically beneath the radial canals or below the stomach or its pouches, often stretching as long bands on to the base of the manubrium. In Octorchidae (fig. 58) each such band is interrupted, forming one mass at the base of the manubrium and another below the radial canal in each radius, in all eight separate gonad-masses, as the name implies. In some Leptomedusae excretory "marginal tubercles" are developed on the ring-canal.

Classification.-As in the Gymnoblastea, the difficulty of uniting the hydroid and medusan systems into one scheme of classification is very great in the present state of our knowledge. In a great many Leptomedusae the hydroid stage is as yet unknown, and it is by no means certain even that they possess one. It is quite possible that some of these medusae will be found to be truly hypogenetic, that is to say, with a life-cycle secondarily simplified by suppression of metagenesis. At present, ten recent and one extinct family of Calyptoblastea (Leptomedusae) may be recognized provisionally:

1. Eucopidae (figs. 55, 59).-Trophosome with stalked hydrothecae; gonosome, free medusae with otocysts and four, rarely six or eight, unbranched radial canals. Two of the commonest British hydroids belong to this family, Obelia and Clytia. Obelia forms numerous polyserial stems of the characteristic zigzag pattern growing up from a creeping basal stolon, and buds the medusa of the same name. In Clytia the polyps arise singly from the stolon, and the medusa is known as Phialidium (fig. 59).
2. Aequoridae.-Trophosome only known in one genus (Polycanna), and similar to the preceding; gonosome, free medusae with otocysts and with at least eight radial canals, often a hundred or more, simple or branched. Aequorea is a common medusa.
3. Thaumantidae.-Trophosome only known in one genus (Thaumantias), similar to that of the Eucopidae; gonosome, free medusae with otocysts inconspicuous or absent, with usually four, sometimes eight, rarely more than eight, radial canals, simple and unbranched, along which the gonads are developed, with numerous tentacles bearing ocelli and with marginal sense-clubs. Laodice and Thaumantias are representative genera.
4. Berenicidae.-Trophosome unknown; gonosome, free medusae, with four or six radial canals, bearing the gonads, with numerous tentacles, between which occur sense-clubs, without otocysts. Berenice, Staurodiscus, \&c.


After Haeckel, System der Medusen, by permission of Gustav Fischer. Fig. 58.-Octorchandra canariensis, from life.
5. Polyorchidae.-Trophosome unknown; gonosome, free medusae of deep form, with radial canals branched in a feathery manner, and bearing gonads on the main canal, but not on the branches, with numerous hollow tentacles bearing ocelli, and without otocysts. Polyorchis, Spirocodon.
6. Campanularidae.-Trophosome as in Eucopidae; gonosome, sessile gonophores. Many common or well-known genera belong here, such as Halecium, Campanularia, Gonothyraea, \&c.
7. Lafoëidae.-Trophosome as in the preceding; gonosome, free medusae or gonophores, the medusae with large open otocysts. The hydroid genus Lafoëa is remarkable for producing gonothecae on the hydrorhiza, each containing a blastostyle which bears a single gonophore; this portion of the colony was formerly regarded as an independent parasitic hydroid, and was named Coppinia. Medusan genera are Mitrocoma, Halopsis, Tiaropsis (fig. 29, \&c.).
(So far as the characters of the trophosome are concerned, the seven preceding families are scarcely distinguishable, and they form a section apart, contrasting sharply with the families next to be mentioned, in none of which are free medusae liberated from the colony, so that only the characters of the trophosome need be considered.)


After E. T. Browne, Proc. Zool. Soc. of London, 1896.
Fig. 59.-Three stages in the development of Phialidium temporarium. a, The youngest stage, is magnified about 22 diam.; $b$, older, is magnified about 8 diam.; $c$, the adult medusa, is magnified.
8. Sertularidae.-Hydrothecae sessile, biserial, alternating or opposite on the stem. Sertularia and Sertularella are two very common genera of this family.
9. Plumularidae.-Hydrothecae sessile, biserial on the main stem, uniserial on the lateral branches or pinnules, which give the colony its characteristic feathery form; with nematophores. A very abundant and prolific family; well-known British genera are Plumularia, Antennularia and Aglaophenia.
10. Hydroceratinidae.-This family contains the single Australian species Clathrozoon wilsoni
11. Dendrograptidae, containing fossil (Silurian) genera, such as Dendrograptus and Thamnograptus, of doubtful affinities.

Order III. Hydrocorallinae.-Metagenetic colony-forming Hydromedusae, in which the polyp-colony forms a massive, calcareous corallum into which the polyps can be retracted; polyp-individuals always of two kinds, gastrozoids and dactylozoids; gonosome either free medusae or sessile gonophores. The trophosome consists of a mass of coenosarcal tubes anastomosing in all planes. The interspaces between the tubes are filled up by a solid mass of lime, consisting chiefly of calcium carbonate, which replaces the chitinous perisarc of ordinary hydroids and forms a stony corallum or coenosteum (fig. 60). The surface of the coenosteum is covered by a layer of common ectoderm, containing large nematocysts, and is perforated by pores of two kinds, gastropores and dactylopores, giving exit to gastrozoids and dactylozoids respectively, which are lodged in vertical pore-canals of wider calibre than the coenosarcal canals of the general network. The coenosteum increases in size by new growth at the surface; and in the deeper, older portions of massive forms the tissues die off after a certain


Fig. 60.-Portion of the calcareous corallum of Millepora nodosa, showing the cyclical arrangement of the pores occupied by the "persons" or hydranths. About twice the natural size. (From Moseley.) time, only the superficial region retaining its vitality down to a certain depth. The living tissues at the surface are cut off from the underlying dead portions by horizontal partitions termed tabulae, which are formed successively as the coenosteum increases in age and size. If the coenosteum of Millepora be broken across, each pore-canal (perhaps better termed a polyp-canal) is seen to be interrupted by a series of transverse partitions, representing successive periods of growth with separation from the underlying dead portions.


Fig. 61.-Enlarged view of the surface of a living Millepora, showing five dactylozooids surrounding a central gastrozooid. (From Moseley.)

Besides the wider vertical pore-canals and the narrower, irregular coenosarcal canals, the coenosteum may contain, in its superficial portion, chambers or ampullae, in which the reproductive zoids (medusae or gonophores) are budded from the coenosarc.

The gastropores and dactylopores are arranged in various ways at the surface, a common pattern being the formation of a cyclosystem (fig. 60), in which a central gastrozoid is surrounded by a ring of dactylozoids (fig. 61). In such a system the
dactylopores may be confluent with the gastropore, so that the entire cyclosystem presents itself as a single aperture subdivided by radiating partitions, thus having a superficial resemblance to a madreporarian coral with its radiating septa (figs. 62 and 63).

The gastrozoids usually bear short capitate tentacles, four, six or twelve in number; but in Astylus (fig. 63) they have no tentacles. The dactylozoids have no mouth; in Milleporidae they have short capitate tentacles, but lack tentacles in Stylasteridae.

The gonosome consists of free medusae in Milleporidae, which are budded from the apex of a dactylozoid in Millepora murrayi, but in other species from the coenosarcal canals. The medusae are produced by direct budding, without an entocodon in the bud. They are liberated in a mature condition, and probably live but a short time, merely sufficient to spread the species. The manubrium bearing the gonads is mouthless, and the umbrella is without tentacles, sense-organs, velum or radial canals. In the Stylasteridae sessile gonophores are formed, always by budding from the coenosarc. In Distichopora the gonophores have radial canals, but in other genera they are sporosacs with no trace of medusoid structure.


Fig. 63.-Portion of the corallum of Astylus subviridis (one of the Stylasteridae), showing cyclosystems placed at intervals on the branches, each with a central gastropore and zone of slit-like dactylopores. (After Moseley.)

Classification.Two families are known:- 60, 61). great.


Fig. 62.-Diagrams illustrating the successive stages in the development of the cyclosystems of the Stylasteridae. (After Moseley.)

1, Sporadopora dichotoma.
2, 3, Allopora nobilis.
4, Allopora profunda.
5, Allopora miniacea.
6, Astylus subviridis.
7, Distichopora coccinea.
$s$, Style.
$d p$, Dactylopore.
$g p$, Gastropore.
$b$, In fig. 6, inner horseshoe-shaped mouth of gastropore.

1. Milleporidae.-Coenosteum massive, irregular in form; pores scattered irregularly or in cyclosystems, without styles, with transverse tabulae; free medusae. A single genus, Millepora (figs.
2. Stylasteridae.-Coenosteum arborescent, sometimes fanlike, with pores only on one face, or on the lateral margins of the branches; gastropores with tabulae only in two genera, but with (except in Astylus) a style, i.e. a conical, thorn-like projection from the base of the pore, sometimes found also in dactylopores; sessile gonophores. Sporadopora has the pores scattered irregularly. Distichopora has the pores arranged in rows. Stylaster has cyclosystems. In Allopora the cyclostems resemble the calyces of Anthozoan corals. In Cryptohelia the cyclosystem is covered by a cap or operculum. In Astylus (fig. 63) styles are absent.

Affinities of the Hydrocorallinae.-There can be no doubt that the forms comprised in this order bear a close relationship to the Hydroidea, especially the sub-order Gymnoblastea, with which they should perhaps be classed in a natural classification. A hydrocoralline may be regarded as a form of hydroid colony in which the coenosarc forms a felt-work ramifying in all planes, and in which the chitinous perisarc is replaced by a massive calcareous skeleton. So far as the trophosome is concerned, the step from an encrusting hydroid such as Hydractinia to the hydrocoralline Millepora is not

Hickson considers that the families Milleporidae and Stylasteridae should stand quite apart from one another and should not be united in one order. The nearest approach to the Stylasteridae is perhaps to be found in Ceratella, with its arborescent trophosome formed of anastomosing coenosarcal tubes supported by a thick perisarc and covered by a common ectoderm. Ceratella stands in much the same relation to the Stylasteridae that Hydractinia does to the Milleporidae, in both cases the chitinous perisarc being replaced by the solid coenosteum to which the hydrocorallines owe the second half of their name.

Order IV. Graptolitoidea (Rhabdophora, Allman).-This order has been constituted for a peculiar group of palaeozoic fossils, which have been interpreted as the remains of the skeletons of Hydrozoa of an extinct type.

A typical graptolite consists of an axis bearing a series of tooth-like projections, like a saw. Each such projection is regarded as representing a cup or hydrotheca, similar to those borne by a calyptoblastic hydroid, such as Sertularia. The supposed hydrothecae may be present on one side of the axis only (monoprionid) or on both sides (diprionid); the first case may be conjectured to be the result of uniserial (helicoid) budding, the second to be produced by biserial (scorpioid) budding. In one division (Retiolitidae) the axis is reticulate. In addition to the stems bearing cups, there are found vesicles associated with them, which have been interpreted as gonothecae or as floats, that is to say, air-bladders, acting as hydrostatic organs for a floating polyp-colony.

Since no graptolites are known living, or, indeed, since palaeozoic times, the interpretation of their structure and affinities must of necessity be extremely conjectural, and it is by no means certain that they are Hydrozoa at all. It can only be said that their organization, so far as the state of their preservation permits it to be ascertained, offers closer analogies with the Hydrozoa, especially the Calyptoblastea, than with any other existing group of the animal kingdom.

See the treatise of Delage and Hérouard (Hydrozoa, [4]), and the article Graptolites.
Order V. Trachylinea.-Hydromedusae without alternation of generations, i.e. without a hydroid phase; the medusa develops directly from the actinula larva, which may, however, multiply by budding. Medusae with sense-organs represented by otocysts derived from modified tentacles (tentaculocysts), containing otoliths of endodermal origin, and innervated from the ex-umbral nerve-ring.

This order, containing the typical oceanic medusae, is divided into two sub-orders.
Sub-order 1. Trachomedusae.-Tentacles given off from the margin of the umbrella, which is entire, i.e. not lobed or indented; tentaculocysts usually enclosed in vesicles; gonads on the radial canals. The medusae of this order are characterized by the tough, rigid consistence of the umbrella, due partly to the dense nature of the mesogloea, partly to the presence of a marginal rim of chondral tissue, consisting of thickened ectoderm containing great numbers of nematocysts, and forming, as it were, a cushion-tyre supporting the edge of the umbrella. Prolongations from the rim of chondral tissue may form clasps or peronia supporting the tentacles. The tentacles are primarily four in number, perradial, alternating with four interradial tentaculocysts, but both tentacles and sense-organs may be multiplied and the primary perradii may be six instead of four (fig. 26). The tentacles are always solid, containing an axis of endoderm-cells resembling notochordal tissue or plant-parenchyma, and are but moderately flexible. The sense-organs are tentaculocysts which are usually enclosed in vesicles and may be sunk far below the surface. The gonads are on the radial canals or on the stomach (Ptychogastridae), and each gonad may be divided into two by a longitudinal sub-umbral muscle-tract. The radial canals are four, six, eight or more, and in some genera blindly-ending centripetal canals are present (fig. 26). The stomach may be drawn out into the manubrium, forming a proboscis ("Magenstiel") of considerable length.

The development of the Trachomedusae, so far as it is known, shows an actinula-stage which is either free (larval) or passed over in the egg (foetal) as in Geryonia; in no case does there appear to be a free planula-stage. The actinula, when free, may multiply by larval budding, but in all cases both the original actinula and all its descendants become converted into medusae, so that there is no alternation of generations. In Gonionemus the actinula becomes attached and polyp-like and reproduces by budding.

The Trachomedusae are divided into the following families:

1. Petasidae (Petachnidae).-Four radial canals, four gonads; stomach not prolonged into the manubrium, which is relatively short; tentaculocysts free. Petasus and other genera make up this family, founded by Haeckel, but no other naturalist has ever seen them, and it is probable that they are simply immature forms of other genera.
2. Olindiadae, with four radial canals and four gonads; manubrium short; ring-canals giving off blind centripetal canals; tentaculocysts enclosed. Olindias mülleri (fig. 64) is a common

Mediterranean species. Other genera are Aglauropsis, Gossea and Gonionemus; the last named bears adhesive suckers on the tentacles. Some doubt attaches to the position of this family. It has been asserted that the tentaculocysts are entirely ectodermal and that either the family should be placed amongst the Leptomedusae, or should form, together with certain Leptomedusae, an entirely distinct order. In Gonionemus, however, the concrement-cells are endodermal.
3. Trachynemidae.-Eight radial canals, eight gonads, stomach not prolonged into manubrium; tentaculocysts enclosed. Rhopalonema, Trachynema, \&c.


After E. T. Browne, Proc. Zool. Soc. of London.

Fig. 65.-Aglantha rosea (Forbes), a British medusa.
4. Ptychogastridae (Pectyllidae).-As in the preceding, but with suckers on the tentacles.
Ptychogastria Allman (= Pectyllis), a deepsea form.
5. Aglauridae.Eight radial canals, two, four or eight gonads; tentacles numerous; tentaculocysts free; stomach prolonged into manubrium. Aglaura, Aglantha (fig. 65), \&c., with eight gonads; Stauraglaura with


After Haeckel, System der Medusen, by permission of Gustav Fischer.

Fig. 64. Olindias mülleri.
6. Geryonidae.-Four or six radial canals; gonads band-like; stomach prolonged into a manubrium of great length; tentaculocysts enclosed. Liriope, \&c., with four radial canals; Geryonia, Carmarina (fig. 26), \&c., with six.
7. Halicreidae.-Eight very broad radial canals; ex-umbrella often provided with lateral outgrowths; tentacles differing in size, but in a single row. Halicreas.
Sub-order 2. Narcomedusae.-Margin of the umbrella-lobed, tentacles arising from the exumbrella at some distance from the margin; tentaculocysts exposed, not enclosed in vesicles; gonads on the sub-umbral floor of the stomach or of the gastric pouches.

The Narcomedusae exhibit peculiarities of form and structure which distinguish them at once from all other Hydromedusae. The umbrella is shallow and has the margin supported by a rim of thickened ectoderm, as in the Trachomedusae, but not so strongly developed. The tentacles are not inserted on the margin of the umbrella, but arise high up on the ex-umbral surface, and the umbrella is prolonged into lobes corresponding to the interspaces between the tentacles. The condition of things can be imagined by supposing that in a medusa primitively of normal build, with tentacles at the margin, the umbrella has grown down past the insertion of the tentacles. As a result of this extension of the umbrellar margin, all structures belonging to this region, namely, the ring-canal, the nerve-rings, and the rim of thickened ectoderm, do not run an even course, but are thrown into festoons, caught up under the insertion of each tentacle in such a way that the ring-canal and its accompaniments form in each notch of the umbrellar


Fig. 66.-Cunina rhododactyla, one of the Narcomedusae. (After Haeckel.)
c, Circular canal.
$h$, "Otoporpae" or centripetal process of the marginal cartilaginous ring connected with tentaculocyst.
$k$, Stomach.
l, Jelly of the disk.
$r$, Radiating canal (pouch of stomach).
$t t$, Tentacles.
margin an inverted $V$, the apex of which corresponds
limbs of the V may run for some distance parallel to one another, and may be fused into one, giving a figure better compared to an inverted Y. Thus the ectodermal rim runs round the edge of each lobe of the umbrella and then passes upwards towards the base of the tentacle from the re-entering angle between two adjacent lobes, to form with its fellow of the next lobe a tentacle-clasp or peronium, i.e. a streak of thickened ectoderm supporting the tentacle. Similarly the ring-canal runs round the edge of the lobe as the so-called festoon-canal, and then runs upwards under the peronium to the base of the tentacle as one of a pair of peronial canals, the limbs of the V-like figure already mentioned. The nerve-rings have a similar course. The tentaculocysts are implanted round the margins of the lobes of the umbrella and may be supported by prolongations of the ectodermal rim termed otoporpae (Gehörspangen). The radial canals are represented by wide gastric pouches, and may be absent, so that the tentacles arise directly from the stomach (Solmaridae). The tentacles are always solid, as in Trachomedusae.

The development of the Narcomedusae is in the main similar to that of the Trachomedusae, but shows some remarkable features. In Aeginopsis a planula is formed by multipolar immigration. The two ends of the planula become greatly lengthened and give rise to the two primary tentacles of the actinula, of which the mouth arises from one side of the planula. Hence the principal axis of the future medusa corresponds, not to the longitudinal axis of the planula, but to a transverse axis. This is in some degree parallel to the cases described above, in which a planula gives rise to the hydrorhiza, and buds a polyp laterally.

In Cunina and allied genera the actinula, formed in the manner described, has a hypostome of great length, quite disproportionate to the size of the body, and is further endowed with the power of producing buds from a stolon arising from the aboral side of the body. In these species the actinula is parasitic upon another medusa; for instance, Cunoctantha octonaria upon Turritopsis, C. proboscidea upon Liriope or Geryonia. The parasite effects a lodgment in the host either by invading it as a free-swimming planula, or, apparently, in other cases, as a spore-embryo which is captured and swallowed as food by the host. The parasitic actinula is found attached to the proboscis of the medusa; it thrusts its greatly elongated hypostome into the mouth of the medusa and nourishes itself upon the food in the digestive cavity of its host. At the same time it produces buds from an aboral stolon. The buds become medusae by the direct method of budding described above. In some cases the buds do not become detached at once, but the stolon continues to grow and to produce more buds, forming a "bud-spike" (Knospenähre), which consists of the axial stolon bearing medusa-buds in all stages of development. In such cases the original parent-actinula does not itself become a medusa, but remains arrested in development and ultimately dies off, so that a true alternation of generations is brought about. It is in these parasitic forms that we meet with the method of reproduction by sporogony described above.

In other Narcomedusae, e.g. Cunoctantha fowleri Browne, buds are formed from the subumbrella on the under side of the stomach pouches, where later the gonads are developed.

Classification.-Three families of Narcomedusae are recognized (see O. Maas [40]):

1. Cunanthidae.-With broad gastric pouches which are simple, i.e. undivided, and "pernemal," i.e. correspond in position with the tentacles. Cunina (fig. 66) with more than eight tentacles; Cunoctantha with eight tentacles, four perradial, four interradial.
2. Aeginidae.-Radii a multiple of four, with radial gastric pouches bifurcated or subdivided; the tentacles are implanted in the notch between the two subdivisions of each (primary) gastric pouch, hence the (secondary) gastric pouches appear to be "internemal" in position, i.e. to alternate in position with the tentacles. Aegina, with four tentacles and eight pouches; Aeginura (fig. 25), with eight tentacles and sixteen pouches; Solmundella (fig. 67), with two tentacles and eight pouches; Aeginopsis (fig. 23), with two or four tentacles and sixteen pouches.
3. Solmaridae.-No gastric pouches; the numerous tentacles arise direct from the stomach, into which also the peronial canals open, so that the ring-canal is cut up into separate festoons. Solmaris, Pegantha, Polyxenia, \&c. To this family should be referred, probably, the genus Hydroctena, described by C.


After O. Maas, Craspedoten Medusen der Siboga Expedition, by permission of E. S. Brill \& Co.

Fig. 67.-Solmundella

## Appendix to the Trachylinae.

Of doubtful position, but commonly referred to the Trachylinae, are the two genera of freshwater medusae, Limnocodium and Limnocnida.

Limnocodium sowerbyi was first discovered in the Victoria regia tank in the Botanic Gardens, Regent's Park, London. Since then it has been discovered in other botanic gardens in various parts of Europe, its two most recent appearances being at Lyons (1901) and Munich (1905), occurring always in tanks in which the Victoria regia is cultivated, a fact which indicates that tropical South America is its original habitat. In the same tanks a small hydroid, very similar to Microhydra, has been found, which bears medusa-buds and is probably the stock from which the medusa is budded. It is a remarkable fact that all specimens of Limnocodium hitherto seen have been males; it may be inferred from this either that only one polyp-stock has been introduced into Europe, from which all the medusae seen hitherto have been budded, or perhaps that the female medusa is a sessile gonophore, as in Pennaria. The male gonads are carried on the radial canals.

Limnocnida tanganyicae was discovered first in Lake Tanganyika, but has since been discovered also in Lake Victoria and in the river Niger. It differs from Limnocodium in having practically no manubrium but a wide mouth two-thirds the diameter of the umbrella across. It buds medusae from the margin of the mouth in May and June, and in August and September the gonads are formed in the place where the buds arose. The hydroid phase, if any, is not known.

Both these medusae have sense-organs of a peculiar type, which are said to contain an endodermal axis like the sense-organs of Trachylinae, but the fact has recently been called in question for Limnocodium by S. Goto, who considers the genus to be allied to Olindias. Allman, on the other hand, referred Limnocodium to the Leptomedusae.

In this connexion must be mentioned, finally, the medusae budded from the fresh-water polyp Microhydra. The polyp-stages of Limnocodium and Microhydra are extremely similar in character. In both cases the hydranth is extremely reduced and has no tentacles, and the polyp forms a colony by budding from the base. In Limnocodium the body secretes a gelatinous mucus to which adhere particles of mud, \&c., forming a protective covering. In Microhydra no such protecting case is formed. In view of the great resemblance between Microhydra and the polyp of Limnocodium, it might be expected that the medusae to which they give origin would also be similar. As yet, however, the medusa of Microhydra has only been seen in an immature condition, but it shows some well-marked differences from Limnocodium, especially in the structure of the tentacles, which furnish useful characters for distinguishing species amongst medusae. The possession of a polyp-stage by Limnocodium and Microhydra furnishes an argument against placing them in the Trachylinae. Their sense-organs require renewed investigations. (Browne [10] and [10a].)

Order VI. Siphonophora.-Pelagic floating Hydrozoa with great differentiation of parts, each performing a special function; generally regarded as colonies showing differentiation of individuals in correspondence with a physiological division of labour.

A typical Siphonophore is a stock or cormus consisting of a number of appendages placed in organic connexion with one another by means of a coenosarc. The coenosarc does not differ in structure from that already described in colonial Hydrozoa. It consists of a hollow tube, or tubes, of which the wall is made up of the two body-layers, ectoderm and endoderm, and the cavity is a continuation of the digestive cavities of the nutritive and other appendages, i.e. of the coelenteron. The coenosarc may consist of a single elongated tube or stolon, forming the stem or axis of the cormus on which, usually, the appendages are arranged in groups termed cormidia; or it may take the form of a compact mass of ramifying, anastomosing tubes, in which case the cormus as a whole has a compact form and cormidia are not distinguishable. In the Disconectae the coenosarc forms a spongy mass, the "centradenia," which is partly hepatic in function, forming the so-called liver, and partly excretory.
structure corresponding to different functions. The cormus is always differentiated into two parts; an upper portion termed the nectosome, in which the appendages are locomotor or hydrostatic in function, that is to say, serve for swimming or floating; and a lower portion termed the siphosome, bearing appendages which are nutritive, reproductive or simply protective in function.

Divergent views have been held by different authors both as regards the nature of the cormus as a whole, and as regards the homologies of the different types of appendages borne by it.

The general theories of Siphonophoran morphology are discussed below, but in enumerating the various types of appendages it is convenient to discuss their morphological interpretation at the same time.


After A. Agassiz, from Lankester's Treatise on Zoology.

Fig. 69.-Porpita, seen from above, showing the pneumatophore and expanded palpons.

In the


Fig. 68.-Diagram showing possible modifications of medusiform and hydriform persons of a colony of Siphonophora. The thick black line represents endoderm, the thinner line ectoderm. (After Allman.)
n, Pneumatocyst.
$k$, Nectocalyces (swimming bells).
l, Hydrophyllium (covering-piece).
$i$, Generative medusiform person.
$g$, Palpon with attached palpacle, $h$.
$e$, Siphon with branched grappling tentacle, $f$.
$m$, Stem.
nectosome one or more of the following types of appendage occur:-

1. Swimming-bells, termed nectocalyces or nectophores (fig. 68, k), absent in Chondrophorida and Cystophorida; they are contractile and resemble, both in appearance, structure and function, the umbrella of a medusa, with radial canals, ring-canal and velum; but they are without manubrium, tentacles or sense-organs, and are always bilaterally symmetrical, a peculiarity of form related with the fact that they are attached on one side to the stem. A given cormus may bear one or several nectocalyces, and by their contractions they propel the colony slowly along, like so many medusae harnessed together. In cases where the cormus has no pneumatophore the topmost swimming bell may contain an oil-reservoir or oleocyst.
2. The pneumatophore or air-bladder (fig. 68, $n$ ), for passive locomotion, forming a float which keeps the cormus at or near the surface of the water. The pneumatophore arises from the ectoderm as a pit or invagination, part of which forms a gas-secreting gland, while the rest gives rise to an air-sack lined by a chitinous cuticle. The orifice of invagination forms a pore which may be closed up or may form a protruding duct or funnel. As in the analogous swimbladder of fishes, the gas in the pneumatophore can be secreted or absorbed, whereby the specific gravity of the body can be diminished or increased, so as to cause it to float nearer the surface or at a deeper level. Never more than one pneumatophore is found in a cormus, and when present it is always situated at the highest point above the swimming bells, if these are present also. In Velella the pneumatophore becomes of complex structure and sends air-tubes, lined by a chitin and resembling tracheae, down into the compact coenosarc, thus evidently serving a respiratory as well as a hydrostatic function.

Divergent views have been held as to the morphological significance of the pneumatophore. E. Haeckel regarded the whole structure as a glandular ectodermal pit formed on the ex-umbral surface of a medusa-person. C. Chun and, more recently, R. Woltereck [59], on the other hand, have shown that the ectodermal pit which gives rise to the pneumatophore represents an entocodon. Hence the cavity of the air-sack is equivalent to a sub-umbral cavity in which no manubrium is formed, and the pore or orifice of invagination would represent the margin of the umbrella. In the wall of the sack is a double layer of endoderm, the space between which is a
continuation of the coelenteron. By coalescence of the endoderm-layers, the coelenteron may be reduced to vessels, usually eight in number, opening into a ring-sinus surrounding the pore. Thus the disposition of the endoderm-cavities is roughly comparable to the gastrovascular system of a medusa.

The difference between the theories of Haeckel and Chun is connected with a further divergence in the interpretation of the stem or axis of the cormus. Haeckel regards it as the equivalent of the manubrium, and as it is implanted on the blind end of the pneumatophore, such a view leads necessarily to the air-sack and gland being a development on the ex-umbral surface of the medusa-person. Chun and Woltereck, on the other hand, regard the stem as a stolo prolifer arising from the aboral pole, that is to say, from the ex-umbrella, similar to that which grows out from the ex-umbral surface of the embryo of the Narcomedusae and produces buds, a view which is certainly supported by the embryological evidence to be adduced shortly.

In the siphosome the following types of appendages occur:-

1. Siphons or nutritive appendages, from which the order takes its name; never absent and usually present in great numbers (fig. 68, e). Each is a tube dilated at or towards the base and containing a mouth at its extremity, leading into a stomach placed in the dilatation already mentioned. The siphons have been compared to the manubrium of a medusa-individual, or to polyps, and hence are sometimes termed gastrozoids.
2. Palpons (fig. 68, g), present in some genera, especially in Physonectae; similar to the siphons but without a mouth, and purely tactile in function, hence sometimes termed dactylozoids. If a distal pore or aperture is present, it is excretory in function; such varieties have been termed "cystons" by Haeckel.
3. Tentacles ("Fangfäden"), always present, and implanted one at the base of each siphon (fig. $68, f$ ). The tentacles of siphonophores may reach a great length and have a complex structure. They may bear accessory filaments or tentilla ( $f$ ), covered thickly with batteries of nematocysts, to which these organisms owe their great powers of offence and defence.
4. Palpacles ("Tastfäden"), occurring together with palpons, one implanted at the base of each palpon (fig. 68, h). Each palpacle is a tactile filament, very extensile, without accessory filaments or nematocysts.
5. Bracts ("hydrophyllia"), occur in Calycophorida and some Physophorida as scale-like appendages protecting other parts (fig. 68, $I$ ). The mesogloea is greatly developed in them and they are often of very tough consistency. By Haeckel they are considered homologous with the umbrella of a medusa.


From G. H. Fowler, after A. Agassiz, Lankester's Treatise on Zoology.
Fig. 70.-Diagram of the structure of Velella, showing the central and peripheral thirds of a half-section of the colony, the middle third being omitted. The ectoderm is indicated by close hatching, the endoderm by light hatching, the mesogloea by thick black lines, the horny skeleton of the pneumatophore and sail by dotting.

BL, Blastostyle.
C, Centradenia.
D, Palpon.
EC, Edge of colony prolonged beyond the pneumatophore.
G, Cavity of the large central siphon.

M, Medusoid gonophores.
PN, Primary central chamber, and $\mathrm{PN}^{\prime}$, concentric chamber of the pneumatophore, showing an opening to the exterior and a "trachea."
S, Sail.
6. Gonostyles, appendages which produce by budding medusae or gonophores, like the blastostyles of a hydroid colony. In their most primitive form they are seen in Velella as "gonosiphons," which possess mouths like the ordinary sterile siphons and bud free medusae. In other forms they have no mouths. They may be branched, so-called "gonodendra," and amongst them may occur special forms of palpons, "gonopalpons." The gonostyles have been compared to the blastostyles of a hydroid colony, or to the manubrium of a medusa which produces free or sessile medusa-buds.
7. Gonophores, produced either on the gonostyles already mentioned or budded, as in hydrocorallines, from the coenosarc, i.e. the stem (fig. 68, i.). They show every transition between free medusae and sporosacs, as already described, for hydroid colonies. Thus in Velella free medusae are produced, which have been described as an independent genus of medusae, Chrysomitra. In other types the medusae may be set free in a mature condition as the so-called "genital swimming bells," comparable to the Globiceps of Pennaria. The most usual condition, however, is that in which sessile medusoid gonophores or sporosacs are produced.


From G. H. Fowler, after G. Cuvier, Lankester's Treatise on Zoology.

Fig. 71.-Upper surface of Velella, showing pneumatophore and sail.

The various types of appendages described in the foregoing may be arranged in groups termed cormidia. In forms with a compact coenosarc such as Velella, Physalia, \&c., the separate cormidia cannot be sharply distinguished, and such a condition is described technically as one with "scattered" cormidia. In forms in which, on the other hand, the coenosarc forms an elongated, tubular axis or stem, the appendages are arranged as regularly recurrent cormidia along it, and the cormidia are then said to be "ordinate." In such cases the oldest cormidia, that is to say, those furthest from the nectosome, may become detached (like the segments or proglottides of a tape-worm) and swim off, each such detached cormidium then becoming a small free cormus which, in many cases, has been given an independent generic name. A cormidium may contain a single nutritive siphon ("monogastric") or several siphons ("polygastric"):

The following are some of the forms of cormidia that occur:-

1. The eudoxome (Calycophorida), consisting of a bract, siphon, tentacle and gonophore; when free it is known as Eudoxia.
2. The ersaeome (Calycophorida), made up of the same appendages as the preceding type but with the addition of a nectocalyx; when free termed Ersaea.
3. The rhodalome of some Rhodalidae, consisting of siphon, tentacle and one or more gonophores.
4. The athorome of Physophora, \&c., consisting of siphon, tentacle, one or more palpons with palpacles, and one or more gonophores.
5. The crystallome of Anthemodes, \&c., similar to the athorome but with the addition of a group of bracts.

Embryology of the Siphonophora.-The fertilized ovum gives rise to a parenchymula, with solid endoderm, which is set free as a freeswimming planula larva, in the manner already described (see Hydrozoa). The planula has its two extremities dissimilar (Bipolaria-larva). The subsequent development is slightly different according as the future cormus is headed by a pneumatophore (Physophorida, Cystophorida) or by a nectocalyx (Calycophorida).
(i.) Physophorida, for example Halistemma (C. Chun, Hydrozoa [1]). The planula becomes elongated and broader towards one pole, at which a pit or invagination of the ectoderm arises. Next the pit closes up to form a vesicle with a pore, and so gives rise to the pneumatophore. From the broader portion of the planula an outgrowth arises which becomes the first tentacle of the cormus. The endoderm of the planula now acquires a cavity, and at the narrower pole a mouth is formed, giving rise to the primary siphon. Thus from the original planula three appendages are, as it were, budded off, while the planula itself mostly gives rise to coenosarc, just as in some hydroids the planula is converted chiefly into hydrorhiza.
(ii.) Calycophorida, for example, Muggiaea. The planula develops, on the whole, in a similar manner, but the ectodermal invagination arises, not at the pole of the planula, but on the side of its broader portion, and gives rise, not to a pneumatophore, but to a nectocalyx, the primary swimming bell or protocodon ("Fallschirm") which is later thrown off and replaced by secondary swimming bells, metacodons, budded from the coenosarc.

From a comparison of the two embryological types there can be no doubt on two points; first,


Fig. 72.-A, Diphyes campanulata; B, a group of appendages (cormidium) of the same Diphyes. (After C.
Gegenbaur.)
a, Axis of the colony.
that the pneumatophore and the protocodon are strictly homologous, and, therefore if the nectocalyx is comparable to the umbrella of a medusa, as seems obvious, the pneumatophore must be so too; secondly, that the coenosarcal axis arises from the ex-umbrella of the medusa and cannot be compared to a manubrium, but is strictly comparable to the "bud-spike" of a Narcomedusan.

Theories of Siphonophore Morphology.-The many theories that have been put forward as to the interpretation of the cormus and the various parts are set forth and discussed in the treatise of Y. Delage and E. Hérouard (Hydrozoa [4]) and more recently by R. Woltereck [59], and only a brief analysis can be given here.


After C. Gegenbaur.
Fig. 73.-Physophora hydrostatica.
$a^{\prime}$, Pneumatocyst.
$t$, Palpons.
a, Axis of the colony.
$m$, Nectocalyx.
$o$, Orifice of nectocalyx.
$n$, Siphon.
$g$, Gonophore.
$i$, Tentacle.

In the first place the cormus has been regarded as a single individual and its appendages as organs. This is the so-called "polyorgan" theory, especially connected with the name of Huxley; but it must be borne in mind that Huxley regarded all the forms produced, in any animal, between one egg-generation and the next, as constituting in the lump one single individual. Huxley, therefore, considered a hydroid colony, for example, as a single individual, and each separate polyp or medusa budded from it as having the value of an organ and not of an individual. Hence Huxley's view is not so different from those held by other authors as it seems to be at first sight.

In more recent years Woltereck [59] has supported Huxley's view of individuality, at the same time drawing a fine distinction between "individual" and "person." The individual is the product of sexual reproduction; a person is an individual of lower rank, which may be produced asexually. A Siphonophore is regarded as a single individual composed of numerous zoids, budded from the primary zoid (siphon) produced from the planula. Any given zoid is a person-zoid if equivalent to the primary zoid, an organ-zoid if equivalent only to a part of it. Woltereck considers the siphonophores most nearly allied to the Narcomedusae, producing like the buds from an aboral stolon, the first bud being represented by the pneumatophore or protocodon, in different cases.

Contrasting, in the second place, with the polyorgan theory are the various "polyperson" theories which interpret the Siphonophore cormus as a colony composed of more or fewer individuals in organic union with one another. On this interpretation there is still room for considerable divergence of opinion as regards detail. To begin with, it is not necessary on the polyperson theory to regard each appendage as a distinct individual; it is still possible to compare appendages with parts of an individual which have become separated from one another by a process of "dislocation of organs." Thus a bract may be regarded, with Haeckel, as a modified umbrella of a medusa, a siphon as its manubrium, and a tentacle as representing a medusan tentacle shifted in attachment from the margin to the sub-umbrella; or a siphon may be compared with a polyp, of which the single tentacle has become shifted so as to be attached to the coenosarc and so on. Some authors prefer, on the other hand, to regard every appendage as a separate individual, or at least as a portion of an individual, of which other portions have been lost or obliterated.

A further divergence of opinion arises from differences in the interpretation of the persons composing the colony. It is possible to regard the cormus (1) as a colony of medusa-persons, (2) as a colony of polyp-persons, (3) as composed partly of one, partly of the other. It is sufficient here to mention briefly the views put forward on this point by C. Chun and R. Haeckel.
cormus is to be compared to a floating hydroid colony. It may be regarded as derived from floating polyps similar to Nemopsis or Pelagohydra, which by budding produce a colony of polyps and also form medusa-buds. The polyp-individuals form the nutritive siphosome or trophosome. The medusa-buds are either fertile or sterile. If fertile they become free medusae or sessile gonophores. If sterile they remain attached and locomotor in function, forming the nectosome, the pneumatophore and swimming-bells.

Haeckel, on the other hand, is in accordance with Balfour in regarding a Siphonophore as a medusome, that is to say, as a colony composed of medusoid persons or organs entirely. Haeckel considers that the Siphonophores have two distinct ancestral lines of evolution:

1. In the Disconanthae, i.e. in such forms as Velella, Porpita, \&c., the ancestor was an eightrayed medusa (Disconula) which acquired a pneumatophore as an ectodermal pit on the exumbrella, and in which the organs (manubrium, tentacles, \&c.) became secondarily multiplied, just as they do in Gastroblasta as the result of incomplete fission. The nearest living allies of the ancestral Disconula are to be sought in the Pectyllidae.


After Haeckel, from Lankester's Treatise on Zoology.
Fig. 74.-Stephalia corona, a young colony.
$p$, Pneumatophore
n, Nectocalyx.
l, Aurophore.
$l o$, Orifice of the aurophore.
$s$, Siphon.
$t$, Tentacle.
2. In the Siphonanthae, i.e. in all other Siphonophores, the ancestral form was a Siphonula, a bilaterally symmetrical Anthomedusa with a single long tentacle (cf. Corymorpha), which became displaced from the margin to the sub-umbrella. The Siphonula produced buds on the manubrium, as many Anthomedusae are known to do, and these by reduction or dislocation of parts gave rise to the various appendages of the colony. Thus the umbrella of the Siphonula became the protocodon, and its manubrium, the axis or stolon, which, by a process of dislocation of organs, escaped, as it were, from the sub-umbrella through a cleft and became secondarily attached to the ex-umbrella. It must be pointed out that, however probable Haeckel's theory may be in other respects, there is not the slightest evidence for any such cleft in the umbrella having been present at any time, and that the embryological evidence, as already pointed out, is all against any homology between the stem and a manubrium, since the primary siphon does not become the stem, which arises from the ex-umbral side of the protocodon and is strictly comparable to a stolon.

Classification.-The Siphonophora may be divided, following Delage and Hérouard, into four sub-orders:
I. Chondrophorida (Disconectae Haeckel, Tracheophysae Chun). With an apical chambered
pneumatophore, from which tracheal tubes may take origin (fig. 70); no nectocalyces or bracts; appendages all on the lower side of the pneumatophore arising from a compact coenosarc, and consisting of a central principal siphon, surrounded by gonosiphons, and these again by tentacles.

Three families: (1) Discalidae, for Discalia and allied genera, deep-sea forms not well known; (2) Porpitidae for the familiar genus Porpita (fig. 69) and its allies; and (3) Velellidae, represented by the well-known genus Velella (figs. 70, 71), common in the Mediterranean and other seas.
II. Calycophorida (Calyconectae, Haeckel). Without pneumatophore, with one, two, rarely more nectocalyces.

Three families: (1) Monophyidae, with a single nectocalyx; examples Muggiaea, sometimes found in British seas, Sphaeronectes, \&c.; (2) Diphyidae, with two nectocalyces; examples Diphyes (fig. 72), Praya, Abyla, \&c.; and (3) Polyphyidae, with numerous nectocalyces; example Hippopodius, Stephanophyes and other genera.


From G. H. Fowler, modified after G. Cuvier and E. Haeckel, Lankester's Treatise on Zoology.
Fig. 75.-A. Physalia, general view, diagrammatic; B, cormidium of Physalia; D, palpon; T, palpacle; G, siphon; GP, gonopalpon; $\mathrm{M} \boldsymbol{o}^{7}$, male gonophore; M ㅇ, female gonophore, ultimately set free.
III. Physophorida (Physonectae + Auronectae, Haeckel). With an apical pneumatophore, not divided into chambers, followed by a series of nectocalyces or bracts.

A great number of families and genera are referred to this group, amongst which may be mentioned specially-(1) Agalmidae, containing the genera Stephanomia, Agalma, Anthemodes, Halistemma, \&c.; (2) Apolemidae, with the genus Apolemia and its allies; (3) Forskaliidae, with Forskalia and allied forms; (4) Physophoridae, for Physophora (fig. 73) and other genera, (5) Anthophysidae, for Anthophysa, Athorybia, \&c.; and lastly the two families (6) Rhodalidae and (7) Stephalidae (fig. 74), constituting the group Auronectae of Haeckel. The Auronectae are peculiar deep-sea forms, little known except from Haeckel's descriptions, in which the large pneumatophore has a peculiar duct, termed the aurophore, placed on its lower side in the midst of a circle of swimming-bells.
IV. Cystophorida (Cystonectae, Haeckel). With a very large pneumatophore not divided into chambers, but without nectocalyces or bracts. Two sections can be distinguished, the Rhizophysina, with long tubular coenosarc-bearing ordinate cormidia, and Physalina, with compact coenosarc-bearing scattered cormidia.

A type of the Rhizophysina is the genus Rhizophysa. The Physalina comprise the families Physalidae and Epibulidae, of which the types are Physalia (figs. 74, 75) and Epibulia, respectively. Physalia, known commonly as the Portuguese man-of-war, is remarkable for its great size, its brilliant colours, and its terrible stinging powers.

Bibliography.-In addition to the works cited below, see the general works cited in the article Hydrozoa, in some of which very full bibliographies will be found.

1. G. J. Allman, "A Monograph of the Gymnoblastic or Tubularian Hydroids," Ray Society (1871-1872); 2. A. Brauer, "Über die Entwickelung von Hydra," Zeitschr. f. wiss. Zool. lii. (1891), pp. 169-216, pls. ix.-xii.; 3. "Über die Entstehung der Geschlechtsprodukte und die Entwickelung von Tubularia mesembryanthemum Allm.," t.c. pp. 551-579, pls. xxxiii.-xxxv.; 4. W. K. Brooks, "The Life-History of the Hydromedusae: a discussion of the Origin of the Medusae, and of the significance of Metagenesis," Mem. Boston Soc. Nat. Hist. iii. (1886), pp. 259-430, pis. xxxvii.-xliv.; 5. "The Sensory Clubs of Cordyli of Laodice," Journ. Morphology, x. (1895), pp. 287-304, pl. xvii.; 6. E. T. Browne, "On British Hydroids and Medusae," Proc. Zool. Soc. (1896), pp. 459-500, pls. xvi., xvii., (1897), pp. 816-835, pls. xlviii. xlix. 12 text-figs.; 7. "Biscayan Medusae," Trans. Linn. Soc. x. (1906), pp. 163-187, pl. xiii.; 8. "Medusae" in Herdman, Rep. Pearl Oyster Fisheries, Gulf of Manaar, iv. (1905), pp. 131-166, 4 pls.; 9. "Hydromedusae with a Revision of the Williadae and Petasidae," Fauna and Geogr. Maldive and Laccadive Archipelagos, ii. (1904), pp. 722-749, pls. liv.-lvii.; 10. "On the Freshwater Medusa liberated by Microhydra ryderi, Potts, and a Comparison with Limnocodium," Quart. Journ. Micr. Sci. I (1906), pp. 635, 645, pl. xxxvii.; 10a. "On the Freshwater Medusa Limnocnida tanganicae" Budgett Memorial Volume (Cambridge, 1908, pp. 471-482, pl. xxviii.; 11. C. Claus, "Über die Struktur der Muskelzellen und über den Körperbau von Mnestra parasites Krohn," Verhandl. zool. bot. Ges. Wien, xxv. (1876), pp. 9-12, pl. i.; 11a. C. Dawydov, "Hydroctena salenskii," Mém. Acad. Imp. St. Pétersbourg (viii.) xiv. No. 9 (1903), 17 pp., 1 pl.; 12. A. Dendy, "On a Free-swimming Hydroid, Pelagohydra mirabilis," n. gen. et sp., Quart. Journ. Micr. Sci. xlvi. (1903), pp. 1-24, pls. i. ii.; 13. H. Driesch, "Tektonische Studien an Hydroidpolypen," (1) Jen. Zeitschr., xxiv. (1890), pp. 189-226, 12 figs.; (2) t.c. pp. 657-688, 6 figs.; (3) ibid. xxv. (1891), pp. 467-479, 3 figs.; 14. G. Duplessis, "On Campanularia volubilis," Soc. Vaud. Bull. 13 (Lausanne, 1874-1875); 15. J. W. Fewkes, "On Mnestra," Amer. Natural., xviii. (1884), pp. 197198, 3 figs.; 16. S. Goto, "Dendrocoryne Inaba, Vertreterin einer neuen Familie der Hydromedusen," Annot. Zool. Tokyo, i. (1897), pp. 93-104, pl. vi., figs. 106-113; 17. "The Craspe dote Medusa Olindias and some of its Natural Allies," Mark Anniversary Volume (New York, 1903), pp. 1-22, 3 pls.; 18. H. Grenacher, "Über die Nesselkapseln von Hydra," Zool. Anz. xviii. (1895), pp. 310-321, 7 figs.; 19. R. T. Günther, "On the Structure and Affinities of Mnestra parasites Krohn; with a revision of the Classification of the Cladonemidae," Mitt. Stat. Neapel, xvi. (1903), pp. 35-62, pls. ii. iii.; 20. E. Haeckel, "Das System der Medusen," Denkschr. med.-nat.-wiss. Ges. (Jena, 1879-1881); 21. "Deep Sea Medusae," in Reports of the Challenger Expedition, Zool. iv. pt. 2 (London, 1882); 22. P. Hallez, "Bougainvillia fruticosa Allm. est le faciès d'eau agitée du Bougainvillia ramosa Van Ben." C.-R. Acad. Sci. Paris, cxl. (1905), pp. 457-459; 23. O. \& R. Hertwig, Der Organismus der Medusen (Jena, 1878), 70 pp., 3 pls.; 24. Das Nervensystem und die Sinnesorgane der Medusen (Leipzig, 1878), 186 pp., 10 pls.; 25. S. J. Hickson, "The Medusae of Millepora," Proc. Roy. Soc. lxvi. (1899), pp. 6-10, 10 figs.; 26. T. Hincks, A History of British Hydroid Zoophytes (2 vols., London, 1868); 27. N. Iwanzov, "Über den Bau, die Wirkungsweise und die Entwickelung der Nesselkapseln von Coelenteraten," Bull. Soc. Imp. Natural, Moscou (1896), pp. 323-355, 4 pls.; 28. C. F. Jickeli, "Der Bau der Hydroidpolypen," (1) Morph. Jahrbuch, viii. (1883), pp. 373-416, pls. xvi.-xviii.; (2) t.c., pp. 580680, pls. xxv.-xxviii.; 29. Albert Lang, "Über die Knospung bei Hydra und einigen Hydropolypen," Zeitschr. f. wiss. Zool. liv. (1892), pp. 365-384, pl. xvii.; 30. Arnold Lang, "Gastroblasta Raffaelei. Eine durch eine Art unvollständiger Theilung entstehende MedusenKolonie," Jena Zeitschr. xix. (1886), pp. 735-762, pls. xx., xxi.; 31. A. Linko, "Observations sur les méduses de la mer Blanche," Trav. Soc. Imp. Nat. St Pétersbourg, xxix. (1899); 32. "Über den Bau der Augen bei den Hydromedusen," Zapiski Imp. Akad. Nauk (Mém. Acad. Imp. Sci.) St Pétersbourg (8) x. 3 (1900), 23 pp., 2 pls.; 33. O. Maas, "Die craspedoten Medusen," in Ergebn. Plankton Expedition, ii. (Kiel and Leipzig, 1893), 107 pp., 8 pls., 3 figs.; 34. "Die Medusen," Mem. Mus. Comp. Zool. Harvard, xxiii. (1897), i.; 35. "On Hydroctena," Zool. Centralbl. xi. (1904), pp. 240-243; 36. "Revision des méduses appartenant aux familles des Cunanthidae et des Aeginidae, et groupement nouveau des genres," Bull. Mus. Monaco, v. (1904), 8 pp.; 37. "Revision der Cannotiden Haeckels," SB. K. Bayer. Akad. xxxiv. (1904), pp. 421-445; 38. "Meduses," Result. Camp. Sci. Monaco, xxviii. (1904), 71 pp., 6 pls.; 39. "Die craspedoten Medusen der Siboga-Expedition," Uitkomst. Siboga-Exped. x. (1905), 84 pp., 14 pls.; 40. "Die arktischen Medusen (ausschliesslich der Polypomedusen)," Fauna arctica, iv. (1906), pp. 479-

526; 41. C. Mereschkowsky, "On a new Genus of Hydroids (Monobrachium) from the White Sea, with a short description of other new Hydroids," Ann. Mag. Nat. Hist. (4) xx. (1877), pp. 220-229, pls. v. vi.; 42. E. Metchinkoft, "Studien über die Entwickelung der Medusen und Siphonophoren," Zeitschr. f. wiss. Zool. xxiv. (1874), pp. 15-83, pls. i.-xii.; 43. "Vergleichendembryologische Studien" (Geryoniden, Cunina), ibid. xxxvi. (1882), pp. 433-458, pl. xxviii.; 44. Embryologische Studien an Medusen (Vienna, 1886), 150 pp., 12 pls., 10 figs.; 45. "Medusologische Mittheilungen," Arb. zool. Inst. Wien, vi. (1886), pp. 237-266, pls. xxii. xxiii.; 46. L. Murbach, "Beiträge zur Kenntnis der Anatomie und Entwickelung der Nesselorgane der Hydroiden," Arch. f. Naturgesch. lx. i. (1894), pp. 217-254, pl. xii.; 47. "Preliminary Note on the Life-History of Gonionemus," Journ. Morph. xi. (1895), pp. 493-496; 48. L. Murbach and C. Shearer, "On Medusae from the Coast of British Columbia and Alaska," Proc. Zool. Soc. (1903), ii. pp. 164-191, pls. xvii.-xxii.; 49. H. F. Perkins, "The Development of Gonionema murbachii," Proc. Acad. Nat. Sci. Philadelphia (1902), pp. 750-790, pls. xxxi-xxxiv.; 50. F. Schaudinn, "Über Haleremita cumulans, n. g. n. sp., einen marinen Hydroidpolypen," SB. Ges. natforsch. Freunde Berlin (1894), pp. 226-234, 8 figs.; 51. F. E. Schulze, "On the Structure and Arrangement of the Soft Parts in Euplectella aspergillum" (Amphibrachium), Tr. R. Soc. Edinburgh, xxix. (1880), pp. 661-673, pl. xvii.; 52. O. Seeliger, "Über das Verhalten der Keimblätter bei der Knospung der Cölenteraten," Zeitschr. f. wiss. Zool. lviii. (1894), pp. 152-188, pls. vii.-ix.; 53. W. B. Spencer, "A new Family of Hydroidea (Clathrozoon), together with a description of the Structure of a new Species of Plumularia," Trans. Roy. Soc. Victoria (1890), pp. 121-140, 7 pls.; 54. M. Ussow, "A new Form of Fresh-water Coelenterate" (Polypodium), Ann. Mag. Nat. Hist. (5) xviii. (1886), pp. 110-124, pl. iv.; 55. E. Vanhöffen, "Versuch einer natürlichen Gruppierung der Anthomedusen," Zool. Anzeiger, xiv. (1891), pp. 439-446; 56. C. Viguier, "Études sur les animaux inférieurs de la baie d'Alger" (Tetraplatia), Arch. Zool. Exp. Gen. viii. (1890), pp. 101142, pls. vii.-ix.; 57. J. Wagner, "Recherches sur l'organisation de Monobrachium parasiticum Méréjk," Arch. biol. x. (1890), pp. 273-309, pls. viii. ix.; 58. A. Weismann, Die Entstehung der Sexualzellen bei den Hydromedusen (Jena, 1883); 59. R. Woltereck, "Beiträge zur Ontogenie und Ableitung des Siphonophorenstocks," Zeitschr. f. wiss. Zool. lxxxii. (1905), pp. 611-637, 21 text-figs.; 60. J. Wulfert, "Die Embryonalentwickelung von Gonothyraea loveni Allm.," Zeitschr. f. wiss. Zool. lxxi. (1902), pp. 296-326, pls. xvi.-xviii.

1 In some cases hydroids have been reared in aquaria from ova of medusae, but these hydroids have not yet been found in the sea (Browne [10 a]).

2 The numbers in square brackets [] refer to the bibliography at the end of this article; but when the number is preceded by the word Hydrozoa, it refers to the bibliography at the end of the article Hydrozoa.

HYDROMETER (Gr. ű $\delta \omega \rho$, water, and $\mu \varepsilon ́ \tau \rho o \nu$, a measure), an instrument for determining the density of bodies, generally of fluids, but in some cases of solids. When a body floats in a fluid under the action of gravity, the weight of the body is equal to that of the fluid which it displaces (see Hydromechanics). It is upon this principle that the hydrometer is constructed, and it obviously admits of two modes of application in the case of fluids: either we may compare the weights of floating bodies which are capable of displacing the same volume of different fluids, or we may compare the volumes of the different fluids which are displaced by the same weight. In the latter case, the densities of the fluids will be inversely proportional to the volumes thus displaced.

The hydrometer is said by Synesius Cyreneus in his fifth letter to have been invented by Hypatia at Alexandria, ${ }^{1}$ but appears to have been neglected until it was reinvented by Robert Boyle, whose "New Essay Instrument," as described in the Phil. Trans. for June 1675, differs in no essential particular from Nicholson's hydrometer. This instrument was devised for the purpose of detecting counterfeit coin, especially guineas and half-guineas. In the first section of the paper (Phil. Trans. No. 115, p. 329) the author refers to a glass instrument exhibited by himself many years before, and "consisting of a bubble furnished with a long and slender stem, which was to be put into several liquors, to compare and estimate their specific gravities." This seems to be the first reference to the hydrometer in modern times.

In fig. 1 C represents the instrument used for guineas, the circular plates A representing plates of lead, which are used as ballast when lighter coins than guineas are examined. B represents "a small glass instrument for estimating the specific gravities of liquors," an account of which was promised by Boyle in the following number of the Phil. Trans., but did not appear.

The instrument represented at B (fig. 1), which is copied from Robert Boyle's sketch in the Phil. Trans. for 1675, is generally known as the common hydrometer. It is usually made of glass, the lower bulb being loaded with mercury or small shot which serves as ballast, causing the instrument to float with the stem vertical. The quantity of mercury or shot inserted depends upon the density of the liquids for which the hydrometer is to be employed, it being essential that the whole of the bulb should be immersed in the heaviest liquid for which the instrument is used, while the length and diameter of the stem must be such that the hydrometer will float in the lightest liquid for which it is required. The stem is usually divided into a number of equal parts, the divisions of the scale being varied in different instruments, according to the purposes for which they are employed.

Let V denote the volume of the instrument immersed (i.e. of liquid displaced) when the surface of the liquid in which the hydrometer floats coincides with the lowest division of the scale, A the area of the transverse section of the stem, $l$ the length of a scale division, n the number of divisions on the stem, and W the weight of the instrument. Suppose the successive divisions of the scale to be numbered $0,1,2 \ldots n$ starting with the lowest, and let $\mathrm{w}_{0}, \mathrm{~W}_{1}, \mathrm{w}_{2} \ldots \mathrm{w}_{\mathrm{n}}$ be the weights of unit volume of the liquids in which the hydrometer sinks to the divisions $0,1,2 \ldots \mathrm{n}$ respectively. Then, by the principle of Archimedes,

$$
\begin{aligned}
& \mathrm{W}=\mathrm{Vw}_{0} ; \text { or } \mathrm{w}_{0}=\mathrm{W} / \mathrm{V} . \text { Also } \\
& \mathrm{W}=(\mathrm{V}+\mathrm{lA}) \mathrm{w}_{1} ; \text { or } \mathrm{w}_{1}=\mathrm{W} /(\mathrm{V}+\mathrm{lA}), \\
& \mathrm{w}_{\mathrm{p} 2}=\mathrm{W} /(\mathrm{V}+\mathrm{plA}), \text { and } \\
& \mathrm{w}_{\mathrm{n}}=\mathrm{W} /(\mathrm{V}+\mathrm{nlA}),
\end{aligned}
$$

or the densities of the several liquids vary inversely as the respective volumes of the instrument immersed in them; and, since the divisions of the scale correspond to equal increments of volume immersed, it follows that the densities of the several liquids in which the instrument sinks to the successive divisions form a harmonic series.

If $\mathrm{V}=\mathrm{NIA}$ then N expresses the ratio of the volume of the instrument up to the zero of the scale to that of one of the scale-divisions. If we suppose the lower part of the instrument replaced by a uniform bar of the same sectional area as the stem and of volume V , the indications of the instrument will be in no respect altered, and the bottom of the bar will be at a distance of N scale-divisions below the zero of the scale.

In this case we have $\mathrm{w}_{\mathrm{p}}=\mathrm{W} /(\mathrm{N}+\mathrm{p}) 1 \mathrm{~A}$; or the density of the liquid varies inversely as $\mathrm{N}+\mathrm{p}$, that is, as the whole number of scale-divisions between the bottom of the tube and the plane of flotation.

If we wish the successive divisions of the scale to correspond to equal increments in the density of the corresponding liquids, then the volumes of the instrument, measured up to the successive divisions of the scale, must form a series in harmonical progression, the lengths of the divisions increasing as we go up the stem.

The greatest density of the liquid for which the instrument described above can be employed is $\mathrm{W} / \mathrm{V}$, while the least density is $\mathrm{W} /(\mathrm{V}+\mathrm{nlA})$, or $\mathrm{W} /(\mathrm{V}+\mathrm{v})$, where v represents the volume of the stem between the extreme divisions of the scale. Now, by increasing v , leaving W and V unchanged, we may increase the range of the instrument indefinitely. But it is clear that if we increase A, the sectional area of the stem, we shall diminish l, the length of a scale-division corresponding to a given variation of density, and thereby proportionately diminish the sensibility of the instrument, while diminishing the section A will increase land proportionately increase the sensibility, but will diminish the range over which the instrument can be employed, unless we increase the length of the stem in the inverse ratio of the sectional area. Hence, to obtain great sensibility along with a considerable range, we require very long slender stems, and to these two objections apply in addition to the question of portability; for, in the first place, an instrument with a very long stem requires a very deep vessel of liquid for its complete immersion, and, in the second place, when most of the stem is above the plane of flotation, the stability of the instrument when floating will be diminished or destroyed. The various devices which have been adopted to overcome this difficulty will be described in the account given of the several hydrometers which have been hitherto generally employed.

The plan commonly adopted to obviate the necessity of inconveniently long stems is to construct a number of hydrometers as nearly alike as may be, but to load them differently, so that the scale-divisions at the bottom of the stem of one hydrometer just overlap those at the
top of the stem of the preceding. By this means a set of six hydrometers, each having a stem rather more than 5 in . long, will be equivalent to a single hydrometer with a stem of 30 in . But, instead of employing a number of instruments differing only in the weights with which they are loaded, we may employ the same instrument, and alter its weight either by adding mercury or shot to the interior (if it can be opened) or by attaching weights to the exterior. These two operations are not quite equivalent, since a weight added to the interior does not affect the volume of liquid displaced when the instrument is immersed up to a given division of the scale, while the addition of weights to the exterior increases the displacement. This difficulty may be met, as in Keene's hydrometer, by having all the weights of precisely the same volume but of different masses, and never using the instrument except with one of these weights attached.

The first hydrometer intended for the determination of the densities of liquids, and furnished with a set of weights to be attached when necessary, was that constructed by Mr Clarke (instrument-maker) and described by J. T. Desaguliers in the Philosophical Transactions for March and April 1730, No. 413, p. 278. The following is Desaguliers's account of the instrument (fig. 2):-
"After having made several fruitless trials with ivory, because it imbibes spirituous liquors, and thereby alters its gravity, he (Mr Clarke) at last made a copper hydrometer, represented in fig. 2, having a brass wire of about 1 in. thick going through, and soldered into the copper ball Bb . The upper part of this wire is filed flat on one side, for the stem of the hydrometer, with a mark at $m$, to which it sinks exactly in proof spirits. There are two other marks, $A$ and $B$, at top and bottom of the stem, to show whether the liquor be $1 / 10$ th above proof (as when it sinks to A), or $1 / 10$ th under proof (as when it emerges to B), when a brass weight such as $C$ has been screwed on to the bottom at c. There are a great many such weights, of different sizes, and marked to be screwed on instead of $C$, for


Fig. 2.-Clarke's Hydrometer. liquors that differ more than $1 / 10$ th from proof, so as to serve for the specific gravities in all such proportions as relate to the mixture of spirituous liquors, in all the variety made use of in trade. There are also other balls for showing the specific gravities quite to common water, which make the instrument perfect in its kind."

Clarke's hydrometer, as afterwards constructed for the purposes of the excise, was provided with thirty-two weights to adapt it to spirits of different specific gravities, and eleven smaller weights, or "weather weights" as they were called, which were attached to the instrument in order to correct for variations of temperature. The weights were adjusted for successive intervals of $5^{\circ} \mathrm{F}$., but for degrees intermediate between these no additional correction was applied. The correction for temperature thus afforded was not sufficiently accurate for excise purposes, and William Speer in his essay on the hydrometer (Tilloch's Phil. Mag., 1802, vol. xiv.) mentions cases in which this imperfect compensation led to the extra duty payable upon spirits which were more than $10 \%$ over proof being demanded on spirits which were purposely diluted to below $10 \%$ over proof in order to avoid the charge. Clarke's hydrometer, however, remained the standard instrument for excise purposes from 1787 until it was displaced by that of Sikes.

Desaguliers himself constructed a hydrometer of the ordinary type for comparing the specific gravities of different kinds of water (Desaguliers's Experimental Philosophy, ii. 234). In order to give great sensibility to the instrument, the large glass ball was made nearly 3 in . in diameter, while the stem consisted of a wire 10 in . in length and only $1 / 40 \mathrm{in}$. in diameter. The instrument weighed 4000 grains, and the addition of a grain caused it to sink through an inch. By altering the quantity of shot in the small balls the instrument could be adapted for liquids other than water.

To an instrument constructed for the same purpose, but on a still larger scale than that of Desaguliers, A. Deparcieux added a small dish on the top of the stem for the reception of the weights necessary to sink the instrument to a convenient depth. The effect of weights placed in such a dish or pan is of course the same as if they were placed within the bulb of the instrument, since they do not alter the volume of that part which is immersed.

The first important improvement in the hydrometer after its reinvention by Boyle was introduced by G. D. Fahrenheit, who adopted the second mode of construction above referred to, arranging his instrument so as always to displace the same volume of liquid, its weight being varied accordingly. Instead of a scale, only a single mark is placed upon the stem, which is very slender, and bears at the top a small scale pan into which weights are placed until the instrument sinks to the mark upon its stem. The volume of the displaced liquid being then always the same, its density will be proportional to the whole weight supported, that is, to the weight of
the instrument together with the weights required to be placed in the scale pan.

Nicholson's hydrometer (fig. 3) combines the characteristics of Fahrenheit's hydrometer and of Boyle's essay instrument. ${ }^{2}$ The following is the description given of it by W. Nicholson in the Manchester Memoirs, ii. 374:-
"AA represents a small scale. It may be taken off at D. Diameter $1 \frac{1}{2}$ in., weight 44 grains.
"B a stem of hardened steel wire. Diameter $1 / 100$ in.
"E a hollow copper globe. Diameter $2 \% / 10$ in. Weight with stem 369 grains.
"FF a stirrup of wire screwed to the globe at C.


Fig. 3.Nicholson's Hydrometer.
"G a small scale, serving likewise as a counterpoise. Diameter $11 / 2 \mathrm{in}$. Weight with stirrup 1634 grains.
"The other dimensions may be had from the drawing, which is one-sixth of the linear magnitude of the instrument itself.
"In the construction it is assumed that the upper scale shall constantly carry 1000 grains when the lower scale is empty, and the instrument sunk in distilled water at the temperature of $60^{\circ}$ Fahr. to the middle of the wire or stem. The length of the stem is arbitrary, as is likewise the distance of the lower scale from the surface of the globe. But, the length of the stem being settled, the lower scale may be made lighter, and, consequently, the globe less, the greater its distance is taken from the surface of the globe; and the contrary."

In comparing the densities of different liquids, it is clear that this instrument is precisely equivalent to that of Fahrenheit, and must be employed in the same manner, weights being placed in the top scale only until the hydrometer sinks to the mark on the wire, when the specific gravity of the liquid will be proportional to the weight of the instrument together with the weights in the scale.

In the subsequent portion of the paper above referred to, Nicholson explains how the instrument may be employed as a thermometer, since, fluids generally expanding more than the solids of which the instrument is constructed, the instrument will sink as the temperature rises.

To determine the density of solids heavier than water with this instrument, let the solid be placed in the upper scale pan, and let the weight now required to cause the instrument to sink in distilled water at standard temperature to the mark B be denoted by w, while W denotes the weight required when the solid is not present. Then $\mathrm{W}-\mathrm{w}$ is the weight of the solid. Now let the solid be placed in the lower pan, care being taken that no bubbles of air remain attached to it , and let $\mathrm{w}_{1}$ be the weight now required in the scale pan. This weight will exceed w in consequence of the water displaced by the solid, and the weight of the water thus displaced will be $\mathrm{W}_{1}-\mathrm{w}$, which is therefore the weight of a volume of water equal to that of the solid. Hence, since the weight of the solid itself is $W-w$, its density must be $(W-w) /\left(w_{1}-w\right)$.

The above example illustrates how Nicholson's or Fahrenheit's hydrometer may be employed as a weighing machine for small weights.

In all hydrometers in which a part only of the instrument is immersed, there is a liability to error in consequence of the surface tension, or capillary action, as it is frequently called, along the line of contact of the instrument and the surface of the liquid (see Capillary Action). This error diminishes as the diameter of the stem is reduced, but is sensible in the case of the thinnest stem which can be employed, and is the chief source of error in the employment of Nicholson's hydrometer, which otherwise would be an instrument of extreme delicacy and precision. The following is Nicholson's statement on this point:-
"One of the greatest difficulties which attends hydrostatical experiments arises from the attraction or repulsion that obtains at the surface of the water. After trying many experiments to obviate the irregularities arising from this cause, I find reason to prefer the simple one of carefully wiping the whole instrument, and especially the stem, with a clean cloth. The weights in the dish must not be esteemed accurate while there is either a cumulus or a cavity in the water round the stem."

It is possible by applying a little oil to the upper part of the bulb of a common or of a Sikes's hydrometer, and carefully placing it in pure water, to cause it to float with the upper part of the bulb and the whole of the stem emerging as indicated in fig. 4 , when it ought properly to sink almost to the top of the stem, the surface tension of the water around the circumference of the circle of contact, $\mathrm{AA}^{\prime}$, providing the additional support required.

The universal hydrometer of G. Atkins, described in the Phil. Mag. for 1808, xxxi. 254, is merely Nicholson's hydrometer with the screw at C projecting through the collar into which it is screwed, and terminating in a sharp point above the cup G. To this point soft bodies lighter than water (which would float if placed in the cup) could be attached, and thus completely immersed. Atkins's instrument was constructed so as to weigh 700 grains, and when immersed to the mark on the stem in distilled water at $60^{\circ} \mathrm{F}$. it carried 300 grains in the upper dish. The hydrometer therefore displaced 1000 grains of distilled water at $60^{\circ} \mathrm{F}$. and hence the specific gravity of any other liquid was at once indicated by adding 700 to the number of grains in the pan required to make the instrument sink to the mark on the stem. The small divisions on the scale corresponded to differences of $1 / 10$ th of a grain in the weight of the instrument.

The "Gravimeter," constructed by Citizen Guyton and described in Nicholson's Journal, 4to, i. 110, differs from Nicholson's instrument in being constructed of glass, and having a cylindrical bulb about 21 centimetres in length and 22 millimetres in diameter. Its weight is so adjusted that an additional weight of 5 grammes must be placed in


Fig. 4. the upper pan to cause the instrument to sink to the mark on the stem in distilled water at the standard temperature. The instrument is provided with an additional piece, or "plongeur," the weight of which exceeds 5 grammes by the weight of water which it displaces; that is to say, it is so constructed as to weigh 5 grammes in water, and consists of a glass envelope filled with mercury. It is clear that the effect of this "plongeur," when placed in the lower pan, is exactly the same as that of the 5 gramme weight in the upper pan. Without the extra 5 grammes the instrument weighs about 20 grammes, and therefore floats in a liquid of specific gravity .8. Thus deprived of its additional weight it may be used for spirits. To use the instrument for liquids of much greater density than water additional weights must be placed in the upper pan, and the "plongeur" is then placed in the lower pan for the purpose of giving to the instrument the requisite stability.

Charles's balance areometer is similar to Nicholson's hydrometer, except that the lower basin admits of inversion, thus enabling the instrument to be employed for solids lighter than water, the inverted basin serving the same purpose as the pointed screw in Atkins's modification of the instrument.

Adie's sliding hydrometer is of the ordinary form, but can be adjusted for liquids of widely differing specific gravities by drawing out a sliding tube, thus changing the volume of the hydrometer while its weight remains constant.

The hydrometer of A. Baumé, which has been extensively used in France, consists of a common hydrometer graduated in the following manner. Certain fixed points were first determined upon the stem of the instrument. The first of these was found by immersing the hydrometer in pure water, and marking the stem at the level of the surface. This formed the zero of the scale. Fifteen standard solutions of pure common salt in water were then prepared, containing respectively $1,2,3, \ldots 15 \%$ (by weight) of dry salt. The hydrometer was plunged in these solutions in order, and the stem having been marked at the several surfaces, the degrees so obtained were numbered $1,2,3, \ldots 15$. These degrees were, when necessary, repeated along the stem by the employment of a pair of compasses till 80 degrees were marked off. The instrument thus adapted to the determination of densities exceeding that of water was called the hydrometer for salts.

The hydrometer intended for densities less than that of water, or the hydrometer for spirits, is constructed on a similar principle. The instrument is so arranged that it floats in pure water with most of the stem above the surface. A solution containing $10 \%$ of pure salt is used to indicate the zero of the scale, and the point at which the instrument floats when immersed in distilled water at $10^{\circ}$ R. ( $541^{1} 2^{\circ}$ F.) is numbered 10 . Equal divisions are then marked off upwards along the stem as far as the 50th degree.
The densities corresponding to the several degrees of Baumé's hydrometer are given by Nicholson (Journal of Philosophy, i. 89) as follows:-

Baumés Hydrometer for Spirits. Temperature $10^{\circ} R$.

| Degrees. | Density. | Degrees. | Density. | Degrees. | Density. |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 10 | 1.000 | 21 | .922 | 31 | .861 |
| 11 | .990 | 22 | .915 | 32 | .856 |
| 12 | .985 | 23 | .909 | 33 | .852 |
| 13 | .977 | 24 | .903 | 34 | .847 |


| 14 | .970 | 25 | .897 | 35 | .842 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 15 | .963 | 26 | .892 | 36 | .837 |
| 16 | .955 | 27 | .886 | 37 | .832 |
| 17 | .949 | 28 | .880 | 38 | .827 |
| 18 | .943 | 29 | .874 | 39 | .822 |
| 19 | .935 | 30 | .867 | 40 | .817 |
| 20 | .928 |  |  |  |  |

Baume's Hydrometer for Salts.

| Degrees. | Density. | Degrees. | Density. | Degrees. | Density. |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 1.000 | 27 | 1.230 | 51 | 1.547 |
| 3 | 1.020 | 30 | 1.261 | 54 | 1.594 |
| 6 | 1.040 | 33 | 1.295 | 57 | 1.659 |
| 9 | 1.064 | 36 | 1.333 | 60 | 1.717 |
| 12 | 1.089 | 39 | 1.373 | 63 | 1.779 |
| 15 | 1.114 | 42 | 1.414 | 66 | 1.848 |
| 18 | 1.140 | 45 | 1.455 | 69 | 1.920 |
| 21 | 1.170 | 48 | 1.500 | 72 | 2.000 |
| 24 | 1.200 |  |  |  |  |

Carrier's hydrometer was very similar to that of Baumé, Cartier having been employed by the latter to construct his instruments for the French revenue. The point at which the instrument floated in distilled water was marked $10^{\circ}$ by Cartier, and $30^{\circ}$ on Carrier's scale corresponded to $32^{\circ}$ on Baumés.

Perhaps the main object for which hydrometers have been constructed is the determination of the value of spirituous liquors, chiefly for revenue purposes. To this end an immense variety of hydrometers have been devised, differing mainly in the character of their scales.

In Speer's hydrometer the stem has the form of an octagonal prism, and upon each of the eight faces a scale is engraved, indicating the percentage strength of the spirit corresponding to the several divisions of the scale, the eight scales being adapted respectively to the temperature $35^{\circ}, 40^{\circ}$, $45^{\circ}, 50^{\circ}, 55^{\circ}, 60^{\circ}, 65^{\circ}$ and $70^{\circ} \mathrm{F}$. Four small pins, which can be inserted into the counterpoise of the instrument, serve to adapt the instrument to the temperatures intermediate between those for which the scales are constructed. William Speer was supervisor and chief assayer of spirits in the port of Dublin. For a more complete account of this instrument see Tilloch's Phil. Mag., xiv. 151.


Fig. 6.

The hydrometer constructed by Jones, of Holborn, consists of a spheroidal bulb with a rectangular stem (fig. 5). Between the bulb and counterpoise is placed a thermometer, which serves to indicate the temperature of the liquid, and the instrument is provided with


Fig. 5.-
Jones's
Hydrometer. three weights which can be attached to the top of the stem. On the four sides of the stem AD are engraved four scales corresponding respectively to the unloaded instrument, and to the instrument loaded with the respective weights. The instrument when unloaded serves for the range from 74 to 47 over proof; when loaded with the first weight it indicates from 46 to 13 over proof, with the second weight from 13 over proof to 29 under proof, and with the third from 29 under proof to pure water, the graduation corresponding to which is marked W at the bottom of the fourth scale. One side of the stem $A D$ is shown in fig. 5, the other three in fig. 6. The thermometer is also provided with four scales corresponding to the scales above mentioned. Each scale has its zero in the middle corresponding to $60^{\circ} \mathrm{F}$. If the mercury in the thermometer stand above this zero the spirit must be reckoned weaker than the hydrometer indicates by the number on the thermometer scale level with the top of the mercury, while if the thermometer indicate a temperature lower than the zero of the scale ( $60^{\circ}$ F.) the spirit must be reckoned stronger by the scale reading. At the side of each of the four scales on the stem of the hydrometer is engraved a set of small numbers indicating the contraction in volume which would be experienced if the requisite amount of water (or spirit) were added to bring the sample tested to the proof strength.

The hydrometer constructed by Dicas of Liverpool is provided with a sliding scale which can be adjusted for different temperatures, and which also indicates the contraction in volume incident on bringing the spirit to proof strength. It is provided with thirty-six different weights which, with the ten divisions on the stem, form a scale from 0 to 370 . The employment of so many weights renders the instrument ill-adapted for practical work where speed is an object.

This instrument was adopted by the United States in 1790, but was subsequently discarded by the Internal Revenue Service for another type. In this latter form the observations have to be made at the standard temperature of $60^{\circ}$ F., at which the graduation 100 corresponds to proof spirit and 200 to absolute alcohol. The need of adjustable weights is avoided by employing a set of five instruments, graduated respectively $0^{\circ}-100^{\circ}, 80^{\circ}-120^{\circ}, 100^{\circ}-140^{\circ}, 130^{\circ}-170^{\circ}$, $160^{\circ}-200^{\circ}$. The reading gives the volume of proof spirit equivalent to the volume of liquor; thus the readings $80^{\circ}$ and $120^{\circ}$ mean that 100 volumes of the test liquors contain the same amount of absolute alcohol as 80 and 120 volumes of proof spirit respectively. Proof spirit is defined in the United States as a mixture of alcohol and water which contains equal volumes of alcohol and water at $60^{\circ} \mathrm{F}$., the alcohol having a specific gravity of 0.7939 at $60^{\circ}$ as compared with water at its maximum density. The specific gravity of proof spirit is 0.93353 at $60^{\circ}$; and 100 volumes of the mixture is made from 50 volumes of absolute alcohol and 53.71 volumes of water.
Quin's universal hydrometer is described in the Transactions of the Society of Arts, viii. 98. It is provided with a sliding rule to adapt it to different temperatures, and has four scales, one of which is graduated for spirits and the other three serve to show the strengths of worts. The peculiarity of the instrument consists in the pyramidal form given to the stem, which renders the scale-divisions more nearly equal in length than they would be on a prismatic stem.

Atkins's hydrometer, as originally constructed, is described in Nicholson's Journal, 8vo, ii. 276. It is made of brass, and is provided with a spheroidal bulb the axis of which is 2 in . in length, the conjugate diameter being $11 / 2 \mathrm{in}$. The whole length of the instrument is 8 in ., the stem square of about $1 / 8-\mathrm{in}$. side, and the weight about 400 grains. It is provided with four weights, marked $1,2,3,4$, and weighing respectively $20,40,61$ and 84 grains, which can be attached to the shank of the instrument at C (fig. 7) and retained there by the fixed weight $B$. The scale engraved upon one face of the stem contains


Fig. 7.-Atkins's Hydrometer. fifty-five divisions, the top and bottom being marked 0 or zero and the alternate intermediate divisions (of which there are twenty-six) being marked with the letters of the alphabet in order. The four weights are so adjusted that, if the instrument floats with the stem emerging as far as the lower division 0 with one of the weights attached, then replacing the weight by the next heavier causes the instrument to sink through the whole length of the scale to the upper division 0 , and the first weight produces the same effect when applied to the naked instrument. The stem is thus virtually extended to five times its length, and the number of divisions increased practically to 272 . When no weight is attached the instrument indicates densities from .806 to .843 ; with No. 1 it registers from .843 to .880 , with No. 2 from .880 to .918 , with No. 3 from .918 to .958 , and with No. 4 from .958 to 1.000 , the temperature being $55^{\circ}$ F. It will thus be seen that the whole length of the stem corresponds to a difference of density of about .04 , and one division to about . 00074 , indicating a difference of little more than $1 / 3 \%$ in the strength of any sample of spirits.

The instrument is provided with a sliding rule, with scales corresponding to the several weights, which indicate the specific gravity corresponding to the several divisions of the hydrometer scale compared with water at $55^{\circ} \mathrm{F}$. The slider upon the rule serves to adjust the scale for different temperatures, and then indicates the strength of the spirit in percentages over or under proof. The slider is also provided with scales, marked respectively Dicas and Clarke, which serve to show the readings which would have been obtained had the instruments of those makers been employed. The line on the scale marked "concentration" indicates the diminution in volume consequent upon reducing the sample to proof strength (if it is over proof, O.P.) or upon reducing proof spirit to the strength of the sample (if it is under proof, U.P.). By applying the several weights in succession in addition to No. 4 the instrument can be employed for liquids heavier than water; and graduations on the other three sides of the stem, together with an additional slide rule, adapt the instrument for the determination of the strength of worts.

Atkins subsequently modified the instrument (Nicholson's Journal, 8vo, iii. 50) by constructing the different weights of different shapes, viz. circular, square, triangular and pentagonal, instead of numbering them 1, 2, 3 and 4 respectively, a figure of the weight being
stamped on the sliding rule opposite to every letter in the series to which it belongs, thus diminishing the probability of mistakes. He also replaced the letters on the stem by the corresponding specific gravities referred to water as unity. Further information concerning these instruments and the state of hydrometry in 1803 will be found in Atkins's pamphlet On the Relation between the Specific Gravities and the Strength of Spirituous Liquors (1803); or Phil. Mag. xvi. 26-33, 205-212, 305-312; xvii. 204-210 and 329-341.

In Gay-Lussac's alcoholometer the scale is divided into 100 parts corresponding to the presence of $1,2, \ldots \%$ by volume of alcohol at $15^{\circ}$ C., the highest division of the scale corresponding to the purest alcohol he could obtain (density .7947) and the lowest division corresponding to pure water. A table provides the necessary corrections for other temperatures.

Tralles's hydrometer differs from Gay-Lussac's only in being graduated at $4^{\circ} \mathrm{C}$. instead of $15^{\circ}$ C., and taking alcohol of density .7939 at $15.5^{\circ}$ C. for pure alcohol instead of .7947 as taken by Gay-Lussac (Keene's Handbook of Hydrometry).

In Beck's hydrometer the zero of the scale corresponds to density 1.000 and the division 30 to density . 850 , and equal divisions on the scale are continued as far as is required in both directions.

In the centesimal hydrometer of Francœur the volume of the stem between successive divisions of the scale is always $1 / 100$ th of the whole volume immersed when the instrument floats in water at $4^{\circ} \mathrm{C}$. In order to graduate the stem the instrument is first weighed, then immersed in distilled water at $4^{\circ}$ C., and the line of flotation marked zero. The first degree is then found by placing on the top of the stem a weight equal to $1 / 100$ th of the weight of the instrument, which increases the volume immersed by $1 / 100$ th of the original volume. The addition to the top of the stem of successive weights, each 11100 th of the weight of the instrument itself, serves to determine the successive degrees. The length of 100 divisions of the scale, or the length of the uniform stem the volume of which would be equal to that of the hydrometer up to the zero graduation, Francœur called the "modulus" of the hydrometer. He constructed his instruments of glass, using different instruments for different portions of the scale (Francœur, Traité d'aréométrie, Paris, 1842).

Dr Boriés of Montpellier constructed a hydrometer which was based upon the results of his experiments on mixtures of alcohol and water. The interval between the points corresponding to pure alcohol and to pure water Boriés divided into 100 equal parts, though the stem was prolonged so as to contain only 10 of these divisions, the other 90 being provided for by the addition of 9 weights to the bottom of the instrument as in Clarke's hydrometer.


Fig. 8.-
Sike's Hydrometer.

The instrument which has now been exclusively used for revenue purposes for nearly a century is that associated with the name of Bartholomew Sikes, who was correspondent to the Board of Excise from 1774 to 1783, and for some time collector of excise for Hertfordshire.

Sikes's hydrometer, on account of its similarity to that of Boriés, appears to have been borrowed from that instrument. It is made of gilded brass or silver, and consists of a spherical ball A (fig. 8), 1.5 in . in diameter, below which is a weight B connected with the ball by a short conical stem C. The stem D is rectangular in section and about $31 / 2 \mathrm{in}$. in length. This is divided into ten equal parts, each of which is subdivided into five. As in Boriés's instrument, a series of 9 weights, each of the form shown at E , serves to extend the scale to 100 principal divisions. In the centre of each weight is a hole capable of admitting the lowest and thickest end of the conical stem C, and a slot is cut into it just wide enough to allow the upper part of the cone to pass. Each weight can thus be dropped on to the lower stem so as to rest on the counterpoise B. The weights are marked $10,20, \ldots 90$; and in using the instrument that weight must be selected which will allow it to float in the liquid with a portion only of the stem submerged. Then the reading of the scale at the line of flotation, added to the number on the weight, gives the reading required. A small supernumerary weight $F$ is added, which can be placed upon the top of the stem. F is so adjusted that when the 60 weight is placed on the lower stem the instrument sinks to the same point in distilled water when F is attached as in proof spirit when $F$ is removed. The best instruments are now constructed for revenue purposes of silver, heavily gilded, because it was found that saccharic acid contained in some spirits attacked brass behind the gilding.

The following table gives the specific gravities corresponding to the principal graduations on Sikes's hydrometer at $60^{\circ} \mathrm{F}$. and $62^{\circ} \mathrm{F}$., together with the corresponding strengths of spirits. The latter are based upon the tables of Charles Gilpin, clerk to the Royal Society, for which the reader is referred to the Phil. Trans. for 1794. Gilpin's work is a model for its accuracy and thoroughness of detail, and his results have scarcely been improved upon by more recent
workers. The merit of Sikes's system lies not so much in the hydrometer as in the complete system of tables by which the readings of the instrument are at once converted into percentage of proof-spirit.

Table showing the Densities corresponding to the Indications of Sike's Hydrometer.

| Sike's Indications. | $60^{\circ} \mathrm{F}$. |  | $62^{\circ} \mathrm{F}$. |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Density. | Proof Spirit per cent. | Density. | Proof Spirit per cent. |
| 0 | . 815297 | 167.0 | . 815400 | 166.5 |
| 1 | . 816956 | 166.1 | . 817059 | 165.6 |
| 2 | . 818621 | 165.3 | . 818725 | 164.8 |
| 3 | . 820294 | 164.5 | . 820397 | 163.9 |
| 4 | . 821973 | 163.6 | . 822077 | 163.1 |
| 5 | . 823659 | 162.7 | . 823763 | 162.3 |
| 6 | . 825352 | 161.8 | . 825457 | 161.4 |
| 7 | . 827052 | 160.9 | . 827157 | 160.5 |
| 8 | . 828759 | 160.0 | . 828864 | 159.6 |
| 9 | . 830473 | 159.1 | . 830578 | 158.7 |
| 10 | . 832195 | 158.2 | . 832300 | 157.8 |
| 11 | . 833888 | 157.3 | . 833993 | 156.8 |
| 12 | . 835587 | 156.4 | . 835692 | 155.9 |
| 13 | . 837294 | 155.5 | . 837400 | 155.0 |
| 14 | . 839008 | 154.6 | . 839114 | 154.0 |
| 15 | . 840729 | 153.7 | . 840835 | 153.1 |
| 16 | . 842458 | 152.7 | . 842564 | 152.1 |
| 17 | . 844193 | 151.7 | . 844299 | 151.1 |
| 18 | . 845936 | 150.7 | . 846042 | 150.1 |
| 19 | . 847685 | 149.7 | . 847792 | 149.1 |
| 20 | . 849442 | 148.7 | . 849549 | 148.1 |
| 20B | . 849393 | 148.7 | . 849500 | 148.1 |
| 21 | . 851122 | 147.6 | . 851229 | 147.1 |
| 22 | . 852857 | 146.6 | . 852964 | 146.1 |
| 23 | . 854599 | 145.6 | . 854707 | 145.1 |
| 24 | . 856348 | 144.6 | . 856456 | 144.0 |
| 25 | . 858105 | 143.5 | . 858213 | 142.9 |
| 26 | . 859869 | 142.4 | . 859978 | 141.8 |
| 27 | . 861640 | 141.3 | . 861749 | 140.8 |
| 28 | . 863419 | 140.2 | . 863528 | 139.7 |
| 29 | . 865204 | 139.1 | . 865313 | 138.5 |
| 30 | . 866998 | 138.0 | . 867107 | 137.4 |
| 30B | . 866991 | 138.0 | . 867100 | 137.4 |
| 31 | . 868755 | 136.9 | . 868865 | 136.2 |
| 32 | . 870526 | 135.7 | . 870636 | 135.1 |
| 33 | . 872305 | 134.5 | . 872415 | 133.9 |
| 34 | . 874090 | 133.4 | . 874200 | 132.8 |
| 35 | . 875883 | 132.2 | . 873994 | 131.6 |
| 36 | . 877684 | 131.0 | . 877995 | 130.4 |
| 37 | . 879492 | 129.8 | . 879603 | 129.1 |
| 38 | . 881307 | 128.5 | . 881419 | 127.9 |
| 39 | . 883129 | 127.3 | . 883241 | 126.7 |
| 40 | . 884960 | 126.0 | . 885072 | 125.4 |
| 40B | . 884888 | 126.0 | . 885000 | 125.4 |
| 41 | . 886689 | 124.8 | . 886801 | 124.2 |
| 42 | . 888497 | 123.5 | . 888609 | 122.9 |
| 43 | . 890312 | 122.2 | . 890425 | 121.6 |
| 44 | . 892135 | 120.9 | . 892248 | 120.3 |
| 45 | . 893965 | 119.6 | . 894078 | 119.0 |
| 46 | . 895803 | 118.3 | . 895916 | 117.6 |
| 47 | . 897647 | 116.9 | . 897761 | 116.3 |
| 48 | . 899509 | 115.6 | . 899614 | 114.9 |
| 49 | . 901360 | 114.2 | . 901417 | 113.5 |
| 50 | . 903229 | 112.8 | . 903343 | 112.1 |
| 50B | . 903186 | 112.8 | . 903300 | 112.1 |
| 51 | . 905024 | 111.4 | . 905138 | 110.7 |
| 52 | . 906869 | 110.0 | . 906983 | 109.3 |


| 53 | . 908722 | 108.6 | . 908837 | 107.9 |
| :---: | :---: | :---: | :---: | :---: |
| 54 | . 910582 | 107.1 | . 910697 | 106.5 |
| 55 | . 912450 | 105.6 | . 912565 | 105.0 |
| 56 | . 914326 | 104.2 | . 914441 | 103.5 |
| 57 | . 916209 | 102.7 | . 916323 | 102.0 |
| 58 | . 918100 | 101.3 | . 918216 | 100.5 |
| 59 | . 919999 | 99.7 | . 820115 | 98.9 |
| 60 | . 921906 | 98.1 | . 922022 | 97.4 |
| 60B | . 921884 | 98.1 | . 922000 | 97.4 |
| 61 | . 923760 | 96.6 | . 923877 | 95.9 |
| 62 | . 925643 | 95.0 | . 925760 | 94.2 |
| 63 | . 927534 | 93.3 | . 927652 | 92.6 |
| 64 | . 929433 | 91.7 | . 929550 | 90.9 |
| 65 | . 931339 | 90.0 | . 931457 | 89.2 |
| 66 | . 933254 | 88.3 | . 933372 | 87.5 |
| 67 | . 935176 | 86.5 | . 935294 | 85.8 |
| 68 | . 937107 | 84.7 | . 937225 | 84.0 |
| 69 | . 939045 | 82.9 | . 939163 | 82.2 |
| 70 | . 940991 | 81.1 | . 941110 | 80.3 |
| 70B | . 940981 | 81.1 | . 941100 | 80.3 |
| 71 | . 942897 | 79.2 | . 943016 | 78.4 |
| 72 | . 944819 | 77.3 | . 944938 | 76.5 |
| 73 | . 946749 | 75.3 | . 946869 | 74.5 |
| 74 | . 948687 | 73.3 | . 948807 | 72.5 |
| 75 | . 950634 | 71.2 | . 950753 | 70.4 |
| 76 | . 952588 | 69.0 | . 952708 | 68.2 |
| 77 | . 954550 | 66.8 | . 954670 | 66.0 |
| 78 | . 956520 | 64.4 | . 956641 | 63.5 |
| 79 | . 958498 | 61.9 | . 958619 | 61.1 |
| 80 | . 960485 | 59.4 | . 960606 | 58.5 |
| 80B | . 960479 | 59.4 | . 960600 | 58.5 |
| 81 | . 962433 | 56.7 | . 962555 | 55.8 |
| 82 | . 964395 | 53.9 | . 964517 | 53.0 |
| 83 | . 966366 | 50.9 | . 966488 | 50.0 |
| 84 | . 968344 | 47.8 | . 968466 | 47.0 |
| 85 | . 970331 | 44.5 | . 970453 | 43.8 |
| 86 | . 972325 | 41.0 | . 972448 | 40.4 |
| 87 | . 974328 | 37.5 | . 974451 | 36.9 |
| 88 | . 976340 | 34.0 | . 976463 | 33.5 |
| 89 | . 978359 | 30.6 | . 978482 | 30.1 |
| 90 | . 980386 | 27.2 | . 980510 | 26.7 |
| 90B | . 980376 | 27.2 | . 980500 | 26.7 |
| 91 | . 982371 | 23.9 | . 982496 | 23.6 |
| 92 | . 984374 | 20.8 | . 984498 | 20.5 |
| 93 | . 986385 | 17.7 | . 986510 | 17.4 |
| 94 | . 988404 | 14.8 | . 988529 | 14.5 |
| 95 | . 990431 | 12.0 | . 990557 | 11.7 |
| 96 | . 992468 | 9.3 | . 992593 | 9.0 |
| 97 | . 994512 | 6.7 | . 994637 | 6.5 |
| 98 | . 996565 | 4.1 | . 996691 | 4.0 |
| 99 | . 998626 | 1.8 | . 998752 | 1.6 |
| 100 | 1.000696 | 0.0 | 1.000822 | 0.0 |

In the above table for Sikes's hydrometer two densities are given corresponding to each of the degrees $20,30,40,50,60,70,80$ and 90 , indicating that the successive weights belonging to the particular instrument for which the table has been calculated do not quite agree. The discrepancy, however, does not produce any sensible error in the strength of the corresponding spirit.

A table which indicates the weight per gallon of spirituous liquors for every degree of Sikes's hydrometer is printed in 23 and 24 Vict. c. 114, schedule B. This table differs slightly from that given above, which has been abridged from the table given in Keene's Handbook of Hydrometry, apparently on account of the equal divisions on Sikes's scale having been taken as corresponding to equal increments of density.

Sikes's hydrometer was established for the purpose of collecting the revenue of the United Kingdom by Act of Parliament, 56 Geo. III. c. 140, by which it was enacted that "all spirits shall be deemed and taken to be of the degree of strength which the said hydrometers called Sikes's hydrometers shall, upon trial by any officer or officers of the customs or excise, denote such spirits to be." This act came into force on January 5, 1817, and was to have remained in force
until August 1, 1818, but was repealed by 58 Geo. III. c. 28, which established Sikes's hydrometer on a permanent footing. By 3 and 4 Will. IV. c. 52 , § 123, it was further enacted that the same instruments and methods should be employed in determining the duty upon imported spirits as should in virtue of any Act of Parliament be employed in the determination of the duty upon spirits distilled at home. It is the practice of the officers of the inland revenue to adjust Sikes's hydrometer at $62^{\circ} \mathrm{F}$., that being the temperature at which the imperial gallon is defined as containing 10 tb avoirdupois of distilled water. The specific gravity of any sample of spirits thus determined, when multiplied by ten, gives the weight in pounds per imperial gallon, and the weight of any bulk of spirits divided by this number gives its volume at once in imperial gallons.

Mr (afterwards Colonel) J. B. Keene, of the Hydrometer Office, London, has constructed an instrument after the model of Sikes's, but provided with twelve weights of different masses but equal volumes, and the instrument is never used without having one of these attached. When loaded with either of the lightest two weights the instrument is specifically lighter than Sikes's hydrometer when unloaded, and it may thus be used for specific gravities as low as that of absolute alcohol. The volume of each weight being the same, the whole volume immersed is always the same when it floats at the same mark whatever weight may be attached.

Besides the above, many hydrometers have been employed for special purposes. Twaddell's hydrometer is adapted for densities greater than that of water. The scale is so arranged that the reading multiplied by 5 and added to 1000 gives the specific gravity with reference to water as 1000. To avoid an inconveniently long stem, different instruments are employed for different parts of the scale as mentioned above.
The lactometer constructed by Dicas of Liverpool is adapted for the determination of the quality of milk. It resembles Sikes's hydrometer in other respects, but is provided with eight weights. It is also provided with a thermometer and slide rule, to reduce the readings to the standard temperature of $55^{\circ} \mathrm{F}$. Any determination of density can be taken only as affording prima facie evidence of the quality of milk, as the removal of cream and the addition of water are operations which tend to compensate each other in their influence on the density of the liquid, so that the lactometer cannot be regarded as a reliable instrument.

The marine hydrometers, as supplied by the British government to the royal navy and the merchant marine, are glass instruments with slender stems, and generally serve to indicate specific gravities from 1.000 to 1.040 . Before being issued they are compared with a standard instrument, and their errors determined. They are employed for taking observations of the density of sea-water.

The salinometer is a hydrometer originally intended to indicate the strength of the brine in marine boilers in which sea-water is employed. Saunders's salinometer consists of a hydrometer which floats in a chamber through which the water from the boiler is allowed to flow in a gentle stream, at a temperature of $200^{\circ} \mathrm{F}$. The peculiarity of the instrument consists in the stream of water, as it enters the hydrometer chamber, being made to impinge against a disk of metal, by which it is broken into drops, thus liberating the steam, which would otherwise disturb the instrument.

The use of Sikes's hydrometer necessitates the employment of a considerable quantity of spirit. For the testing of spirits in bulk no more convenient instrument has been devised, but where very small quantities are available more suitable laboratory methods must be adopted.

In England, the Finance Act 1907 (7 Ed. VII. c. 13), section 4, provides as follows: (1) The Commissioners of Customs and the Commissioners of Inland Revenue may jointly make regulations authorizing the use of any means described in the regulations for ascertaining for any purpose the strength or weight of spirits. (2) Where under any enactment Sykes's (sic) Hydrometer is directed to be used or may be used for the purpose of ascertaining the strength or weight of spirits, any means so authorized by regulations may be used instead of Sykes's Hydrometer and references to Sykes's Hydrometer in any enactment shall be construed accordingly. (3) Any regulations made under this section shall be published in the London, Edinburgh and Dublin Gazette, and shall take effect from the date of publication, or such later date as may be mentioned in the regulations for the purpose. (4) The expression "spirits" in this section has the same meaning as in the Spirits Act 1880.

In Nicholson's Journal, iii. 89, Citizen Eusebe Salverte calls attention to the poem "De Ponderibus et Mensuris" generally ascribed to Rhemnius Fannius Palaemon, and consequently 300 years older than Hypatia, in which the hydrometer is described and attributed to Archimedes.

HYDROPATHY, the name given, from the Greek, to the "water-cure," or the treatment of disease by water, used outwardly and inwardly. Like many descriptive names, the word "hydropathy" is defective and even misleading, the active agents in the treatment being heat and cold, of which water is little more than the vehicle, and not the only one. Thermotherapeutics (or thermotherapy) is a term less open to objection.

Hydropathy, as a formal system, dates from about 1829, when Vincenz Priessnitz (18011851), a farmer of Gräfenberg in Silesia, Austria, began his public career in the paternal homestead, extended so as to accommodate the increasing numbers attracted by the fame of his cures. Two English works, however, on the medical uses of water had been translated into German in the century preceding the rise of the movement under Priessnitz. One of these was by Sir John Floyer (1649-1734), a physician of Lichfield, who, struck by the remedial use of certain springs by the neighbouring peasantry, investigated the history of cold bathing, and published in 1702 his $\Psi_{u \chi \rho 0 \lambda o u \sigma i ́ \alpha, ~ o r ~ t h e ~ H i s t o r y ~ o f ~ C o l d ~ B a t h i n g, ~ b o t h ~ A n c i e n t ~ a n d ~ M o d e r n . " ~}^{\text {. }}$ The book ran through six editions within a few years, and the translation was largely drawn upon by Dr J. S. Hahn of Silesia, in a work published in 1738, On the Healing Virtues of Cold Water, Inwardly and Outwardly applied, as proved by Experience. The other work was that of Dr James Currie (1756-1805) of Liverpool, entitled Medical Reports on the Effects of Water, Cold and Warm, as a remedy in Fevers and other Diseases, published in 1797, and soon after translated into German by Michaelis (1801) and Hegewisch (1807). It was highly popular, and first placed the subject on a scientific basis. Hahn's writings had meanwhile created much enthusiasm among his countrymen, societies having been everywhere formed to promote the medicinal and dietetic use of water; and in 1804 Professor Örtel of Ansbach republished them and quickened the popular movement by unqualified commendation of water drinking as a remedy for all diseases. In him the rising Priessnitz found a zealous advocate, and doubtless an instructor also.

At Gräfenberg, to which the fame of Priessnitz drew people of every rank and many countries, medical men were conspicuous by their numbers, some being attracted by curiosity, others by the desire of knowledge, but the majority by the hope of cure for ailments which had as yet proved incurable. Many records of experiences at Gräfenberg were published, all more or less favourable to the claims of Priessnitz, and some enthusiastic in their estimate of his genius and penetration; Captain Claridge introduced hydropathy into England in 1840, his writings and lectures, and later those of Sir W. Erasmus Wilson (1809-1884), James Manby Gully (18081883) and Edward Johnson, making numerous converts, and filling the establishments opened soon after at Malvern and elsewhere. In Germany, France and America hydropathic establishments multiplied with great rapidity. Antagonism ran high between the old practice and the new. Unsparing condemnation was heaped by each on the other; and a legal prosecution, leading to a royal commission of inquiry, served but to make Priessnitz and his system stand higher in public estimation.

Increasing popularity diminished before long that timidity which had in great measure prevented trial of the new method from being made on the weaker and more serious class of cases, and had caused hydropathists to occupy themselves mainly with a sturdy order of chronic invalids well able to bear a rigorous regimen and the severities of unrestricted crisis. The need of a radical adaptation to the former class was first adequately recognized by John Smedley, a manufacturer of Derbyshire, who, impressed in his own person with the severities as well as the benefits of "the cold water cure," practised among his workpeople a milder form of hydropathy, and began about 1852 a new era in its history, founding at Matlock a counterpart of the establishment at Gräfenberg.

Ernst Brand (1826-1897) of Berlin, Räljen and Theodor von Jürgensen of Kiel, and Karl Liebermeister (1833-1901) of Basel, between 1860 and 1870, employed the cooling bath in abdominal typhus with striking results, and led to its introduction to England by Dr Wilson Fox. In the Franco-German war the cooling bath was largely employed, in conjunction frequently with quinine; and it now holds a recognized position in the treatment of hyperpyrexia. The wet sheet pack has become part of medical practice; the Turkish bath, introduced by David Urquhart (1805-1877) into England on his return from the East, and ardently adopted by Dr Richard Barter (1802-1870) of Cork, has become a public institution, and, with the "morning tub" and the general practice of water drinking, is the most noteworthy of the many contributions by hydropathy to public health (see Baths, ad fin.).

The appliances and arrangements by means of which heat and cold are brought to bear on the economy are-(a) Packings, hot and cold, general and local, sweating and cooling; (b) hot air and steam baths; (c) general baths, of hot water and cold; (d) sitz, spinal, head and foot baths; (e) bandages (or compresses), wet and dry; also ( $f$ ) fomentations and poultices, hot and cold, sinapisms, stupes, rubbings and water potations, hot and cold.
(a) Packings.-The full pack consists of a wet sheet enveloping the body, with a number of dry
blankets packed tightly over it, including a macintosh covering or not. In an hour or less these are removed and a general bath administered. The pack is a derivative, sedative, sudorific and stimulator of cutaneous excretion. There are numerous modifications of it, notably the cooling pack, where the wrappings are loose and scanty, permitting evaporation, and the application of indefinite duration, the sheet being rewetted as it dries; this is of great value in protracted febrile conditions. There are also local packs, to trunk, limbs or head separately, which are derivative, soothing or stimulating, according to circumstance and detail.
(b) Hot air baths, the chief of which is the Turkish (properly, the Roman) bath, consisting of two or more chambers ranging in temperature from $120^{\circ}$ to $212^{\circ}$ or higher, but mainly used at $150^{\circ}$ for curative purposes. Exposure is from twenty minutes up to two hours according to the effect sought, and is followed by a general bath, and occasionally by soaping and shampooing. It is stimulating, derivative, depurative, sudorific and alterative, powerfully promoting tissue change by increase of the natural waste and repair. It determines the blood to the surface, reducing internal congestions, is a potent diaphoretic, and, through the extremes of heat and cold, is an effective nervous and vascular stimulant and tonic. Morbid growths and secretions, as also the uraemic, gouty and rheumatic diathesis, are beneficially influenced by it. The full pack and Turkish bath have between them usurped the place and bettered the function of the once familiar hot bath. The Russian or steam bath and the lamp bath are primitive and inferior varieties of the modern Turkish bath, the atmosphere of which cannot be too dry and pure.
(c) General baths comprise the rain (or needle), spray (or rose), shower, shallow, plunge, douche, wave and common morning sponge baths, with the dripping sheet, and hot and cold spongings, and are combinations, as a rule, of hot and cold water. They are stimulating, tonic, derivative and detergent.
(d) Local baths comprise the sitz (or sitting), douche (or spouting), spinal, foot and head baths, of hot or cold water, singly or in combination, successive or alternate. The sitz, head and foot baths are used "flowing" on occasion. The application of cold by "Leiter's tubes" is effective for reducing inflammation (e.g. in meningitis and in sunstroke); in these a network of metal or indiarubber tubing is fitted to the part affected, and cold water kept continuously flowing through them. Rapid alternations of hot and cold water have a powerful effect in vascular stasis and lethargy of the nervous system and absorbents, yielding valuable results in local congestions and chronic inflammations.
(e) Bandages (or compresses) are of two kinds,-cooling, of wet material left exposed for evaporation, used in local inflammations and fevers; and heating, of the same, covered with waterproof material, used in congestion, external or internal, for short or long periods. Poultices, warm, of bread, linseed, bran, \&c., changed but twice in twenty-four hours, are identical in action with the heating bandage, and superior only in the greater warmth and consequent vital activity their closer application to the skin ensures.
(f) Fomentations and poultices, hot or cold, sinapisms, stupes, rubefacients, irritants, frictions, kneadings, calisthenics, gymnastics, electricity, \&c., are adjuncts largely employed.

Bibliography.-Among the numerous earlier works on hydropathy, the following are worth mention: Balbirnie, Water Cure in Consumption (1847), Hydropathic Aphorisms (1856) and A Plea for the Turkish Bath (1862); Beni-Barde, Traité d'hydrothérapie (1874); Claridge, Cold Water Cure, or Hydropathy (1841), Facts and Evidence in Support of Hydropathy (1843) and Cold Water, Tepid Water and Friction Cure (1849); Dunlop, Philosophy of the Bath (1873); Floyer, Psychrolousia, or the History of Cold-Bathing, \&c. (1702); J. S. Hahn (Schweidnitz), Observations on the Healing Virtues of Cold Water (1738); Hunter, Hydropathy for Home Use (1879); E. W. Lane, Hydropathy, or the Natural System of Medical Treatment (1857); R. J. Lane, Life at the Water Cure (1851); Shew, Hydropathic Family Physician (1857); Smedley, Practical Hydropathy (1879); Smethurst, Hydrotherapia, or the Water Cure (1843); Wainwright, Inquiry into the Nature and Use of Baths (1737); Weiss, Handbook of Hydropathy (1844); Wilson Principles and Practice of the Cold Water Cure (1854) and The Water Cure (1859). A useful recent work dealing comprehensively with the subject is Richard Metcalfe's Rise and Progress of Hydropathy (1906).

HYDROPHOBIA (Gr. ǔठ $\omega \rho$, water, and фó $\beta o s$, fear; so called from the symptom of dread of water), or Rabies (Lat. for "madness"), an acute disease, occurring chiefly in certain of the lower animals, particularly the canine species, and liable to be communicated by them to other animals and to man.
In Dogs, \&c.-The occurrence of rabies in the fox, wolf, hyaena, jackal, raccoon, badger and skunk has been asserted; but there is every probability that it is originally a disease of the dog.

It is communicated by inoculation to nearly all, if not all, warm-blooded creatures. The transmission from one animal to another only certainly takes place through inoculation with viruliferous matters. The malady is generally characterized at a certain stage by an irrepressible desire in the animal to act offensively with its natural weapons-dogs and other carnivora attacking with their teeth, herbivora with their hoofs or horns, and birds with their beaks, when excited ever so slightly. In the absence of excitement the malady may run its course without any fit of fury or madness.

Symptoms.-The disease has been divided into three stages or periods, and has also been described as appearing in at least two forms, according to the peculiarities of the symptoms. But, as a rule, one period of the disease does not pass suddenly into another, the transition being almost imperceptible; and the forms do not differ essentially from each other, but appear merely to constitute varieties of the same disease, due to the natural disposition of the animal, or other modifying circumstances. These forms have been designated true or furious rabies (Fr. rage vrai; Ger. rasende Wuth) and dumb rabies (Fr. rage mue; Ger. stille Wuth).

The malady does not commence with fury and madness, but in a strange and anomalous change in the habits of the dog: it becomes dull, gloomy, and taciturn, and seeks to isolate itself in out-of-the-way places, retiring beneath chairs and to odd corners. But in its retirement it cannot rest: it is uneasy and fidgety, and no sooner has it lain down than suddenly it jumps up in an agitated manner, walks backwards and forwards several times, again lies down and assumes a sleeping attitude, but has only maintained it for a few minutes when it is once more moving about. Again it retires to its corner, to the farthest recess it can find, and huddles itself up into a heap, with its head concealed beneath its chest and fore-paws. This state of continual agitation and inquietude is in striking contrast with its ordinary habits, and should therefore receive attention. Not unfrequently there are a few moments when the creature appears more lively than usual, and displays an extraordinary amount of affection. Sometimes there is a disposition to gather up straw, thread, bits of wood, \&c., which are industriously carried away; a tendency to lick anything cold, as iron, stones, \&c., is also observed in many instances; and there is also a desire evinced to lick other animals. Sexual excitement is also frequently an early symptom. At this period no disposition to bite is observed; the animal is docile with its master and obeys his voice, though not so readily as before, nor with the same pleased countenance. There is something strange in the expression of its face, and the voice of its owner is scarcely able to make it change from a sudden gloominess to its usual animated aspect. These symptoms gradually become more marked; the restlessness and agitation increase. If on straw the dog scatters and pulls it about with its paws, and if in a room it scratches and tumbles the cushions or rugs on which it usually lies. It is incessantly on the move, rambling about, scratching the ground, sniffing in corners and at the doors, as if on the scent or seeking for something. It indulges in strange movements, as if affected by some mental influences or a prey to hallucinations. When not excited by any external influence it will remain for a brief period perfectly still and attentive, as if watching something, or following the movements of some creature on the wall; then it will suddenly dart forward and snap at the vacant air, as if pursuing an annoying object, or endeavouring to seize a fly. At another time it throws itself, yelling and furious, against the wall, as if it heard threatening voices on the other side, or was bent on attacking an enemy. Nevertheless, the animal is still docile and submissive, for its master's voice will bring it out of its frenzy. But the saliva is already virulent, and the excessive affection which it evinces at intervals, by licking the hands or face of those it loves, renders the danger very great should there be a wound or abrasion. Until a late period in the disease the master's voice has a powerful influence over the animal. When it has escaped from all control and wanders erratically abroad, ferocious and restless, and haunted by horrid phantoms, the familiar voice yet exerts its influence, and it is rare indeed that it attacks its master.

There is no dread of water in the rabid dog; the animal is generally thirsty, and if water be offered will lap it with avidity, and swallow it at the commencement of the disease. And when, at a later period, the constriction about the throat-symptomatic of the disease-renders swallowing difficult, the dog will none the less endeavour to drink, and the lappings are as frequent and prolonged when deglutition becomes impossible. So little dread has the rabid dog of water that it will ford streams and swim rivers; and when in the ferocious stage it will even do this in order to attack other creatures on the opposite side.

At the commencement of the disease the dog does not usually refuse to eat, and some animals are voracious to an unusual degree. But in a short time it becomes fastidious, only eating what it usually has a special predilection for. Soon, however, this gives place to a most characteristic symptom-either the taste becomes extremely depraved or the dog has a fatal and imperious desire to bite and ingest everything. The litter of its kennel, wool from cushions, carpets, stockings, slippers, wood, grass, earth, stones, glass, horse-dung, even its own faeces and urine, or whatever else may come in its way, are devoured. On examination of the body of a dog which has died of rabies it is so common to find in the stomach a quantity of dissimilar and strange matters on which the teeth have been exercised that, if there was nothing known of the animal's history, there would be strong evidence of its having been affected with the disease. When a dog, then, is observed to gnaw and eat suchlike matters, though it exhibits no tendency

The mad dog does not usually foam at the mouth to any great extent at first. The mucus of the mouth is not much increased in quantity, but it soon becomes thicker, viscid, and glutinous, and adheres to the angles of the mouth, fauces and teeth. It is at this period that the thirst is most ardent, and the dog sometimes furiously attempts to detach the saliva with its paws; and if after a while it loses its balance in these attempts and tumbles over, there can no longer be any doubt as to the nature of the malady. There is another symptom connected with the mouth in that form of the disease named "dumb madness" which has frequently proved deceptive. The lower jaw drops in consequence of paralysis of its muscles, and the mouth remains open. The interior is dry from the air passing continually over it, and assumes a deep red tint, somewhat masked by patches of dust or earth, which more especially adhere to the upper surface of the tongue and to the lips. The strange alteration produced in the dog's physiognomy by its constantly open mouth and the dark colour of the interior is rendered still more characteristic by the dull, sad, or dead expression of the animal's eyes. In this condition the creature is not very dangerous, because generally it could not bite if it tried-indeed there does not appear to be much desire to bite in dumb madness; but the saliva is none the less virulent, and accidental inoculations with it, through imprudent handling, will prove as fatal as in the furious form. The mouth should not be touched,-numerous deaths having occurred through people thinking the dog had some foreign substance lodged in its throat, and thrusting their fingers down to remove it. The sensation of tightness which seems to exist at the throat causes the dog to act as if a bone were fixed between its teeth or towards the back of its mouth, and to employ its forepaws as if to dislodge it. This is a very deceptive symptom, and may prove equally dangerous if caution be not observed. Vomiting of blood or a chocolate-coloured fluid is witnessed in some cases, and has been supposed to be due to the foreign substances in the stomach, which abrade the lining membrane; this, however, is not correct, as it has been observed in man.

The voice of the rabid dog is very peculiar, and so characteristic that to those acquainted with it nothing more is needed to prove the presence of the disease. Those who have heard it once or twice never forget its signification. Owing to the alterations taking place in the larynx the voice becomes hoarse, cracked and stridulous, like that of a child affected with croup-the "voix du coq," as the French have it. A preliminary bark is made in a somewhat elevated tone and with open mouth; this is immediately succeeded by five, six or eight decreasing howls, emitted when the animal is sitting or standing, and always with the nose elevated, which seem to come from the depths of the throat, the jaws not coming together and closing the mouth during such emission, as in the healthy bark. This alteration in the voice is frequently the first observable indication of the malady, and should at once attract attention. In dumb madness the voice is frequently lost from the very commencement-hence the designation.

The sensibility of the mad dog appears to be considerably diminished, and the animal appears to have lost the faculty of expressing the sensations it experiences: it is mute under the infliction of pain, though there can be no doubt that it still has peripheral sensation to some extent. Burning, beating and wounding produce much less effect than in health, and the animal will even mutilate itself with its teeth. Suspicion, therefore, should always strongly attach to a dog which does not manifest a certain susceptibility to painful impressions and receives punishment without any cry or complaint. There is also reason for apprehension when a dog bites itself persistently in any part of its body. A rabid dog is usually stirred to fury at the sight of one of its own species; this test has been resorted to by Henrie Marie Bouley (1814-1885) to dissipate doubts as to the existence of the disease when the diagnosis is otherwise uncertain. As soon as the suspected animal, if it is really rabid, finds itself in the presence of another of its species it at once assumes the aggressive, and, if allowed, will bite furiously. All rabid animals indeed become excited, exasperated, and furious at the sight of a dog, and attack it with their natural weapons, even the timid sheep when rabid butts furiously at the enemy before which in health it would have fled in terror. This inversion of sentiment is sometimes valuable in diagnosing the malady; it is so common that it may be said to be present in every case of rabies. When, therefore, a dog, contrary to its habits and natural inclination, becomes suddenly aggressive to other dogs, it is time to take precautions.

In the large majority of instances the dog is inoffensive in the early period of the disease to those to whom it is familiar. It then flies from its home and either dies, is killed as "mad," or returns in a miserable plight, and in an advanced stage of the malady, when the desire to bite is irresistible. It is in the early stage that sequestration and suppressive measures are most valuable. The dogs which propagate the disease are usually those that have escaped from their owners. After two or three days, frequently in about twelve hours, more serious and alarming symptoms appear, ferocious instincts are developed, and the desire to do injury is irrepressible. The animal has an indefinable expression of sombre melancholy and cruelty. The eyes have their pupils dilated, and emit flashes of light when they are not dull and heavy; they always appear so fierce as to produce terror in the beholder; they are red, and their sensibility to light is increased; and wrinkles, which sometimes appear on the forehead, add to the repulsive aspect of the animal. If caged it flies at the spectator, emitting its characteristic howl or bark, and seizing the iron bars with its teeth, and if a stick be thrust before it this is grasped and
gnawed. This fury is soon succeeded by lassitude, when the animal remains insensible to every excitement. Then all at once it rouses up again, and another paroxysm of fury commences. The first paroxysm is usually the most intense, and the fits vary in duration from some hours to a day, and even longer; they are ordinarily briefer in trained and pet dogs than in those which are less domesticated, but in all the remission is so complete after the first paroxysm that the animals appear to be almost well, if not in perfect health. During the paroxysms respiration is hurried and laboured, but tranquil during the remissions. There is an increase of temperature, and the pulse is quick and hard. When the animal is kept in a dark place and not excited, the fits of fury are not observed. Sometimes it is agitated and restless in the manner already described. It never becomes really furious or aggressive unless excited by external objects-the most potent of these, as has been said, being another dog, which, however, if it be admitted to its cage, it may not at once attack. The attacked animal rarely retaliates, but usually responds to the bites by acute yells, which contrast strangely with the silent anger of the aggressor, and tries to hide its head with its paws or beneath the straw. These repeated paroxysms hurry the course of the disease. The secretion and flowing of a large quantity of saliva from the mouth are usually only witnessed in cases in which swallowing has become impossible, the mouth being generally dry. At times the tongue, nose and whole head appear swollen. Other dogs frequently shun one which is rabid, as if aware of their danger.
The rabid dog, if lodged in a room or kept in a house, is continually endeavouring to escape; and when it makes its escape it goes freely forward, as if impelled by some irresistible force. It travels considerable distances in a short time, perhaps attacking every living creature it meets -preferring dogs, however, to other animals, and these to mankind; cats, sheep, cattle and horses are particularly liable to be injured. It attacks in silence, and never utters a snarl or a cry of anger; should it chance to be hurt in return it emits no cry or howl of pain. The degree of ferocity appears to be related to natural disposition and training. Some dogs, for instance, will only snap or give a slight bite in passing, while others will bite furiously, tearing the objects presented to them, or which they meet in their way, and sometimes with such violence as to injure their mouth and break their teeth, or even their jaws. If chained, they will in some cases gnaw the chain until their teeth are worn away and the bones laid bare. The rabid dog does not continue its progress very long. Exhausted by fatigue and the paroxysms of madness excited in it by the objects it meets, as well as by hunger, thirst, and also, no doubt, by the malady, its limbs soon become feeble; the rate of travelling is lessened and the walk is unsteady, while its drooping tall, head inclined towards the ground, open mouth, and protruded tongue (of a leaden colour or covered with dust) give the distressed creature a very striking and characteristic physiognomy. In this condition, however, it is much less to be dreaded than in its early fits of fury, since it is no longer capable or desirous of altering its course or going out of its way to attack an animal or a man not immediately in the path. It is very probable that its fast-failing vision, deadened scent, and generally diminished perception prevent its being so readily impressed or excited by surrounding objects as it previously was. To each paroxysm, which is always of short duration, there succeeds a degree of exhaustion as great as the fits have been violent and oft repeated. This compels the animal to stop; then it shelters itself in obscure places-frequently in ditches by the roadside-and lies there in a somnolescent state for perhaps hours. There is great danger, nevertheless, in disturbing the dog at this period; for when roused from its torpor it has sometimes sufficient strength to inflict a bite. This period, which may be termed the second stage, is as variable in its duration as the first, but it rarely exceeds three or four days. The above-described phenomena gradually merge into those of the third or last period, when symptoms of paralysis appear, which are speedily followed by death. During the remission in the paroxysms these paralytic symptoms are more particularly manifested in the hind limbs, which appear as if unable to support the animal's weight, and cause it to stagger about; or the lower jaw becomes more or less drooping, leaving the parched mouth partially open. Emaciation rapidly sets in, and the paroxysms diminish in intensity, while the remissions become less marked. The physiognomy assumes a still more sinister and repulsive aspect; the hair is dull and erect; the flanks are retracted; the eyes lose their lustre and are buried in the orbits, the pupil being dilated, and the cornea dull and semi-opaque; very often, even at an early period, the eyes squint, and this adds still more to the terrifying appearance of the poor dog. The voice, if at all heard, is husky, the breathing laborious, and the pulse hurried and irregular. Gradually the paralysis increases, and the posterior extremities are dragged as if the animal's back were broken, until at length it becomes general; it is then the prelude to death. Or the dog remains lying in a state of stupor, and can only raise itself with difficulty on the fore-limbs when greatly excited. In this condition it may yet endeavour to bite at objects within its reach. At times convulsions of a tetanic character appear in certain muscles; at other times these are general. A comatose condition ensues, and the rabid dog, if permitted to die naturally, perishes, in the great majority of cases, from paralysis and asphyxia.
In dumb madness there is paralysis of the lower jaw, which imparts a curious and very characteristic physiognomy to the dog; the voice is also lost, and the animal can neither eat nor drink. In this condition the creature remains with its jaw pendent and the mouth consequently wide open, showing the flaccid or swollen tongue covered with brownish matter, and a stringy gelatinous-looking saliva lying between it and the lower lip and coating the fauces, which sometimes appear to be inflamed. Though the animal is unable to swallow fluids, the desire to
drink is nevertheless intense; for the creature will thrust its face into the vessel of water in futile attempts to obtain relief, even until the approach of death. Water may be poured down its throat without inducing a paroxysm. The general physiognomy and demeanour of the poor creature inspire the beholder with pity rather than fear. The symptoms due to cerebral excitement are less marked than in the furious form of the disease; the agitation is not so considerable, and the restlessness, tendency to run away, and desire to bite are nearly absent; generally the animal is quite passive. Not unfrequently one or both eyes squint, and it is only when very much excited that the dog may contrive to close its mouth. Sometimes there is swelling about the pharynx and the neck; when the tongue shares in this complication it hangs out of the mouth. In certain cases there is a catarrhal condition of the membrane lining the nasal cavities, larynx, and bronchi; sometimes the animal testifies to the existence of abdominal pain, and the faeces are then soft or fluid. The other symptoms-such as the rapid exhaustion and emaciation, paralysis of the posterior limbs towards the termination of the disease, as well as the rapidity with which it runs its course-are the same as in the furious form.
The simultaneous occurrence of furious and dumb madness has frequently been observed in packs of fox-hounds. Dumb madness differs, then, from the furious type in the paralysis of the lower jaw, which hinders the dog from biting, save in very exceptional circumstances; the ferocious instincts are also in abeyance; and there is no tendency to aggression. It has been calculated that from 15 to $20 \%$ of rabid dogs have this particular form of the disease. Puppies and young dogs chiefly have furious rabies.

These are the symptoms of rabies in the dog; but it is not likely, nor is it necessary, that they will all be present in every case. In other species the symptoms differ more or less from those manifested by the dog, but they are generally marked by a change in the manner and habits of the creatures affected, with strong indications of nervous disturbance, in the majority of species amounting to ferociousness and a desire to injure, timid creatures becoming bold and aggressive.

In Human Beings.-The disease of hydrophobia has been known from early times, and is alluded to in the works of Aristotle, Xenophon, Plutarch, Virgil, Horace, Ovid and many others, as well as in those of the early writers on medicine. Celsus gives detailed instructions respecting the treatment of men who have been bitten by rabid dogs, and dwells on the dangers attending such wounds. After recommending suction of the bitten part by means of a dry cupping glass, and thereafter the application of the actual cautery or of strong caustics, and the employment of baths and various internal remedies, he says: "Idque cum ita per triduum factum est, tutus esse homo a periculo videtur. Solet autem ex eo vulnere, ubi parum occursum est, aquae timor nasci, ט́ठро甲оßí $\alpha \nu$ Graeci appellant. Miserrimum genus morbi; in quo simul aeger et siti et aquae metu cruciatur; quo oppressis in angusto spes est." Subsequently Galen described minutely the phenomena of hydrophobia, and recommended the excision of the wounded part as a protection against the disease. Throughout many succeeding centuries little or nothing was added to the facts which the early physicians had made known upon the subject. The malady was regarded with universal horror and dread, and the unfortunate sufferers were generally abandoned by all around them and left to their terrible fate. In later times the investigations of Boerhaave, Gerard van Swieten (1700-1772), John Hunter, François Magendie (1783-1855), Gilbert Breschet (1784-1845), Virchow, Albert Reder, as also of William Youatt (1776-1847), George Fleming, Meynell, Karl Hertwig (1798-1881), and others, have furnished important information; but all these were put into the shade by the researches of Pasteur.

The disease is communicated by the secretions of the mouth of the affected animal entering a wound or abrasion of the human skin or mucous membrane. In the great majority of cases ( $90 \%$ ) this is due to the bite of a rabid dog, but bites of rabid cats, wolves, foxes, jackals, \&c. are occasionally the means of conveying the disease. Numerous popular fallacies still prevail on the subject of hydrophobia. Thus it is supposed that the bite of an angry dog may produce the disease, and all the more if the animal should subsequently develop symptoms of rabies. The ground for this erroneous notion is the fact, which is unquestionable, that animals in whom rabies is in the stage of incubation, during which there are few if any symptoms, may by their bites convey the disease, though fortunately during this early stage they are little disposed to bite. The bite of a non-rabid animal, however enraged, cannot give rise to hydrophobia.

The period of incubation of the disease, or that time which elapses between the introduction of the virus and the development of the symptoms, appears to vary in a remarkable degree, being in some cases as short as a fortnight, and in others as long as several months or even years. On an average it seems to be from about six weeks to three months, but it mainly depends on the part bitten; bites on the head are the most dangerous. The incubation period is also said to be shorter in children. The rare instances of the appearance of hydrophobia many years after the introduction of the poison are always more or less open to question as to subsequent inoculation.

When the disease is about to declare itself it not unfrequently happens that the wound, which had quickly and entirely healed after the bite, begins to exhibit evidence of irritation or inflammatory action, or at least to be the seat of morbid sensations such as numbness, tingling or itching. The symptoms characterizing the premonitory stage are great mental depression and disquietude, together with restlessness and a kind of indefinite fear. There is an unusual tendency to talk, and the articulation is abrupt and rapid. Although in some instances the patients will not acknowledge that they have been previously bitten, and deny it with great obstinacy, yet generally they are well aware of the nature of their malady, and speak despairingly of its consequences. There is in this early stage a certain amount of constitutional disturbance showing itself by feverishness, loss of appetite, sleeplessness, headache, great nervous excitability, respiration of a peculiar sighing or sobbing character, and even occasionally a noticeable aversion to liquids. These symptoms-constituting what is termed the melancholic stage-continue in general for one or two days, when they are succeeded by the stage of excitement in which all the characteristic phenomena of the malady are fully developed. Sometimes the disease first shows itself in this stage, without antecedent symptoms.
The agitation of the sufferer now becomes greatly increased, and the countenance exhibits anxiety and terror. There is noticed a marked embarrassment of the breathing, but the most striking and terrible features of this stage are the effects produced by attempts to swallow fluids. The patient suffers from thirst and desires eagerly to drink, but on making the effort is seized with a most violent suffocative paroxysm produced by spasm of the muscles of swallowing and breathing, which continues for several seconds, and is succeeded by a feeling of intense alarm and distress. With great caution and determination the attempt is renewed, but only to be followed with a repetition of the seizure, until the unhappy sufferer ceases from sheer dread to try to quench the thirst which torments him. Indeed the very thought of doing so suffices to bring on a choking paroxysm, as does also the sound of the running of water. The patient is extremely sensitive to any kind of external impression; a bright light, a loud noise, a breath of cool air, contact with any one, are all apt to bring on one of these seizures. But besides these suffocative attacks there also occur general convulsions affecting the whole muscular system of the body, and occasionally a condition of tetanic spasm. These various paroxysms increase in frequency and severity with the advance of the disease, but alternate with intervals of comparative quiet, in which, however, there is intense anxiety and more or less constant difficulty of breathing, accompanied with a peculiar sonorous expiration, which has suggested the notion that the patient barks like a dog. In many instances there is great mental disturbance, with fits of maniacal excitement, in which he strikes at every one about him, and accuses them of being the cause of his sufferings-these attacks being succeeded by calm intervals in which he expresses great regret for his violent behaviour. During all this stage of the disease the patient is tormented with a viscid secretion accumulating in his mouth, which from dread of swallowing he is constantly spitting about him. There may also be noticed snapping movements of the jaws as if he were attempting to bite, but these are in reality a manifestation of the spasmodic action which affects the muscles generally. There is no great amount of fever, but there is constipation, diminished flow of urine, and often sexual excitement.

After two or three days of suffering of the most terrible description the patient succumbs, death taking place either in a paroxysm of choking, or on the other hand in a tranquil manner from exhaustion, all the symptoms having abated, and the power of swallowing returned before the end. The duration of the disease from the first declaration of the symptoms is generally from three to five days.

Apart from the inoculation method (see below), the treatment of most avail is that which is directed towards preventing the absorption of the poison into the system. This may be accomplished by excision of the part involved in the bite of the rabid animal, or, where this from its locality is impracticable, in the application to the wound of some chemical agent which will destroy the activity of the virus, such as potassa fusa, lunar caustic (nitrate of silver), or the actual cautery in the form of a red-hot wire. The part should be thoroughly acted on by these agents, no matter what amount of temporary suffering this may occasion. Such applications should be resorted to immediately after the bite has been inflicted, or as soon thereafter as possible. Further, even though many hours or days should elapse, these local remedies should still be applied; for if, as appears probable, some at least of the virus remains for long at the injured part, the removal or effectual destruction of this may prevent the dread consequences of its absorption. Every effort should be made to tranquillize and reassure the patient.

Two special points of interest have arisen in recent years in connexion with this disease. One is the Pasteur treatment by inoculation with rabic virus (see also Parasitic Diseases), and the other was the attempt of the government to exterminate rabies in the British Isles by muzzling dogs.

The Pasteur treatment was first applied to human beings in 1885 after prolonged investigation and experimental trial on animals. It is based on the fact that a virus, capable of giving rabies by inoculation, can be extracted from the tissues of a rabid

## Pasteur treatment.

 animal and then intensified or attenuated at pleasure. It appears that the strength of the rabic virus, as determined by inoculation, is constant in the same species of animal, but is modified by passing through another species. For instance, the natural virus of dogs is always of the same strength, but when inoculated into monkeys it becomes weakened, and the process of attenuation can be carried on by passing the virus through a succession of monkeys, until it loses the power of causing death. If this weakened virus is then passed back through guinea-pigs, dogs or rabbits, it regains its former strength. Again, if it be passed through a succession of dogs it becomes intensified up to a maximum of strength which is called the virus fixe. Pasteur further discovered that the strength can be modified by temperature and by keeping the dried tissues of a rabid animal containing the virus. Thus, if the spinal cord of a rabid dog be preserved in a dry state, the virus loses strength day by day. The system of treatment consists in making an emulsion of the cord and graduating the strength of the dose by using a succession of cords, which have been kept for a progressively diminishing length of time. Those which have been kept for fourteen days are used as a starting-point, yielding virus of a minimum strength. They are followed by preparations of diminishing age and increasing strength, day by day, up to the maximum, which is three days old. These are successively injected into the circulatory system. The principle is the artificial acquisition by the patient of resistance to the rabic virus, which is presumed to be already in the system but has not yet become active, by accustoming him gradually to its toxic effect, beginning with a weak form and progressively increasing the dose. It is not exactly treatment of the disease, because it is useless or nearly so when the disease has commenced, nor is it exactly preventive, for the patient has already been bitten. It must be regarded as a kind of anticipatory cure. The cords are cut into sections and preserved dry in sterilized flasks plugged with cotton-wool. Another method of preparing the inoculatory virus, which has been devised by Guido Tizzoni and Eugenio Centanni, consists in subjecting the virus fixe to peptic digestion by diluted gastric juice for varying periods of time.The first patient was treated by Pasteur's system in July 1885. He was successively inoculated with emulsions made from cords that had been kept fourteen and ten days, then eleven and eight days, then eight, seven, six days, and so on. Two forms of treatment are now used-(1) the "simple," in which the course from weak to strong virus is extended over nine days; (2) the "intensive," in which the maximum is reached in seven days. The latter is used in cases of very bad bites and those of some standing, in which it is desirable to lose no time. Two days are compressed into one at the commencement by making injections morning and evening instead of once a day, so that the fifth-day cord is reached in four days instead of six, as in the "simple" treatment. When the maximum-the third-day cord-is reached the injections are continued with fifth-, fourth-, and third-day cords. The whole course is fifteen days in the simple treatment and twenty-one in the intensive. The doses injected range from 1 to 3 cubic centimetres. Injections are made alternately into the right and left flanks. The following table shows the number treated from 1886 to 1905, with the mortality.

| Year. | Patients <br> Treated. | Deaths. | Mortality <br> per cent. |
| :---: | :---: | :---: | :---: |
| 1886 | 2671 | 25 | .94 |
| 1887 | 1770 | 14 | .79 |
| 1888 | 1622 | 9 | .55 |
| 1889 | 1830 | 7 | .38 |
| 1890 | 1540 | 5 | .32 |
| 1891 | 1559 | 4 | .25 |
| 1892 | 1790 | 4 | .22 |
| 1893 | 1648 | 6 | .36 |
| 1894 | 1387 | 7 | .50 |
| 1895 | 1520 | 5 | .33 |
| 1896 | 1308 | 4 | .30 |
| 1897 | 1521 | 6 | .39 |
| 1898 | 1465 | 3 | .20 |
| 1899 | 1614 | 4 | .25 |
| 1900 | 1419 | 10 | .70 |
| 1901 | 1318 | 5 | .37 |
| 1902 | 1105 | 2 | .18 |
| 1903 | 630 | 4 | .65 |
| 1904 | 757 | 5 | .66 |
| 1905 | 727 | 4 | .54 |

These figures do not include cases which develop hydrophobia during treatment or within fifteen days after treatment is completed, for it is held that persons who die within that period
have their nervous centres invaded by virus before the cure has time to act. The true mortality should therefore be considerably higher. For instance, in 1898 three deaths came within this category, which just doubles the mortality; and in 1899 the additional deaths were six, bringing the mortality up to two-and-a-half times that indicated in the table. When, however, the additional deaths are included the results remain sufficiently striking, if two assumptions are granted-(1) that all the persons treated have been bitten by rabid animals; (2) that a large proportion of persons so bitten usually have hydrophobia. Unfortunately, both these assumptions lack proof, and therefore the evidence of the efficacy of the treatment cannot be said to satisfy a strictly scientific standard. With regard to the first point, the patients are divided into three categories-(1) those bitten by an animal the rabidity of which is proved by the development of rabies in other animals bitten by it or inoculated from its spinal cord; (2) those bitten by an animal pronounced rabid on a veterinary examination; (3) those bitten by an animal suspected of being rabid. The number of patients in each category in 1898 was (1) 141, (2) 855 , (3) 469 ; and in 1899 it was (1) 152, (2) 1099, (3) 363 . As might be expected, the vast majority came under the second and third heads, in which the evidence of rabidity is doubtful or altogether lacking. With regard to the second point, the proportion of persons bitten by rabid animals who ordinarily develop hydrophobia has only been "estimated" from very inadequate data. Otto Bollinger from a series of collected statistics states that before the introduction of the Pasteur treatment, of patients bitten by dogs undoubtedly rabid $47 \%$ died, the rate being $33 \%$ in those whose wounds had been cauterized and $83 \%$ when there had been no local treatment. If the number of rabid dogs be compared with the deaths from hydrophobia in any year or series of years, it can hardly be very high. For instance, in 1895, 668 dogs, besides other animals, were killed and certified to be rabid in England, and the deaths from hydrophobia were twenty. Of course this proves nothing, as the number of persons bitten is not known, but the difference between the amount of rabies and of hydrophobia is suggestively great in view of the marked propensity of rabid dogs to bite, nor is it accounted for by the fact that some of the persons bitten were treated at the Institut Pasteur. A comparison of the annual mortality from hydrophobia in France before and after the introduction of the treatment would afford decisive evidence as to its efficacy; but unfortunately no such comparison can be made for lack of vital statistics in that country. The experience of the Paris hospitals, however, points to a decided diminution of mortality. On the whole it must be said, in the absence of further data, that the Pasteur treatment certainly diminishes the danger of hydrophobia from the bites of rabid animals.

More recently treatment with an anti-rabic serum has been suggested (see Parasitic Diseases). Victor Babes and Lepp and later Guido Tizzoni and Eugenio Centanni have worked out a method of serum treatment curative and protective. In this method not the rabic poison itself, as in the Pasteur treatment, but the protective substance formed is injected into the tissues. The serum of a vaccinated animal is capable of neutralizing the power of the virus of rabies not only when mixed with the virus before injection but even when injected simultaneously or within twenty-four hours after the introduction of the virus. These authors showed that the serum of a rabbit protects a rabbit better than does the serum of a dog, and vice versa. At the end of twenty days' injections they found they could obtain such a large quantity of anti-rabic substance in the serum of an animal, that even 1 part of serum to 25,000 of the body weight would protect an animal. This process differs from that of Pasteur in so far as that in place of promoting the formation of the antidote within the body of the patient, by a process of vaccination with progressively stronger and stronger virus, this part of the process is carried on in an animal, Babes using the dog and Centanni the sheep, the blood serum of which is injected. This method of vaccination is useful as a protective to those in charge of kennels.

The attempt to stamp out rabies in Great Britain was an experiment undertaken by the government in the public interest. The principal means adopted were the muzzling of dogs in infected areas, and prolonged quarantine for imported animals. The efficacy of

## Muzzling order in England.

 dog-muzzling in checking the spread of rabies and diminishing its prevalence has been repeatedly proved in various countries. Liable as other animals may be to the disease, in England at least the dog is pre-eminently the vehicle of contagion and the great source of danger to human beings. There is a difference of opinion on the way in which muzzling acts, though there can be none as to the effect it produces in reducing rabies. Probably it acts rather by securing the destruction of ownerless and stray-which generally includes rabid—dogs than by preventing biting; for though it may prevent snapping, even the wire-cage muzzle does not prevent furious dogs from biting, and it is healthy, not rabid, dogs that wear the muzzle. It has therefore been suggested that a collar would have the same effect, if all collarless dogs were seized; but the evidence goes to show that it has not, perhaps because rabid dogs are more likely to stray from home with their collars, which are constantly worn, than with muzzles which are not, and so escape seizure. Moreover, it is much easier for the police to see whether a dog is wearing a muzzle or not than it is to make sure about the collar. However this may be, the muzzle has proved moreefficacious, but it was not applied systematically in England until a late date. Sometimes the regulations were in the hands of the government, and sometimes they were left to local authorities; in either case they were allowed to lapse as soon as rabies had died down. In April 1897 the Board of Agriculture entered on a systematic attempt to exterminate rabies by the means indicated. The plan was to enforce muzzling over large areas in which the disease existed, and to maintain it for six months after the occurrence of the last case. In spite of much opposition and criticism, this was resolutely carried out under Mr Walter Long, the responsible minister, and met with great success. By the spring of 1899-that is, in two years-the disease had disappeared in Great Britain, except for one area in Wales; and, with this exception, muzzling was everywhere relaxed in October 1899. It was taken off in Wales also in the following May, no case having occurred since November 1899. Rabies was then pronounced extinct. During the summer of 1900, however, it reappeared in Wales, and several counties were again placed under the order. The year 1901 was the third in succession in which no death from hydrophobia was registered in the United Kingdom. In the ten years preceding 1899, 104 deaths were registered, the death-rate reaching 30 in 1889 and averaging 29 annually. In 1902 two deaths from hydrophobia were registered. From that date to June 1909 (the latest available for the purpose of this article) no death from hydrophobia was notified in the United Kingdom.

See Annales de l'Institut Pasteur, from 1886; Journal of the Board of Agriculture, 1899; Makins, "Hydrophobia," in Treves's System of Surgery; Woodhead, "Rabies," in Allbutt's System of Medicine.

HYDROSPHERE (Gr. Ǔ $\delta \omega \rho$, water, and $\sigma \varphi \alpha i ̃ \rho \alpha$, sphere), in physical geography, a name given to the whole mass of the water of the oceans, which fills the depressions in the earth's crust, and covers nearly three-quarters of its surface. The name is used in distinction from the atmosphere, the earth's envelope of air, the lithosphere (Gr. $\lambda$ í $\theta$ os, rock) or solid crust of the earth, and the centrosphere or interior mass within the crust. To these "spheres" some writers add, by figurative usage, the terms "biosphere," or life-sphere, to cover all living things, both animals and plants, and "psychosphere," or mind-sphere, covering all the products of human intelligence.

HYDROSTATICS (Gr. úठw , water, and the root $\sigma \tau \alpha$-, to cause to stand), the branch of hydromechanics which discusses the equilibrium of fluids (see Hydromechanics).

HYDROXYLAMINE, $\mathrm{NH}_{2} \mathrm{OH}$, or hydroxy-ammonia, a compound prepared in 1865 by W. C. Lossen by the reduction of ethyl nitrate with tin and hydrochloric acid. In 1870 E. Ludwig and T. H. Hein (Chem. Centralblatt, 1870, 1, p. 340) obtained it by passing nitric oxide through a series of bottles containing tin and hydrochloric acid, to which a small quantity of platinum tetrachloride has been added; the acid liquid is poured off when the operation is completed, and sulphuretted hydrogen is passed in; the tin sulphide is filtered off and the filtrate evaporated. The residue is extracted by absolute alcohol, which dissolves the hydroxylamine hydrochloride and a little ammonium chloride; this last substance is removed as ammonium platino-chloride, and the residual hydroxylamine hydrochloride is recrystallized. E. Divers obtains it by mixing cold saturated solutions containing one molecular proportion of sodium nitrate, and two molecular proportions of acid sodium sulphite, and then adding a saturated solution of potassium chloride to the mixture. After standing for twenty-four hours, hydroxylamine potassium disulphonate crystallizes out. This is boiled for some hours with water and the solution cooled, when potassium sulphate separates first, and then hydroxylamine sulphate. E. Tafel (Zeit. anorg. Chem., 1902, 31, p. 289) patented an electrolytic process, wherein $50 \%$ sulphuric acid is treated in a divided cell provided with a cathode of amalgamated lead, $50 \%$ nitric acid being gradually run into the cathode compartment. Pure
anhydrous hydroxylamine has been obtained by C. A. Lobry de Bruyn from the hydrochloride, by dissolving it in absolute methyl alcohol and then adding sodium methylate. The precipitated sodium chloride is filtered, and the solution of hydroxylamine distilled in order to remove methyl alcohol, and finally fractionated under reduced pressure. The free base is a colourless, odourless, crystalline solid, melting at about $30^{\circ} \mathrm{C}$., and boiling at $58^{\circ} \mathrm{C}$. (under a pressure of 22 mm .). It deliquesces and oxidizes on exposure, inflames in dry chlorine and is reduced to ammonia by zinc dust. Its aqueous solution is strongly alkaline, and with acids it forms welldefined stable salts. E. Ebler and E. Schott (J. pr. Chem., 1908, 78, p. 289) regard it as acting with the formula $\mathrm{NH}_{2} \cdot \mathrm{OH}$ towards bases, and as $\mathrm{NH}_{3}: \mathrm{O}$ towards acids, the salts in the latter case being of the oxonium type. It is a strong reducing agent, giving a precipitate of cuprous oxide from alkaline copper solutions at ordinary temperature, converting mercuric chloride to mercurous chloride, and precipitating metallic silver from solutions of silver salts. With aldehydes and ketones it forms oximes (q.v.). W. R. Dunstan (Jour. Chem. Soc., 1899, 75, p. 792) found that the addition of methyl iodide to a methyl alcohol solution of hydroxylamine resulted in the formation of trimethyloxamine, $\mathrm{N}\left(\mathrm{CH}_{3}\right)_{3} \mathrm{O}$.

Many substituted hydroxylamines are known, substitution taking place either in the $\alpha$ or $\beta$ position $\left(\mathrm{NH}_{2} \cdot \mathrm{OH}\right)$. $\beta$-phenylhydroxyl-amine, $\mathrm{C}_{6} \mathrm{H}_{5} \mathrm{NH} \cdot \mathrm{OH} \cdot$, is obtained in the reduction of nitrobenzene in neutral solution (e.g. by the action of the aluminium-mercury couple and water), but better, according to C. Goldschmidt (Ber., 1896, 29, p. 2307) by dissolving nitrobenzene in ten times its weight of ether containing a few cubic centimetres of water, and heating with excess of zinc dust and anhydrous calcium chloride for three hours on a water bath. It also appears as an intermediate product in the electrolytic reduction of nitrobenzene in sulphuric acid solution. By gentle oxidation it yields nitrosobenzene. Derivatives of the type $\mathrm{R}_{2} \mathrm{~N} \cdot \mathrm{OH}$ result in the action of the Grignard reagent on amyl nitrite. Dihydroxy-ammonia or nitroxyl, $\mathrm{NH}(\mathrm{OH})_{2}$, a very unstable and highly reactive substance, has been especially studied by A. Angeli (see A. W. Stewart, Recent Advances in Physical and Inorganic Chemistry, 1909).

HYDROZOA, one of the most widely spread and prolific groups of aquatic animals. They are for the most part marine in habitat, but a familiar fresh-water form is the common Hydra of ponds and ditches, which gives origin to the name of the class. The Hydrozoa comprise the hydroids, so abundant on all shores, most of which resemble vegetable organisms to the unassisted eye; the hydrocorallines, which, as their name implies, have a massive stony skeleton and resemble corals; the jelly-fishes so called; and the Siphonophora, of which the species best known by repute is the so-called "Portuguese man-of-war" (Physalia), dreaded by sailors on account of its terrible stinging powers.

In external form and appearance the Hydrozoa exhibit such striking differences that there would seem at first sight to be little in common between the more divergent members of the group. Nevertheless there is no other class in the animal kingdom with better marked characteristics, or with more uniform morphological peculiarities underlying the utmost diversity of superficial characters.

All Hydrozoa, in the first place, exhibit the three structural features distinctive of the Coelentera (q.v.). (1) The body is built up of two layers only, an external protective and sensory layer, the ectoderm, and an internal digestive layer, the endoderm. (2) The body contains but a single internal cavity, the coelenteron or gastrovascular space, which may be greatly ramified, but is not shut off into cavities distinct from the central digestive space. (3) The generative cells are produced in either the ectoderm or endoderm, and not in a third layer arising in the embryo, distinct from the two primary layers; in other words, there is no mesoderm or coelom.

To these three characters the Hydrozoa add a fourth which is distinctive of the subdivision of the Coelenterata termed the Cnidaria; that is to say, they always possess peculiar stinging organs known as nettle-cells, or nematocysts (Cnidae), each produced in a cell forming an integral part of the animal's tissues. The Hydrozoa are thus shown to belong to the group of Coelenterata Cnidaria, and it remains to consider more fully their distinctive features, and in particular those which mark them off from the other main division of the Cnidaria, the Anthozoa (q.v.), comprising the corals and sea-anemones.

The great diversity, to which reference has already been made, in the form and structure of the Hydrozoa is due to two principal causes. In the first place, we find in this group two distinct types of person or individual, the polyp and the medusa ( $q q . v$. .), each capable of a wide range of variations; and when both polyp and medusa occur in the life-cycle of the same species, as is frequently the case, the result is an alternation of generations of a type peculiarly
characteristic of the class. In the second place, the power of non-sexual reproduction by budding is practically of universal occurrence among the Hydrozoa, and by the buds failing to separate from the parent stock, colonies are produced, more or less complicated in structure and often of great size. We find that polyps may either bud other polyps or may produce medusae, and that medusae may bud medusae, though never, apparently, polyps. Hence we have a primary subdivision of the colonies of Hydrozoa into those produced by budding of polyps and those produced by budding of medusae. The former may contain polyp-persons and medusa-persons, either one kind alone or both kinds combined; the latter will contain only medusa-persons variously modified.

The morphology of the Hydrozoa reduces itself, therefore, to a consideration of the morphology of the polyp, of the medusa and of the colony. Putting aside the last-named, for a detailed account of which see Hydromedusae, we can best deal with the peculiarities of the polyp and medusa from a developmental point of view.

In the development of the Hydrozoa, and indeed of the Cnidaria generally, the egg usually gives rise to an oval larva which swims about by means of a coating of cilia on the surface of the body. This very characteristic larva is termed a planula, but though very uniform externally, the planulae of different species, or of the same species at different periods, do not always represent the same stage of embryonic development internally. On examining more minutely the course of the development, it is found that the ovum goes through the usual process of cleavage, always total and regular in this group, and so gives rise to a hollow sphere or ovoid with the wall composed of a single layer of cells, and containing a spacious cavity, the blastocoele or segmentation-cavity. This is the blastula stage occurring universally in all Metazoa, probably representing an ancestral Protozoan colony in phylogeny. Next the blastula gives rise to an internal mass of cells (fig. 1, hy) which come from the wall either by immigration (fig. 1, A) or by splitting off (delamination). The formation of an inner cell-mass converts the single-layered blastula (monoblastula) into a double-layered embryo (diblastula) which may be termed a parenchymula, since at first the inner cell-mass forms an irregular parenchyma which may entirely fill up and obliterate the segmentation cavity (fig. 1, B). At a later stage, however, the cells of the inner mass arrange themselves in a definite layer surrounding an internal cavity (fig. 1, C, al), which soon acquires an opening to the exterior at one pole, and so forms the characteristic embryonic stage of all Enterozoa known as the gastrula (fig. 2). In this stage the body is composed of two layers, ectoderm (d) externally, and endoderm ( $c$ ) internally, surrounding a central cavity, the archenteron ( $b$ ), which communicates with the exterior by a pore (a), the blastopore.


From Balfour, after Kowalewsky.
Fig. 1.-Formation of the Diblastula of Eucope (one of the Calyptoblastic Hydromedusae) by immigration. A, B, C, three successive stages. ep, Ectoderm; hy, endoderm; al, enteric cavity.

Thus a planula larva may be a blastula, or but slightly advanced beyond this stage, or it may be (and most usually is) a parenchymula; or in some cases (Scyphomedusae) it may be a gastrula. It should be added that the process of development, the gastrulation as it is termed, may be shortened by the immigration of cells taking place at one pole only, and in a connected layer with orderly arrangement, so that the gastrula stage is reached at once from the blastula without any intervening parenchymula stage. This is a process of gastrulation by invagination which is found in all animals above the Coelenterata, but which is very rare in the Cnidaria, and is known only in the Scyphomedusae amongst the Hydrozoa.


From Gegenbaur's

After the gastrula stage, which is found as a developmental stage in all Enterozoa, the embryo of the Hydrozoa proceeds to develop characters which are peculiar to the Coelenterata only. Round the blastopore hollow outgrowths, variable in number, arise by the evagination of the entire body-wall, both ectoderm and endoderm. Each outgrowth contains a prolongation of the archenteric cavity (compare figs. 2 and 3, A). In this way is formed a ring of tentacles, the most characteristic organs of the Cnidaria. They surround a region which is termed the peristome, and which contains in the centre the blastopore, which becomes the adult mouth. The archenteron becomes the gastrovascular system or coelenteron. Between the ectoderm and endoderm a gelatinous supporting layer, termed the mesogloea, makes its appearance. The gastrula has now become an actinula, which may be termed the distinctive larva of the Cnidaria, and doubtless represents in a transitory manner the common ancestor of the group. In no case known, however, does the actinula become the adult, sexually mature individual, but always undergoes further modifications, whereby it develops into either a polyp or a medusa.

To become a polyp, the actinula (fig. 3, A) becomes attached to some firm object by the pole farthest from the mouth, and its growth preponderates in the direction of the principal axis, that is to say, the axis passing through the mouth (fig. 3, $a-b$ ). As a result the body becomes columnar in form (fig. 3, B), and without further change passes into the characteristic polypform (see Polyp).


Fig. 3.-Diagram showing the change of the Actinula (A) into a Polyp (B); a-b, principal (vertical) axis; $c$ - $d$, horizontal axis. The endoderm is shaded, the ectoderm is left clear.


Fig. 4.-Diagram showing the change of the Actinula into a Medusa. A, Vertical section of the actinula; $a-b$ and $c-d$ as in fig. 3, B, transitional stage, showing preponderating growth in the horizontal plane. C, C', D, D', two types of medusa organization; C and D are composite sections, showing a radius ( R ) on one side, an interradius (IR) on the other; $\mathrm{C}^{\prime}$ and $\mathrm{D}^{\prime}$ are plans; the mouth and manubrium are indicated at the centre, leading into the gastral cavity subdivided by the four areas of concrescence in each interradius (IR). $t$, tentacle; $g$.p, gastric pouch; r.c, radial canal not present in C and $\mathrm{C}^{\prime}$; c.c, circular or ring-canal; e.l, endoderm-lamella formed by concrescence. For a more detailed diagram of medusa-structure see article Medusa.

It is convenient to distinguish two types of polyp by the names hydro polyp and anthopolyp, characteristic of the Hydrozoa and Anthozoa respectively. In the hydropolyp the body is typically elongated, the height of the column being far greater than the diameter. The peristome is relatively small and the mouth is generally raised on a projecting spout or hypostome. The ectoderm loses entirely the ciliation which it had in the planula and actinula stages and commonly secretes on its external surface a protective or supporting investment, the perisarc. Contrasting with this, the anthopolyp is generally of squat form, the diameter often exceeding the height; the peristome is wide, a hypostome is lacking, and the ectoderm, or so much of it as is exposed, i.e. not covered by secretion of skeletal or other investment, retains its ciliation throughout life. The internal structural differences are even more characteristic. In the hydropolyp the blastopore of the embryo forms the adult mouth situated at the extremity of the hypostome, and the ectoderm and endoderm meet at this point. In the anthopolyp the blastopore is carried inwards by an in-pushing of the body-wall of the region of the peristome, so that the adult mouth is an opening leading into a short ectodermal oesophagus or stomodaeum, at the bottom of which is the blastopore. Further, in the hydropolyp the digestive cavity either remains simple and undivided and circular in transverse section, or may show ridges projecting internally, which in this case are formed of endoderm alone, without any participation of the mesogloea. In the anthopolyp, on the other hand, the digestive cavity is always subdivided by so-called mesenteries, in-growths of the endoderm containing vertical lamellae of mesogloea (see Anthozoa). In short, the hydropolyp is characterized by a more simple type of organization than the anthopolyp, and is in most respects less modified from the
actinula type of structure.
Returning now to the actinula, this form may, as already stated, develop into a medusa, a type of individual found only in the Hydrozoa, as here understood. To become a medusa, the actinula grows scarcely at all in the direction of the principal axis, but greatly along a plane at right angles to it. Thus the body becomes umbrella-shaped, the concave side representing the peristome, and the convex side the column, of the polyp. Hence the tentacles are found at the edge of the umbrella, and the hypostome forms usually a projecting tube, with the mouth at the extremity, forming the manubrium or handle of the umbrella. The medusa has a pronounced radial symmetry, and the positions of the primary tentacles, usually four in number, mark out the so-called radii, alternating with which are four interradii. The ectoderm retains its ciliation only in the sensory organs. The mesogloea becomes enormously increased in quantity (hence the popular name "jelly-fish"), and in correlation with this the endoderm-layer lining the coelenteron becomes pressed together in the interradial areas and undergoes concrescence, forming a more or less complicated gastrovascular system (see MEdusA). It is sufficient to state here that the medusa is usually a free-swimming animal, floating mouth downwards on the open seas, but in some cases it may be attached by its aboral pole, like a polyp, to some firm basis, either temporarily or permanently.

Thus the development of the two types of individual seen in the Hydrozoa may be summarized as follows:-


This development, though probably representing the primitive sequence of events, is never actually found in its full extent, but is always abbreviated by omission or elimination of one or more of the stages. We have already seen that the parenchymula stage is passed over when the gastrulation is of the invaginate type. On the other hand, the parenchymula may develop directly into the actinula or even into the polyp, with suppression of the intervening steps. Great apparent differences may also be brought about by variations in the period at which the embryo is set free as a larva, and since two free-swimming stages, planula and actinula, are unnecessary, one or other of them is always suppressed. A good example of this is seen in two common genera of British hydroids, Cordylophora and Tabularia. In Cordylophora the embryo is set free at the parenchymula stage as a planula which fixes itself and develops into a polyp, both gastrula and actinula stages being suppressed. In Tubularia, on the other hand, the parenchymula develops into an actinula within the maternal tissues, and is then set free, creeps about for a time, and after fixing itself, changes into a polyp; hence in this case the planulastage, as a free larva, is entirely suppressed.

The Hydrozoa may be defined, therefore, as Cnidaria in which two types of individual, the polyp and the medusa, may be present, each type developed along divergent lines from the primitive actinula form. The polyp (hydropolyp) is of simple structure and never has an ectodernal oesophagus or mesenteries. ${ }^{1}$ The general ectoderm loses its cilia, which persist only in the sensory cells, and it frequently secretes external protective or supporting structures. An internal mesogloeal skeleton is not found.

The class is divisible into two main divisions or sub-classes, Hydromedusae and Scyphomedusae, of which definitions and detailed systematic accounts will be found under these headings.

General Works on Hydrozoa.-C. Chun, "Coelenterata (Hohlthiere)," Bronn's Klassen und Ordnungen des Thier-Reichs ii. 2 (1889 et seq.); Y. Delage, and E. Hérouard, Traité de zoologie concrète, ii. part 2, Les Coelentérés (1901); G. H. Fowler, "The Hydromedusae and Scyphomedusae" in E. R. Lankester's Treatise on Zoology, ii. chapters iv. and v. (1900); S. J. Hickson, "Coelenterata and Ctenophora," Cambridge Natural History, i. chapters x.-xv. (1906).

HYENA, a name applicable to all the representatives of the mammalian family Hyaenidae, a group of Carnivora (q.v.) allied to the civets. From all other large Carnivora except the African hunting-dog, hyenas are distinguished by having only four toes on each foot, and are further characterized by the length of the fore-legs as compared with the hind pair, the non-retractile claws, and the enormous strength of the jaws and teeth, which enables them to break the hardest bones and to retain what they have seized with unrelaxing grip.


Fig. 1.-The Striped Hyena (Hyaena striata).


Fig. 2.-The Spotted Hyena (Hyaena crocuta).

The striped hyena (Hyaena striata) is the most widely distributed species, being found throughout India, Persia, Asia Minor, and North and East Africa, the East African form constituting a distinct race, H. striata schillingsi; while there are also several distinct Asiatic races. The species resembles a wolf in size, and is greyish-brown In colour, marked with indistinct longitudinal stripes of a darker hue, while the legs are transversely striped. The hairs on the body are long, especially on the ridge of the neck and back, where they form a distinct mane, which is continued along the tail. Nocturnal in habits, it prefers by day the gloom of caves and ruins, or of the burrows which it occasionally forms, and issues forth at sunset, when it commences its unearthly howling. When the animal is excited, the howl changes into what has been compared to demoniac laughter, whence the name of "laughing-hyena." These creatures feed chiefly on carrion, and thus perform useful service by devouring remains which might otherwise pollute the air. Even human dead are not safe from their attacks, their powerful claws enabling them to gain access to newly interred bodies in cemeteries. Occasionally (writes Dr W. T. Blanford) sheep or goats, and more often dogs, are carried off, and the latter, at all events, are often taken alive to the animal's den. This species appears to be solitary in habits, and it is rare to meet with more than two together. The cowardice of this hyena is proverbial; despite its powerful teeth, it rarely attempts to defend itself. A very different animal is the spotted hyena, Hyaena (Crocuta) crocuta, which has the sectorial teeth of a more cat-like type, and is marked by dark-brown spots on a yellowish ground, while the mane is much less distinct. At the Cape it was formerly common, and occasionally committed
great havoc among the cattle, while it did not hesitate to enter the Kaffir dwellings at night and carry off children sleeping by their mothers. By persistent trapping and shooting, its numbers have now been considerably reduced, with the result, however, of making it exceedingly wary, so that it is not readily caught in any trap with which it has had an opportunity of becoming acquainted. Its range extends from Abyssinia to the Cape. The Abyssinian form has been regarded as a distinct species, under the name of $H$. liontiewi, but this, like various more southern forms, is but regarded as a local race. The brown hyena (H. brunnea) is South African, ranging to Angola on the west and Kilimanjaro on the east. In size it resembles the striped hyena, but differs in appearance, owing to the fringe of long hair covering the neck and fore part of the back. The general hue is ashy-brown, with the hair lighter on the neck (forming a collar), chest and belly; while the legs are banded with dark brown. This species is not often seen, as it remains concealed during the day. Those frequenting the coast feed on dead fish, crabs and an occasional stranded whale, though they are also a danger to the sheep and cattle kraal. Strand-wolf is the local name at the Cape.

Although hyenas are now confined to the warmer regions of the Old World, fossil remains show that they had a more northerly range during Tertiary times; the European cave-hyena being a form of the spotted species, known as $H$. crocuta spelaea. Fossil hyenas occur in the Lower Pliocene of Greece, China, India, \&c.; while remains indistinguishable from those of the striped species have been found in the Upper Pliocene of England and Italy.

HYÈRES, a town in the department of the Var in S.E. France, 11 m . by rail E. of Toulon. In 1906 the population of the commune was 17,790 , of the town 10,464 ; the population of the former was more than doubled in the last decade of the 19 th century. Hyères is celebrated (as is also its fashionable suburb, Costebelle, nearer the seashore) as a winter health resort. The town proper is situated about $21 / 2 \mathrm{~m}$. from the seashore, and on the south-western slope of a steep hill ( 669 ft ., belonging to the Maurettes chain, 961 ft .), which is one of the westernmost spurs of the thickly wooded Montagnes des Maures. It is sheltered from the north-east and east winds, but is exposed to the cold north-west wind or mistral. Towards the south and south-east a fertile plain, once famous for its orange groves, but now mainly covered by vineyards and farms, stretches to the sea, while to the south-west, across a narrow valley, rises a cluster of low hills, on which is the suburb of Costebelle. The older portion of the town is still surrounded, on the north and east, by its ancient, though dilapidated medieval walls, and is a labyrinth of steep and dirty streets. The more modern quarter which has grown up at the southern foot of the hill has handsome broad boulevards and villas, many of them with beautiful gardens, filled with semi-tropical plants. Among the objects of interest in the old town are: the house (Rue Rabaton, 7) where J. B. Massillon (1663-1742), the famous pulpit orator, was born; the parish church of St Louis, built originally in the 13th century by the Cordelier or Franciscan friars, but completely restored in the earlier part of the 19th century; and the site of the old château, on the summit of the hill, now occupied by a villa. The plain between the new town and the sea is occupied by large nurseries, an excellent jardin d'acclimatation, and many market gardens, which supply Paris and London with early fruits and vegetables, especially artichokes, as well as with roses in winter. There are extensive salt beds (salines) both on the peninsula of Giens, S. of the town, and also E. of the town. To the east of the Giens peninsula is the fine natural harbour of Hyères, as well as three thinly populated islands (the Stoechades of the ancients), Porquerolles, Port Cros and Le Levant, which are grouped together under the common name of Îles d'Hyères.

The town of Hyères seems to have been founded in the 10th century, as a place of defence against pirates, and takes its name from the aires (hierbo in the Provençal dialect), or threshing-floors for corn, which then occupied its site. It passed from the possession of the viscounts of Marseilles to Charles of Anjou, count of Provence, and brother of St Louis (the latter landed here in 1254, on his return from Egypt). The château was dismantled by Henri IV., but thanks to its walls, the town resisted in 1707 an attack made by the duke of Savoy.

See Ch. Lenthéric, La Provence Maritime ancienne et moderne (chap. 5) (Paris, 1880).
(W. A. B. C.)

HYGIEIA, in Greek mythology, the goddess of health. It seems probable that she was originally an abstraction, subsequently personified, rather than an independent divinity of very ancient date. The question of the original home of her worship has been much discussed. The oldest traces of it, so far as is known at present, are to be found at Titane in the territory of Sicyon, where she was worshipped together with Asclepius, to whom she appears completely assimilated, not an independent personality. Her cult was not introduced at Epidaurus till a late date, and therefore, when in 420 b.c. the worship of Asclepius was introduced at Athens coupled with that of Hygieia, it is not to be inferred that she accompanied him from Epidaurus, or that she is a Peloponnesian importation at all. It is most probable that she was invented at the time of the introduction of Asclepius, after the sufferings caused by the plague had directed special attention to sanitary matters. The already existing worship of Athena Hygieia had nothing to do with Hygieia the goddess of health, but merely denoted the recognition of the power of healing as one of the attributes of Athena, which gradually became crystallized into a concrete personality. At first no special relationship existed between Asclepius and Hygieia, but gradually she came to be regarded as his daughter, the place of his wife being already secured by Epione. Later Orphic hymns, however, and Herodas iv. 1-9, make her the wife of Asclepius. The cult of Hygieia then spread concurrently with that of Asclepius, and was introduced at Rome from Epidaurus in 293, by which time she may have been admitted (which was not the case before) into the Epidaurian family of the god. Her proper name as a Romanized Greek importation was Valetudo, but she was gradually identified with Salus, an older genuine Italian divinity, to whom a temple had already been erected in 302. While in classical times Asclepius and Hygieia are simply the god and goddess of health, in the declining years of paganism they are protecting divinities generally, who preserve mankind not only from sickness but from all dangers on land and sea. In works of art Hygieia is represented, together with Asclepius, as a maiden of benevolent appearance, wearing the chiton and giving food or drink to a serpent out of a dish.

See the article by H. Lechat in Daremberg and Saglio's Dictionnaire des antiquités, with full references to authorities; and E. Thrämer in Roscher's Lexikon der Mythologie, with a special section on the modern theories of Hygieia.

HYGIENE (Fr. hygiène, from Gr. ט́үı $\alpha$ véıv, to be healthy), the science of preserving health, its practical aim being to render "growth more perfect, decay less rapid, life more vigorous, death more remote." The subject is thus a very wide one, embracing all the agencies which affect the physical and mental well-being of man, and it requires acquaintance with such diverse sciences as physics, chemistry, geology, engineering, architecture, meteorology, epidemiology, bacteriology and statistics. On the personal or individual side it involves consideration of the character and quality of food and of water and other beverages; of clothing; of work, exercise and sleep; of personal cleanliness, of special habits, such as the use of tobacco, narcotics, \&c.; and of control of sexual and other passions. In its more general and public aspects it must take cognizance of meteorological conditions, roughly included under the term climate; of the site or soil on which dwellings are placed; of the character, materials and arrangement of dwellings, whether regarded individually or in relation to other houses among which they stand; of their heating and ventilation; of the removal of excreta and other effete matters; of medical knowledge relating to the incidence and prevention of disease; and of the disposal of the dead.

These topics will be found treated in such articles as Dietetics, Food, Food-Preservation, Adulteration, Water, Heating, Ventilation, Sewerage, Bacteriology, Housing, Cremation, \&c. For legal enactments which concern the sanitary well-being of the community, see Public Health.

HYGINUS, eighth pope. It was during his pontificate (c. 137-140) that the gnostic heresies began to manifest themselves at Rome.

HYGINUS (surnamed Gromaticus, from gruma, a surveyor's measuring-rod), Latin writer on land-surveying, flourished in the reign of Trajan (A.D. 98-117). Fragments of a work on legal boundaries attributed to him will be found in C. F. Lachmann, Gromatici Veteres, i. (1848).

A treatise on Castrametation (De Munitionibus Castrorum), also attributed to him, is probably of later date, about the 3rd century a.d. (ed. W. Gemoll, 1879; A. von Domaszewski, 1887).

HYGINUS, GAIUS JULIUS, Latin author, a native of Spain (or Alexandria), was a pupil of the famous Cornelius Alexander Polyhistor and a freedman of Augustus, by whom he was made superintendent of the Palatine library (Suetonius, De Grammaticis, 20). He is said to have fallen into great poverty in his old age, and to have been supported by the historian Clodius Licinus. He was a voluminous author, and his works included topographical and biographical treatises, commentaries on Helvius Cinna and the poems of Virgil, and disquisitions on agriculture and bee-keeping. All these are lost.

Under the name of Hyginus two school treatises on mythology are extant: (1) Fabularum Liber, some 300 mythological legends and celestial genealogies, valuable for the use made by the author of the works of Greek tragedians now lost; (2) De Astronomia, usually called Poetica Astronomica, containing an elementary treatise on astronomy and the myths connected with the stars, chiefly based on the K $\alpha \tau \alpha \sigma \tau \varepsilon \rho เ \sigma \mu$ oí of Eratosthenes. Both are abridgments and both are by the same hand; but the style and Latinity and the elementary mistakes (especially in the rendering of the Greek originals) are held to prove that they cannot have been the work of so distinguished a scholar as C. Julius Hyginus. It is suggested that these treatises are an abridgment (made in the latter half of the 2nd century) of the Genealogiae of Hyginus by an unknown grammarian, who added a complete treatise on mythology.

Editions.-Fabulae, by M. Schmidt (1872); De Astronomia, by B. Bunte (1875); see also Bunte, De C. Julii Hygini, Augusti Liberti, Vita et Scriptis (1846).
 absolute or relative amount of moisture in the atmosphere; an instrument which only qualitatively determines changes in the humidity is termed a "hygroscope." The earlier instruments generally depended for their action on the contraction or extension of substances when exposed to varying degrees of moisture; catgut, hair, twisted cords and wooden laths, all of which contract with an increase in the humidity and vice versa, being the most favoured materials. The familiar "weather house" exemplifies this property. This toy consists of a house provided with two doors, through which either a man or woman appears according as the weather is about to be wet or fine. This action is effected by fixing a catgut thread to the base on which the figures are mounted, in such a manner that contraction of the thread rotates the figures so that the man appears and extension so that the woman appears.

Many of the early forms are described in C. Hutton, Math. and Phil. Dictionary (1815). The modern instruments, which utilize other principles, are described in Meteorology: II. Methods and Apparatus.

HYKSOS, or "Shepherd Kings," the name of the earliest invaders of Egypt of whom we have definite evidence in tradition. Josephus (c. Apion. i. 14), who identifies the Hyksos with the Israelites, preserves a passage from the second book of Manetho giving an account of them. (It may be that Josephus had it, not direct from Manetho's writings, but through the garbled version of some Alexandrine compiler.) In outline it is as follows. In the days of a king of Egypt named Timaeus the land was suddenly invaded from the east by men of ignoble race, who conquered it without a struggle, destroyed cities and temples, and slew or enslaved the inhabitants. At length they elected a king named Salatis, who, residing at Memphis, made all Egypt tributary, and established garrisons in different parts, especially eastwards, fearing the

Assyrians. He built also a great fortress at Avaris, in the Sethroite nome, east of the Bubastite branch of the Nile. Salatis was followed in succession by Beon, Apachnas, Apophis, Jannas and Asses. These six kings reigned 198 years and 10 months, and all aimed at extirpating the Egyptians. Their whole race was named Hyksos, i.e. "shepherd kings," and some say they were Arabs (another explanation found by Josephus is "captive shepherds"). When they and their successors had held Egypt for 511 years, the kings of the Thebais and other parts of Egypt rebelled, and a long and mighty war began. Misphragmuthosis worsted the "Shepherds" and shut them up in Avaris; and his son Thutmosis, failing to capture the stronghold, allowed them to depart; whereupon they went forth, 240,000 in number, established themselves in Judea and built Jerusalem.

In Manetho's list of kings, the six above named (with many variations in detail) form the XVth dynasty, and are called "six foreign Phoenician kings." The XVIth dynasty is of thirty-two "Hellenic (sic?) shepherd kings," the seventeenth is of "shepherds and Theban kings" (reigning simultaneously). The lists vary greatly in different versions, but the above seems the most reasonable selection of readings to be made. For "Hellenic" see below. The supposed connexion with the Israelites has made the problem of the Hyksos attractive, but light is coming upon it very slowly. In 1847 E. de Rougé proved from a fragment of a story in the papyri of the British Museum, that Apopi was one of the latest of the Hyksos kings, corresponding to Aphobis; he was king of the "pest" and suppressed the worship of the Egyptian gods, and endeavoured to make the Egyptians worship his god Setekh or Seti; at the same time an Egyptian named Seqenenrē reigned in Thebes, more or less subject to Aphobis. The city of Hawari (Avaris) was also mentioned in the fragment.

In 1850 a record of the capture of this city from the Hyksos by Ahmosi, the founder of the eighteenth dynasty, was discovered by the same scholar. A large class of monuments was afterwards attributed to the Hyksos, probably in error. Some statues and sphinxes, found in 1861 by Mariette at Tanis (in the north-east of the Delta), which had been usurped by later kings, had peculiar "un-Egyptian" features. One of these bore the name of Apopi engraved lightly on the shoulder; this was evidently a usurper's mark, but from the whole circumstances it was concluded that these, and others of the same type of features found elsewhere, must have belonged to the Hyksos. This view held the field until 1893, when Golénischeff produced an inferior example bearing its original name, which showed that in this case it represented Amenemhe III. In consequence it is now generally believed that they all belong to the twelfth dynasty. Meanwhile a headless statue of a king named Khyan, found at Bubastis, was attributed on various grounds to the Hyksos, the soundest arguments being his foreign name and the boastful un-Egyptian epithet "beloved of his ka," where "beloved of Ptah" or some other god was to be expected. His name was immediately afterwards recognized on a lion found as far away from Egypt as Bagdad. Flinders Petrie then pointed out a group of kings named on scarabs of peculiar type, which, including Khyan, he attributed to the period between the Old Kingdom and the New, while others were in favour of assigning them all to the Hyksos, whose appellation seemed to be recognizable in the title Hek-khos, "ruler of the barbarians," borne by Khyan. The extraordinary importance of Khyan was further shown by the discovery of his name on a jar-lid at Cnossus in Crete. Semitic features were pointed out in the supposed Hyksos names, and Petrie was convinced of their date by his excavations of 1905-1906 in the eastern Delta. Avaris is generally assigned to the region towards Pelusium on the strength of its being located in the Sethroite nome by Josephus, but Petrie thinks it was at Tell el-Yahudiyeh (Yehudia), where Hyksos scarabs are common. From the remains of fortifications there he argues that the Hyksos were uncivilized desert people, skilled in the use of the bow, and must thus have destroyed by their archery the Egyptian armies trained to fight hand-to-hand; further, that their hordes were centered in Syria, but were driven thence by a superior force in the East to take refuge in the islands and became a sea-power-whence the strange description "Hellenic" in Manetho, which most editors have corrected to $\alpha \lambda \lambda$ oí, "others." Besides the statue of Khyan, blocks of granite with the name of Apopi have been found in Upper Egypt at Gebelen and in Lower Egypt at Bubastis. The celebrated Rhind mathematical papyrus was copied in the reign of an Apopi from an original of the time of Amenemhe III. Large numbers of Hyksos scarabs are found in Upper and Lower Egypt, and they are not unknown in Palestine. Khyan's monuments, inconspicuous as they are, actually extend over a wider area-from Bagdad to Cnossus-than those of any other Egyptian king.

It is certain that this mysterious people were Asiatic, for they are called so by the Egyptians. Though Seth was an Egyptian god, as god of the Hyksos he represents some Asiatic deity. The possibility of a connexion between the Hyksos and the Israelites is still admitted in some quarters. Hatred of these impious foreigners, of which there is some trace in more than one text, aroused amongst the Egyptians (as nothing ever did before or since) that martial spirit which carried the armies of Tethmosis to the Euphrates.

HYLAS, In Greek legend, son of Theiodamas, king of the Dryopians in Thessaly, the favourite of Heracles and his companion on the Argonautic expedition. Having gone ashore at Kios in Mysia to fetch water, he was carried off by the nymphs of the spring in which he dipped his pitcher. Heracles sought him in vain, and the answer of Hylas to his thrice-repeated cry was lost in the depths of the water. Ever afterwards, in memory of the threat of Heracles to ravage the land if Hylas were not found, the inhabitants of Kios every year on a stated day roamed the mountains, shouting aloud for Hylas (Apollonius Rhodius i. 1207; Theocritus xiii.; Strabo xii. 564; Propertius i. 20; Virgil, Ecl. vi. 43). But, although the legend is first told in Alexandrian times, the "cry of Hylas" occurs long before as the "Mysian cry" in Aeschylus (Persae, 1054), and in Aristophanes (Plutus, 1127) "to cry Hylas" is used proverbially of seeking something in vain. Hylas, like Adonis and Hyacinthus, represents the fresh vegetation of spring, or the water of a fountain, which dries up under the heat of summer. It is suggested that Hylas was a harvest deity and that the ceremony gone through by the Kians was a harvest festival, at which the figure of a boy was thrown into the water, signifying the dying vegetation-spirit of the year.

See G. Türk in Breslauer Philologische Abhandlungen, vii. (1895); W. Mannhardt, Mythologische Forschungen (1884).

HYLOZOISM (Gr. ü $\eta$, matter, $\zeta \omega \eta ́, ~ l i f e)$, in philosophy, a term applied to any system which explains all life, whether physical or mental, as ultimately derived from matter ("cosmic matter," Weldstoff). Such a view of existence has been common throughout the history of thought, and especially among physical scientists. Thus the Ionian school of philosophy, which began with Thales, sought for the beginning of all things in various material substances, water, air, fire (see Ionian School). These substances were regarded as being in some sense alive, and taking some active part in the development of being. This primitive hylozoism reappeared in modified forms in medieval and Renaissance thought, and in modern times the doctrine of materialistic monism is its representative. Between modern materialism and hylozoism proper there is, however, the distinction that the ancients, however vaguely, conceived the elemental matter as being in some sense animate if not actually conscious and conative.

HYMEN, or Hymenaeus, originally the name of the song sung at marriages among the Greeks. As usual the name gradually produced the idea of an actual person whose adventures gave rise to the custom of this song. He occurs often in association with Linus and Ialemus, who represent similar personifications, and is generally called a son of Apollo and a Muse. As the son of Dionysus and Aphrodite, he was regarded as a god of fruitfulness. In Attic legend he was a beautiful youth who, being in love with a girl, followed her in a procession to Eleusis disguised as a woman, and saved the whole band from pirates. As reward he obtained the girl in marriage, and his happy married life caused him ever afterwards to be invoked in marriage songs (Servius on Virgil, Aen. i. 651). According to another story, he was a youth who was killed by the fall of his house on his wedding day; hence he was invoked, to propitiate him and avert a similar fate from others (Servius, loc. cit.). He is represented in works of art as an effeminatelooking, winged youth, carrying a bridal torch and wearing a nuptial veil. The marriage song was sung, with musical accompaniment, during the procession of the bride from her parents' house to that of the bridegroom, Hymenaeus being invoked at the end of each portion.

See R. Schmidt, De Hymenaeo el Talasio (1886), and J. A. Hild in Daremberg and Saglis's Dictionnaire des antiquités.
 classification for one of the most important orders of the class Hexapoda (q.v.). The order was founded by Linnaeus (Systema Naturae, 1735), and is still recognized by all naturalists in the sense proposed by him, to include the saw-flies, gall-flies, ichneumon-flies and their allies, ants, wasps and bees. The relationship of the Hymenoptera to other orders of insects is discussed in the article Hexapoda, but it may be mentioned here that in structure the highest members of the order are remarkably specialized, and that in the perfection of their instincts they stand at the head of all insects and indeed of all invertebrate animals. About 30,000 species of Hymenoptera are now known.


After C. L. Marlatt, Bur. Ent. Bull. 3, N.S., U.S. Dept. Agric.
Fig. 1.-A, Front of head of Saw-fly (Pachynematus); $a$, labrum; $b$, clypeus; $c$, vertex; $d$, $d$, antennal cavities. C and D, Mandibles. E, First maxilla; $a$, cardo; $b$, stipes; $c$, galea; $d$, lacinia; $e$, palp. B, Second maxillae (Labium); $a$, mentum; $b$, ligula (between the two galeae); $c, c$, palps. Magnified.


Fig. 2.-Jaws of Hive-bee (Apis mellifica). Magnified about $61 / 2$ times. $a$, mandible; $b, c$, palp and lacinia of first maxilla; $d, e, g, h$, mentum, palp, fused laciniae (ligula or "tongue") and galea of 2nd maxillae.


After C. Janet, Mem. Soc. Zool. France (1898).
Fig. 3.-Median section through mid-body of female Red Ant (Myrmica rubra). H, Head; 1, 2, 3, the thoracic segments; i., ii., the first and second abdominal segments; i., being the propodeum.

Characters.-In all Hymenoptera the mandibles (fig. 1, C, D) are well developed, being adapted, as in the more lowly winged insects, such as the Orthoptera, for biting. The more generalized Hymenoptera have the second maxillae but slightly modified, their inner lobes being fused to form a ligula (fig. 1, B, b). In the higher families this structure becomes elongated (fig. 2, $g$ ) so as to form an elaborate sucking-organ or "tongue." These insects are able, therefore, to bite as well as to suck, whereas most insects which have acquired the power of suction have lost that of biting. Both fore- and hind-wings are usually present, both pairs being membranous, the hind-wings small and not folded when at rest, each provided along the costa with a row of curved hooks which catch on to a fold along the dorsum of the adjacent forewing during flight. A large number of Hymenoptera are, however, entirely wingless-at least as regards one sex or form of the species. One of the most remarkable features is the close union of the foremost abdominal segment (fig. 3, i.) with the metathorax, of which it often seems to form a part, the apparent first abdominal segment being, in such case, really the second (fig. 3, ii.). The true first segment, which undergoes a more or less complete fusion with the thorax is known as the "median segment" or propodeum. In female Hymenoptera the typical insectan ovipositor with its three pairs of processes is well developed, and in the higher families this organ becomes functional as a sting (fig. 5), -used for offence and defence. As regards their life history, all Hymenoptera undergo a "complete" metamorphosis. The larva is soft-skinned (eruciform), being either a caterpillar (fig. 6, b) or a legless grub (fig. 7, a), and the pupa is free (fig. $7, c$ ), i.e. with the appendages not fixed to the body, as is the case in the pupa of most moths.


Fig. 4.-Fore-Wings of Hymenoptera.

1. Tenthredinidae (Hylotoma) - 1, marginal; 2, appendicular; 3, 4, 5, 6, radial or submarginal; 7, 8, 9, median or discoidal; 10, sub-costal; 11, 12, cubital or branchial; and 13, anal or lanceolate cellules; $a, b, c$, submarginal nervures; $d$, basal nervures; $e, f$, recurrent nervures; st, stigma; co, costa.
2. Cynipidae (Cynips).
3. Chalcididae (Perilampus).
4. Proctotrypidae (Codrus).
5. Mymaridae (Mymar).
6. Braconidae (Bracon).
7. Ichneumonidae (Trogus).
8. Chrysididae (Cleptes).
9. Formicidae (Formica).
10. Vespidae (Vespa).
11. Apidae (Apathus).

Structure.-The head of a hymenopterous insect bears three simple eyes (ocelli) on the front and vertex in addition to the large compound eyes. The feelers are generally simple in type, rarely showing serrations or prominent appendages; but one or two basal segments are frequently differentiated to form an elongate "scape," the remaining segments-carried at an elbowed angle to the scape-making up the "flagellum"; the segments of the flagellum often bear complex sensory organs. The general characters of the jaws have been mentioned above, and in detail there is great variation in these organs among the different families. The sucking tongue of the Hymenoptera has often been compared with the hypopharynx of other insects. According to D. Sharp, however, the hypopharynx is present in all Hymenoptera as a distinct structure at the base of the "tongue," which must be regarded as representing the fused laciniae of the second maxillae. In the thorax the pronotum and prosternum are closely associated with the mesothorax, but the pleura of the prothorax are usually shifted far forwards, so that the fore-legs are inserted just behind the head. A pair of small plates-the tegulae-are very generally present at the bases of the fore-wings. The union of the first abdominal segment with the metathorax has been already mentioned. The second (so-called "first") abdominal segment is often very constricted, forming the "waist" so characteristic of wasps and ants for example. The constriction of this segment and its very perfect articulation with the propodeum give great mobility to the abdomen, so that the ovipositor or sting can be used with the greatest possible accuracy and effect.

Mention has already been made of the series of curved hooks along the costa of the hindwing; by means of this arrangement the two wings of a side are firmly joined together during flight, which thus becomes particularly accurate. The wings in the Hymenoptera show a marked reduction in the number of nervures as compared with more primitive insects. The main median nervure, and usually also the sub-costal become united with the radial, while the branches of
radial, median and cubital nervures pursuing a transverse or recurrent course across the wing, divide its area into a number of areolets or "cells," that are of importance in classification. Among many of the smaller Hymenoptera we find that the wings are almost destitute of nervures. In the hind-wings-on account of their reduced size-the nervures are even more reduced than in the fore-wings.
The legs of Hymenoptera are of the typical insectan form, and the foot is usually composed of five segments. In many families the trochanter appears to be represented by two small segments, there being thus an extra joint in the leg. It is almost certain that the distal of these two segments really belongs to the thigh, but the ordinary nomenclature will be used in the present article, as this character is of great importance in discriminating families, and the two segments in question are referred to the trochanter by most systematic writers.


After C. Janet, Aiguillon de la Myrmica rubra (Paris, 1898).
Fig. 5.-Ovipositor or Sting of Red Ant (Myrmica rubra) Queen. Magnified. The right sheath C (outer process of the ninth abdominal segment-9) is shown in connexion with the guide B formed by the inner processes of the 9th segment. The stylet A (process of the 8th abdominal segment-8) is turned over to show its groove $a$, which works along the tongue or rail $b$.

The typical insectan ovipositor, so well developed among the Hymenoptera, consists of three pairs of processes (gonapophyses) two of which belong to the ninth abdominal segment and one to the eighth. The latter are the cutting or piercing stylets (fig. 5, A) of the ovipositor, while the two outer processes of the ninth segment are modified into sheaths or feelers (fig. 5, C) and the two inner processes form a guide (fig. 5, B) on which the stylets work, tongues or rails on the "guide" fitting accurately into longitudinal grooves on the stylet. In the different families of the Hymenoptera, there are various modifications of the ovipositor, in accord with the habits of the insects and the purposes to which the organ is put. The sting of wasps, ants and bees is a modified ovipositor and is used for egg-laying by the fertile females, as well as for defence. Most male Hymenoptera have processes which form claspers or genital armature. These processes are not altogether homologous with those of the ovipositor, being formed by inner and outer lobes of a pair of structures on the ninth abdominal segment.

Many points of interest are to be noted in the internal structure of the Hymenoptera. The gullet leads into a moderate-sized crop, and several pairs of salivary glands open into the mouth. The crop is followed by a proventriculus which, in the higher Hymenoptera, forms the so-called "honey stomach," by the contraction of whose wails the solid and liquid food can be separated, passed on into the digestive stomach, or held in the crop ready for regurgitation into the mouth. Behind the digestive stomach are situated, as usual, intestine and rectum, and the number of kidney (Malpighian) tubes varies from only six to over a hundred, being usually great.

In the female, each ovary consists of a large number of ovarian tubes, in which swollen chambers containing the egg-cells alternate with smaller chambers enclosing nutrient material. In connexion with the ovipositor are two poison-glands, one acid and the other alkaline in its secretion. The acid gland consists of one, two or more tubes, with a cellular coat of several layers, opening into a reservoir whence the duct leads to the exterior. The alkaline gland is an irregular tube with a single cellular layer, its duct opening alongside that of the acid reservoir. These glands are most strongly developed when the ovipositor is modified into a sting.

Development.-Parthenogenesis is of normal occurrence in the life-cycle of many Hymenoptera. There are species of gall-fly in which males are unknown, the unfertilized eggs always developing into females. On the other hand, in certain saw-flies and among the higher families, the unfertilized eggs, capable of development, usually give rise to male insects (see Bee). The larvae of most saw-flies feeding on the leaves of plants are caterpillars (fig. $6, b$ ) with numerous abdominal pro-legs, but in most families of Hymenoptera the egg is laid in such a
situation that an abundant food-supply is assured without exertion on the part of the larva, which is consequently a legless grub, usually white in colour, and with soft flexible cuticle (fig. 7, a). The organs and instincts for egg-laying and food-providing are perhaps the most remarkable features in the economy of the Hymenoptera. Gall-fly grubs are provided with vegetable food through the eggs being laid by the mother insect within plant tissues. The ichneumon pierces the body of a caterpillar and lays her eggs where the grubs will find abundant animal food. A digging-wasp hunts for insect prey and buries it with the egg, while a true wasp feeds her brood with captured insects, as a bird her fledglings. Bees store honey and pollen to serve as food for their young. Thus we find throughout the order a degree of care for offspring unreached by other insects, and this family-life has, in the best known of the Hymenoptera-ants, wasps and bees-developed into an elaborate social organization.

Social Life.-The development of a true insect society among the Hymenoptera is dependent on a differentiation among the females between individuals with well-developed ovaries ("queens") whose special function is reproduction; and individuals with reduced or aborted ovaries ("workers") whose duty is to build the nest, to gather food and to tend and feed the larvae. Among the wasps the workers may only differ from the queens in size, and individuals intermediate between the two forms of female may be met with. Further, the queen wasp, and also the queen humble-bee, commences unaided the work of building and founding a new nest, being afterwards helped by her daughters (the workers) when these have been developed. In the hive-bee and among ants, on the other hand, there are constant structural distinctions between queen and worker, and the function of the queen bee in a hive is confined to egglaying, the labour of the community being entirely done by the workers. Many ants possess several different forms of worker, adapted for special duties. Details of this fascinating subject are given in the special articles Ant, Bee and WASP (q.v.).

Habits and Distribution.-Reference has been already made to the various methods of feeding practised by Hymenoptera in the larval stage, and the care taken of or for the young throughout the order leads in many cases to the gathering of such food by the mother or nurse. Thus, wasps catch flies; worker ants make raids and carry off weak insects of many kinds; bees gather nectar from flowers and transform it into honey within their stomachs-largely for the sake of feeding the larvae in the nest. The feeding habits of the adult may agree with that of the larva, or differ, as in the ease of wasps which feed their grubs on flies, but eat principally vegetable food themselves. The nest-building habit is similarly variable. Digging wasps make simple holes in the ground; many burrowing bees form branching tunnels; other bees excavate timber or make their brood-chambers in hollow plant-stems; wasps work up with their saliva vegetable fibres bitten off tree-bark to make paper; social bees produce from glands in their own bodies the wax whence their nest-chambers are built. The inquiline habit ("cuckooparasitism"), when one species makes use of the labour of another by invading the nest and laying her eggs there, is of frequent occurrence among Hymenoptera; and in some cases the larva of the intruder is not content with taking the store of food provided, but attacks and devours the larva of the host.

Most Hymenoptera are of moderate or small size, the giants of the order-certain saw-flies and tropical digging-wasps-never reach the bulk attained by the largest beetles, while the wing-spread is narrow compared with that of many dragon-flies and moths. On the other hand, there are thousands of very small species, and the tiny "fairy-flies" (Mymaridae), whose larvae live as parasites in the eggs of various insects, are excessively minute for creatures of such complex organization. Hymenoptera are probably less widely distributed than Aptera, Coleoptera or Diptera, but they are to be found in all except the most inhospitable regions of the globe. The order is, with few exceptions, terrestrial or aerial in habit. Comparatively only a few species are, for part of their lives, denizens of fresh water; these, as larvae, are parasitic on the eggs or larvae of other aquatic insects, the little hymenopteron, Polynema natans, one of the "fairy-flies"-swims through the water by strokes of her delicate wings in search of a dragon-fly's egg in which to lay her own egg, while the rare Agriotypus dives after the case of a caddis-worm. It is of interest that the waters have been invaded by the parasitic group of the Hymenoptera, since in number of species this is by far the largest of the order. No group of terrestrial insects escapes their attacks-even larvae boring in wood are detected by ichneumon flies with excessively long ovipositors. Not a few cases are known in which a parasitic larva is itself pierced by the ovipositor of a "hyperparasite," and even the offspring of the latter may itself fall a victim to the attack of a "tertiary parasite."

Fossil History.-Very little is known of the history of the Hymenoptera previous to the Tertiary epoch, early in which, as we know from the evidence of many Oligocene and Miocene fossils, all the more important families had been differentiated. Fragments of wings from the Lias and Oolitic beds have been referred to ants and bees, but the true nature of these remains is doubtful.

Classification.-Linnaeus divided the Hymenoptera into two sections-the Terebrantia, whose
females possess a cutting or piercing ovipositor, and the Aculeata, in which the female organ is modified into a sting. This nomenclature was adopted by P. A. Latreille and has been in general use until the present day. A closely similar division of the order results from T. Hartig's character drawn from the trochanter-whether of two segments or undivided-the groups being termed respectively Ditrocha and Monotrocha. But the most natural division is obtained by the separation of the saw-flies as a primitive sub-order, characterized by the imperfect union of the first abdominal segment with the thorax, and by the broad base of the abdomen, so that there is no median constriction or "waist," and by the presence of thoracic legs-usually also of abdominal pro-legs-in the larva. All the other families of Hymenoptera, including the gall-flies, ichneumons and aculeates, have the first abdominal segment closely united with the thorax, the second abdominal segment constricted so as to form a narrow stalk or "waist," and legless larvae without a hinder outlet to the food-canal. These two sub-orders are usually known as the Sessiliventra and Petioliventra respectively, but the names Symphyta and Apocrita proposed in 1867 by C. Gerstaecker have priority, and should not be replaced.

## Symphyta.

This sub-order, characterized by the "sessile," broad-based abdomen, whose first segment is imperfectly united with the thorax, and by the usually caterpillar-like larvae with legs, includes the various groups of saw-flies. Three leading families may be mentioned. The Cephidae, or stem saw-flies, have an elongate pronotum, a compressed abdomen, and a single spine on the shin of the fore-leg. The soft, white larvae have the thoracic legs very small and feed in the stems of various plants. Cephus pygmaeus is a well-known enemy of corn crops. The Siricidae ("wood-wasps") are large elongate insects also with one spine on each fore-shin, but with the pronotum closely joined to the mesothorax. The ovipositor is long and prominent, enabling the female insect to lay her eggs in the wood of trees, where the white larvae, whose legs are excessively short, tunnel and feed. These insects are adorned with bands of black and yellow, or with bright metallic colours, and on account of their large size and formidable ovipositors they often cause needless alarm to persons unfamiliar with their habits. The Tenthredinidae, or true saw-flies, are distinguished by two spines on each fore-shin, while the larvae are usually caterpillars, with three pairs of thoracic legs, and from six to eight pairs of abdominal pro-legs the latter not possessing the hooks found on the pro-legs of lepidopterous caterpillars. Most saw-fly larvae devour leaves, and the beautifully serrate processes of the ovipositor are well adapted for egg-laying in plant tissues. Some saw-fly larvae are protected by a slimy secretion (fig. 6, c) and a few live concealed in galls. In the form of the feelers, the wing-neuration and minor structural details there is much diversity among the saw-flies. They have been usually regarded as a single family, but W. H. Ashmead has lately differentiated eleven families of them.

## Apocrita.

This sub-order includes the vast majority of the Hymenoptera, characterized by the narrowly constricted waist in the adult and by the legless condition of the larva. The trochanter is simple in some genera and divided in others. With regard to the minor divisions of this group, great difference of opinion has prevailed among students. In his recent classification Ashmead (1901) recognizes seventy-nine families arranged under eight "super-families." The number of species included in this division is enormous, and the multiplication of families is, to some extent, a natural result of increasingly close study. But the distinctions between many of these rest on comparatively slight characters, and it is likely that the future discovery of new genera may abolish many among such distinctions as may now be drawn. It seems advisable, therefore, in the present article to retain the wider conception of the family that has hitherto contented most writers on the Hymenoptera. Ashmead's "super-families" have, however, been adopted asfounded on definite structural characters-they probably indicate relationship more nearly than the older divisions founded mostly on habit. The Cynipoidea include the gall-flies and their parasitic relations. In the Chalcidoidea, Ichneumonoidea and Proctotrypoidea will be found nearly all the "parasitic Hymenoptera" of older classifications. The Formicoidea are the ants. The group of Fossores, or "digging-wasps," is divided by Ashmead, one section forming the Sphecoidea, while the other, together with the Chrysidae and the true wasps, make up the Vespoidea. The Apoidea consists of the bees only.


After Marlatt, Ent. Circ. 26, U.S. Dept. Agric.
Fig. 6.-a, Pear Saw-fly (Eriocampoides limacina); b, larva without, and $c$, with its slimy protective coat; $e$, cocoon; $f$, larva before pupation; $g$, pupa, magnified; $d$, leaves with larvae.


After Howard, Ent. Tech. Bull. 5 U.S. Dept. Agric
Fig. 7.-Chalcid (Dibrachys boucheanus), a hyper-parasite.
a, Larva.
$d$, Its head more highly magnified.
$b$, Female fly.

Cynipoidea.-In this division the ovipositor issues from the ventral surface of the abdomen; the pronotum reaches back to the tegulae; the trochanter has two segments; the fore-wing (fig. 4,2 ) has no stigma, but one or two areolets. The feelers with twelve to fifteen segments are thread-like and straight. All the insects included in this group are small and form two familiesthe Cynipidae and the Figitidae. They are the "gall-flies," many of the species laying eggs in various plant-tissues where the presence of the larva causes the formation of a pathological growth or gall, always of a definite form and characteristic of the species; the "oak-apple" and the bedeguar of the rose are familiar examples. Other flies of this group have the inquiline habit, laying their eggs in the galls of other species, while others again pierce the cuticle of maggots or aphids, in whose bodies their larvae live as parasites.

Chalcidoidea.-This division resembles the Cynipoidea in the position of the ovipositor, and in the two segmented trochanters. The fore-wing also has no stigma, and the whole wing is almost destitute of nervures and areolets, while the pronotum does not reach back to the tegulae, and the feelers are elbowed (fig. 7). The vast majority of this group, including nearly 5000 known species, are usually reckoned as a single family, the Chalcididae, comprising small insects, often of bright metallic colours, whose larvae are parasitic in insects of various orders. The "figinsects," whose presence in ripening figs is believed essential to the proper development of the fruit, belong to Blastophaga and other genera of this family. They are remarkable in having wingless males and winged females. The "polyembryonic" development of an Encyrtus, as studied by P. Marchal, is highly remarkable. The female lays her egg in the egg of a small ermine moth (Hyponomeuta) and the egg gives rise not to a single embryo but to a hundred, which develop as the host-caterpillar develops, being found at a later stage within the latter enveloped in a flexible tube.

The Mymaridae or "fairy-flies" are distinguished from the Chalcididae by their narrow fringed
wings (figs. 4,5) and by the situation of the ovipositor just in front of the tip of the abdomen. They are among the most minute of all insects and their larvae are probably all parasitic in insects' eggs.

Ichneumonoidea.-The ten thousand known species included in this group agree with the Cynipoidea and Chalcidoidea in the position of the ovipositor and in the jointed trochanters, but are distinguished by the fore-wing possessing a distinct stigma and usually a typical series of nervures and areolets (figs. 4, 8). Many of the species are of fair size. They lay their eggs (fig. 8) in the bodies of insects and their larvae belonging to various orders. A few small families such as the Evaniidae and the Stephanidae are included here, but the vast majority of the group fall into two large families, the Ichneumonidae and the Braconidae, the former distinguished by the presence of two median (or discoidal) cells in the fore-wing (figs. 4, 7), while the latter has only one (figs. 4, 6). Not a


After Riley and Howard, Insect Life, vol. i.
Fig. 8.-Ichneumon Fly (Rhyssa persuasoria) ovipositing. few of these insects, however, are entirely wingless. On account of their work in destroying plant-eating insects, the ichneumon-flies are of great economic importance.

Proctotrypoidea.-This group may be distinguished from the preceding by the position of the ovipositor at the extreme apex of the abdomen, and from the groups that follow (with very few exceptions) by the jointed trochanters of the legs. The pronotum reaches back to the tegulae. The Pelecinidae-included here by Ashmead-are large insects with remarkably elongate abdomens and undivided trochanters. All the other members of the group may be regarded as forming a single family-the Proctotrypidae, including an immense number of small parasitic Hymenoptera, not a few of which are wingless. Of special interest are the transformations of Platygaster, belonging to this family, discovered by M. Ganin, and familiarized to English readers through the writings of Sir J. Lubbock (Lord Avebury). The first larva is broad in front and tapers behind to a "tail" provided with two divergent processes, so that it resembles a small crustacean. It lives in the grub of a gall-midge and it ultimately becomes changed into the usual white and fleshy hymenopterous larva. The four succeeding sections, in which the ovipositor is modified into a sting (always exserted from the tip of the abdomen) and the trochanters are with few exceptions simple, form the Aculeata of Linnaeus.

Formicoidea.-The ants which form this group are readily distinguished by the differentiation of the females into winged "queens" and wingless "workers." The pronotum extends back to the wing-bases, and the "waist" is greatly constricted and marked by one or two "nodes." The differentiation of the females leads to a complex social life, the nesting habits of ants and the various industries that they pursue being of surpassing interest (see Ant).

Vespoidea.-This section includes a number of families characterized by the backward extension of the prothorax to the tegulae and distinguished from the ants by the absence of "nodes" at the base of the abdomen. The true wasps have the fore-wings folded lengthwise when at rest and the fore-legs of normal build-not specialized for digging. The Vespidae or social wasps have "queens" and "workers" like the ants, but both these forms of female are winged; the claws on their fret are simple. In the Eumenidae or solitary wasps the female sex is undifferentiated, and the foot claws are toothed. (For the habits of these insects see Wasp.) The Chrysididae or ruby wasps are small insects with a very hard cuticle exhibiting brilliant metallic colours-blue, green and crimson. Only three or four abdominal segments are visible, the hinder segments being slender and retracted to form a telescope-like tube in which the ovipositor lies. When the ovipositor is brought into use this tube is thrust out. The eggs are laid in the nests of various bees and wasps, the chrysid larva living as a "cuckoo" parasite. The Trigonalidae, a small family whose larvae are parasitic in wasps' nests, also probably belong here.

The other families of the Vespoidea belong to the series of "Fossores" or digging-wasps. In two of the families-the Mutillidae and Thynnidae-the females are wingless and the larvae live as parasites in the larvae of other insects; the female Mutilla enters bumble-bees' nests and lays her eggs in the bee-grubs. In the other families both sexes are winged, and the instinct and industry of the females are among the most wonderful in the Hymenoptera. They make burrows wherein they place insects or spiders which they have caught and stung, laying their eggs beside the victim so that the young larvae find themselves in presence of an abundant and appropriate food-supply. Valuable observations on the habits of these insects are due to J. H. Fabre and G. W. and E. Peckham. The prey is sometimes stung in the neighbourhood of the nerve ganglia, so that it is paralysed but not killed, the grub of the fossorial wasp devouring its victim alive; but this instinct varies in perfection, and in many cases the larva flourishes equally whether its prey be killed or not. The females have a wonderful power of finding their burrows on returning from their hunting expeditions. Among the Vespoid families of fossorial wasps, the

Pompilidae are the most important. They are recognizable by their slender and elongate hindlegs; many of them provision their burrows with spiders. The Sapygidae are parasitic on bees, while the Scoliidae are large, robust and hairy insects, many of which prey upon the grubs of chafers.

Sphecoidea.-In this division are included the rest of the "digging-wasps," distinguished from the Vespoidea by the short pronotum not reaching backward to the tegulae. They have usually been reckoned as forming a single, very large family-the Sphegidae-but ten or twelve subdivisions of the group are regarded as distinct families by Ashmead and others. Great diversity is shown in the details of structure, habits and nature of the prey. Species of Sphex, studied by Fabre, provisioned their brood-chambers with crickets. Pelopoeus hunts spiders, while Ammophila catches caterpillars for the benefit of her young. Fabre states that the lastnamed insect uses a stone for the temporary closing of her burrow, and the Peckhams have seen a female Ammophila take a stone between her mandibles and use it as a hammer for pounding down the earth over her finished nest. The habits of Bembex are of especial interest. The female, instead of provisioning her burrow with a supply of food that will suffice the larva for its whole life, brings fresh flies with which she regularly feeds her young. In this instinct we have a correspondence with the habits of social wasps and bees. Yet it may be thought that the usual instinct of the "digging-wasps" to capture and store up food in an underground burrow for the benefit of offspring which they will never see is even more surprising. The habit of some genera is to catch the prey before making their tunnel, but more frequently the insect digs her nest, and then hunts for prey to put into it.

Apoidea.-The bees which make up this group agree with the Sphecoidea in the short pronotum, but may be distinguished from all other Hymenoptera by the widened first tarsal segment and the plumose hairs on head and body. They are usually regarded as forming a single family-the Apidae-but there is very great diversity in structural details, and Ashmead divides them into fourteen families. The "tongue," for example, is short and obtuse or emarginate in Colletes and Prosopis, while in all other bees it is pointed at the tip. But in Andrena and its allies it is comparatively short, while in the higher genera, such as Apis and Bombus, it is elongate and flexible, forming a most elaborate and perfect organ for taking liquid food. Bees feed on honey and pollen. Most of the genera are "solitary" in habit, the female sex being undifferentiated; but among the humble-bees and hive-bees we find, as in social wasps and ants, the occurrence of workers, and the consequent elaboration of a wonderful insectsociety. (See Bee.)

Bibliography.-The literature of several special families of the Hymenoptera will be found under the articles Ant, Bee, Ichneumon-Fly, Wasp, \&c., referred to above. Among earlier students on structure may be mentioned P. A. Latreille, Familles naturelles du règne animal (Paris, 1825), who recognized the nature of the "median segment." C. Gerstaecker (Arch. f. Naturg. xx., 1867) and F. Brauer (Sitzb. K. Akad. Wiss. Wien. lxxxv., 1883) should also be consulted on this subject. For internal anatomy, specially the digestive organs, see L. Dufour, Mém. savants étrangers, vii. (1841), and Ann. Sci. Nat. Zool. (4), i. 1854. For nervous system H. Viallanes, Ann. Sci. Nat. Zool. (7), ii. iv. 1886-1887, and F. C. Kenyon, Journ. Comp. Neurol. vi., 1896. For poison and other glands, see L. Bordas, Ann. Sci. Nat. Zool. (7) xix., 1895. For the sting and ovipositor H. Dewitz, Zeits. wiss. Zool. xxv., 1874, xxviii., 1877, and F. Zander, ib. lxvi., 1899. For male genital armature S. A. Peytoureau, Morphologie de l'armure génitale des insectes (Bordeaux, 1895), and E. Zander, Zeits. wiss. Zool. lxvii., 1900. The systematic student of Hymenoptera is greatly helped by C. G. de Dalla Torre's Catalogus Hymenopterorum (10 vols., Leipzig, 1893-1902). For general classifications see F. W. Konow, Entom. Nachtr. (1897), and W. H. Ashmead, Proc. U.S. Nat. Mus. xxiii., 1901; the latter paper deals also especially with the Ichneumonoidea of the globe. For habits and life histories of Hymenoptera see J. Lubbock (Lord Avebury), Ants, Bees and Wasps (9th ed., London, 1889); C. Janet, Études sur les fourmis, les guêpes et les abeilles (Paris, \&c., 1893 and onwards); and G. W. and E. G. Peckham, Instincts and Habits of Solitary Wasps (Madison, Wis. U.S.A., 1898). Monographs of most of the families of British Hymenoptera have now been published. For saw-flies and gall-flies, see P. Cameron's British Phytophagous Hymenoptera (4 vols., London, Roy. Soc., 1882-1893). For Ichneumonoidea, C. Morley's Ichneumons of Great Britain (Plymouth, 1903, \&c.), and T. A. Marshall's "British Braconidae," Trans. Entom. Soc., 1885-1899. The smaller parasitic Hymenoptera have been neglected in this country since A. H. Haliday's classical papers Entom. Mag. i.-v., (1833-1838) but Ashmead's "North American Proctotrypidae" (Bull. U.S. Nat. Mus. xlv., 1893) is valuable for the European student. For the Fossores, wasps, ants and bees see E. Saunders, Hymenoptera Aculeata of the British Islands (London, 1896). Exhaustive references to general systematic works will be found in de Dalla Torre's Catalogue mentioned above. Of special value to English students are C. T. Bingham's Fauna of British India, "Hymenoptera" (London, 1897 and onwards), and P. Cameron's volumes on Hymenoptera in the Biologia Centrali-Americana. F. Smith's Catalogues of Hymenoptera in the British Museum (London, 1853-1859) are well worthy of study.
(G. H. C.)

HYMETTUS (Ital. Monte Matto, hence the modern name Trello Vouni), a mountain in Attica, bounding the Athenian plain on the S.E. Height, 3370 ft . It was famous in ancient times for its bees, which gathered honey of peculiar flavour from its aromatic herbs; their fame still persists. The spring mentioned by Ovid (Ars Amat. iii. 687) is probably to be recognized near the monastery of Syriani or Kaesariani on the western slope. This may be identical with that known
 often has a bluish tinge, was used extensively for building in ancient Athens, and also, in early times, for sculpture; but the white marble of Pentelicus was preferred for both purposes.

See E. Dodwell, Classical and Topographical Tour (1819), i. 483.

HYMNS.-1. Classical Hymnody.-The word "hymn" (úpvos) was employed by the ancient Greeks ${ }^{1}$ to signify a song or poem composed in honour of gods, heroes or famous men, or to be recited on some joyful, mournful or solemn occasion. Polymnia was the name of their lyric muse. Homer makes Alcinous entertain Odysseus with a "hymn" of the minstrel Demodocus, on the capture of Troy by the wooden horse. The Works and Days of Hesiod begins with an invocation to the Muses to address hymns to Zeus, and in his Theogonia he speaks of them as singing or inspiring "hymns" to all the divinities, and of the bard as "their servant, hymning the glories of men of old, and of the gods of Olympus." Pindar calls by this name odes, like his own, in praise of conquerors at the public games of Greece. The Athenian dramatists (Euripides most frequently) use the word and its cognate verbs in a similar manner; they also describe by them metrical oracles and apophthegms, martial, festal and hymeneal songs, dirges and lamentations or incantations of woe.

Hellenic hymns, according to this conception of them, have come down to us, some from a very early and others from a late period of Greek classical literature. Those which passed by the name of Homer ${ }^{2}$ were already old in the time of Thucydides. They are mythological poems (several of them long), in hexameter verse-some very interesting. That to Apollo contains a traditionary history of the origin and progress of the Delphic worship; those on Hermes and on Dionysus are marked by much liveliness and poetical fancy. Hymns of a like general character, but of less interest (though these also embody some fine poetical traditions of the Greek mythology, such as the story of Teiresias, and that of the wanderings of Leto), were written in the 3rd century before Christ, by Callimachus of Cyrene. Cleanthes, the successor of Zeno, composed (also in hexameters) an "excellent and devout hymn" (as it is justly called by Cudworth, in his Intellectual System) to Zeus, which is preserved in the Eclogae of Stobaeus, and from which Aratus borrowed the words, "For we are also His offspring," quoted by St Paul at Athens. The so-called Orphic hymns, in hexameter verse, styled $\tau \varepsilon \lambda \varepsilon \tau \alpha i ́$, or hymns of initiation into the "mysteries" of the Hellenic religion, are productions of the Alexandrian school,-as to which learned men are not agreed whether they are earlier or later than the Christian era.

The Romans did not adopt the word "hymn"; nor have we many Latin poems of the classical age to which it can properly be applied. There are, however, a few-such as the simple and graceful "Dianae sumus in fide" ("Dian's votaries are we") of Catullus, and "Dianam tenerae dicite virgines" ("Sing to Dian, gentle maidens") of Horace-which approach much more nearly than anything Hellenic to the form and character of modern hymnody.
2. Hebrew Hymnody.-For the origin and idea of Christian hymnody we must look, not to Gentile, but to Hebrew sources. St Augustine's definition of a hymn, generally accepted by Christian antiquity, may be summed up in the words, "praise to God with song" ("cum cantico"); Bede understood the "canticum" as properly requiring metre; though he thought that what in its original language was a true hymn might retain that character in an unmetrical translation. Modern use has enlarged the definition; Roman Catholic writers extend it to the praises of saints; and the word now comprehends rhythmical prose as well as verse, and prayer and spiritual meditation as well as praise.

The modern distinction between psalms and hymns is arbitrary (see Psalms). The former word was used by the LXX. as a generic designation, probably because it implied an accompaniment by the psaltery (said by Eusebius to have been of very ancient use in the East) or other instruments. The cognate verb "psallere" has been constantly applied to hymns, both in the Eastern and in the Western Church; and the same compositions which they described generically as "psalms" were also called by the LXX. "odes" (i.e. songs) and "hymns." The latter word occurs, e.g. in Ps. lxxii. 20 ("the hymns of David the son of Jesse"), in Ps. lxv. 1, and also in the Greek titles of the 6th, 54th, 55th, 67th and 76th (this numbering of the psalms being
that of the English version, not of the LXX.). The 44th chapter of Ecclesiasticus, "Let us now praise famous men," \&c., is entitled in the Greek $\pi \alpha \tau \varepsilon ์ \rho \omega \nu$ úpvos, "The Fathers' Hymn." Bede speaks of the whole book of Psalms as called "liber hymnorum," by the universal consent of Hebrews, Greeks and Latins.

In the New Testament we find our Lord and His apostles singing a hymn (ú $\mu \nu \eta ́ \sigma \alpha \nu \tau \varepsilon \varsigma$ $\dot{\varepsilon} \xi \tilde{\eta} \lambda \theta \circ v$ ), after the institution of the Lord's Supper; St Paul and Silas doing the same (úpvouv tòv $\theta \varepsilon o ́ v$ ) in their prison at Philippi; St James recommending psalm-singing ( $\psi \alpha \lambda \lambda \varepsilon ́ \tau \omega$ ), and St
 Paul also, in the 14th chapter of the first epistle to the Corinthians, speaks of singing ( $\psi \alpha \lambda \tilde{\omega}$ )
 to the assemblies of the Corinthian Christians for common worship. All the words thus used were applied by the LXX. to the Davidical psalms; it is therefore possible that these only may be intended, in the different places to which we have referred. But there are in St Paul's epistles several passages (Eph. v. 14; 1 Tim. iii. 16; 1 Tim. vi. 15, 16; 2 Tim. ii. 11, 12) which have so much of the form and character of later Oriental hymnody as to have been supposed by Michaelis and others to be extracts from original hymns of the Apostolic age. Two of them are apparently introduced as quotations, though not found elsewhere in the Scriptures. A third has not only rhythm, but rhyme. The thanksgiving prayer of the assembled disciples, recorded in Acts iv., is both in substance and in manner poetical; and in the canticles, "Magnificat," "Benedictus," \&c., which manifestly followed the form and style of Hebrew poetry, hymns or songs, proper for liturgical use, have always been recognized by the church.
3. Eastern Church Hymnody.-The hymn of our Lord, the precepts of the apostles, the angelic song at the nativity, and "Benedicite omnia opera" are referred to in a curious metrical prologue to the hymnary of the Mozarabic Breviary as precedents for the practice of the Western Church. In this respect, however, the Western Church followed the Eastern, in which hymnody prevailed from the earliest times.

Philo describes the Theraputae ( $q . v$. ) of the neighbourhood of Alexandria as composers of original hymns, which (as well as old) were sung at their great religious festivals-the people listening in silence till they came to the closing strains, or refrains, at the end

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 of a hymn or stanza (the "acroteleutia" and "ephymnia"), in which all, women as well as men, heartily joined. These songs, he says, were in various metres (for which he uses a number of technical terms); some were choral, some not; and they were divided into variously constructed strophes or stanzas. Eusebius, who thought that the Theraputae were communities of Christians, says that the Christian practice of his own day was in exact accordance with this description.The practice, not only of singing hymns, but of singing them antiphonally, appears, from the well-known letter of Pliny to Trajan, to have been established in the Bithynian churches at the beginning of the 2nd century. They were accustomed "stato die ante lucem

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 convenire, carmenque Christo, quasi Deo, dicere secum invicem." This agrees well, in point of time, with the tradition recorded by the historian Socrates, that Ignatius (who suffered martyrdom about a.D. 107) was led by a vision or dream of angels singing hymns in that manner to the Holy Trinity to introduce antiphonal singing into the church of Antioch, from which it quickly spread to other churches. There seems to be an allusion to choral singing in the epistle of Ignatius himself to the Romans, where he exhorts them, "Xopòऽ $ү \varepsilon \lambda \omega \delta i ́ \alpha \nu " ~(" h a v i n g ~ f o r m e d ~ t h e m s e l v e s ~ i n t o ~ a ~ c h o i r "), ~ t o ~ " s i n g ~ p r a i s e ~ t o ~$ the Father in Christ Jesus." A statement of Theodoret has sometimes been supposed to refer the origin of antiphonal singing to a much later date; but this seems to relate only to the singing of Old Testament Psalms (тף̀v $\Delta \alpha u$ ıбıкฑ̀ $\nu \mu \lambda \omega \delta i ́ \alpha \nu$ ), the alternate chanting of which, by a choir divided into two parts, was (according to that statement) first introduced into the church of Antioch by two monks famous in the history of their time, Flavianus and Diodorus, under the emperor Constantius II.Other evidence of the use of hymns in the 2nd century is contained in a fragment of Caius, preserved by Eusebius, which refers to "all the psalms and odes written by faithful brethren from the beginning," as "hymning Christ, the Word of God, as God." Tertullian

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 also, in his description of the "Agapae," or love-feasts, of his day, says that, after washing hands and bringing in lights, each man was invited to come forward and sing to God's praise something either taken from the Scriptures or of his own composition ("ut quisque de Sacris Scripturis vel proprio ingenio potest"). George Bull, bishop of St David's, believed one of those primitive compositions to be the hymn appended by Clement of Alexandria to his Paedagogus; and Archbishop Ussher considered the ancient morning and evening hymns, of which the use was enjoined by the Apostolical Constitutions, and which are also mentioned in the "Tract on Virginity" printed with the works of St Athanasius, and in St Basil's treatise upon the Holy Spirit, to belong to the same family. Clement's hymn, in a short anapaestic metre, beginning $\sigma \tau o ́ \mu \iota \circ \nu \pi \omega \lambda \omega \nu \dot{\alpha} \delta \alpha \tilde{\omega} \nu$ (or, accordingto some editions, $\beta \alpha \sigma \iota \lambda \varepsilon \tilde{\cup} \dot{\alpha} \gamma i ́ \omega \nu$, $\lambda o ́ \gamma \varepsilon \pi \alpha \nu \delta \alpha \mu \alpha ́ \tau \omega \rho$ —translated by the Rev. A. Chatfield, "O Thou, the King of Saints, all-conquering Word"), is rapid, spirited and well-adapted for singing. The Greek "Morning Hymn" (which, as divided into verses by Archbishop Ussher in his treatise De Symbolis, has a majestic rhythm, resembling a choric or dithyrambic strophe) is the original form of "Gloria in Excelsis," still said or sung, with some variations, in all branches of the church which have not relinquished the use of liturgies. The Latin form of this hymn (of which that in the English communion office is an exact translation) is said, by Bede and other ancient writers, to have been brought into use at Rome by Pope Telesphorus, as early as the time of the emperor Hadrian. A third, the Vesper or "Lamp-lighting" hymn (" $\varphi \tilde{\omega} \varsigma$ i $\lambda \alpha \rho o ̀ v ~ \alpha ́ \gamma i ́ \alpha \varsigma ~ \delta o ́ \xi \eta ऽ "-~$ translated by Canon Bright "Light of Gladness, Beam Divine"), holds its place 3rd century. to this day in the services of the Greek rite. In the 3rd century Origen seems to have had in his mind the words of some other hymns or hymn of like character, when he says (in his treatise Against Celsus): "We glorify in hymns God and His only begotten Son; as do also the Sun, the Moon, the Stars and all the host of heaven. All these, in one Divine chorus, with the just among men, glorify in hymns God who is over all, and His only begotten Son." So highly were these compositions esteemed in the Syrian churches that the council which deposed Paul of Samosata from the see of Antioch in the time of Aurelian justified that act, in its synodical letter to the bishops of Rome and Alexandria, on this ground (among others) that he had prohibited the use of hymns of that kind, by uninspired writers, addressed to Christ.

After the conversion of Constantine, the progress of hymnody became closely connected with church controversies. There had been in Edessa, at the end of the 2nd or early in the 3rd century, a Gnostic writer of conspicuous ability, named Bardesanes, who was succeeded, as the head of his sect or school, by his son Harmonius. Both father and son wrote hymns, and set them to agreeable melodies, which acquired, and in the 4th century still retained, much local popularity. Ephraem Syrus, the first voluminous hymn-writer whose works remain to us, thinking that the same melodies might be made useful to the faith, if adapted to more orthodox words, composed to them a large number of hymns in the Syriac language, principally in tetrasyllabic, pentasyllable and heptasyllabic metres, divided into strophes of from 4 to 12, 16 and even 20 lines each. When a strophe contained five lines, the fifth was generally an "ephymnium," detached in sense, and consisting of a prayer, invocation, doxology or the like, to be sung antiphonally, either in full chorus or by a separate part of the choir. The Syriac Chrestomathy of August Hahn (Leipzig, 1825), and the third volume of H. A. Daniel's Thesaurus Hymnologicus (Leipzig, 1841-1856), contain specimens of these hymns. Some of them have been translated into (unmetrical) English by the Rev. Henry Burgess (Select Metrical Hymns of Ephrem Syrus, \&c., 1853). A considerable number of those so translated are on subjects connected with death, resurrection, judgment, \&c., and display not only Christian faith and hope, but much simplicity and tenderness of natural feeling. Theodoret speaks of the spiritual songs of Ephraem as very sweet and profitable, and as adding much, in his (Theodoret's) time, to the brightness of the commemorations of martyrs in the Syrian Church.

The Greek hymnody contemporary with Ephraem followed, with some licence, classical models. One of its favourite metres was the Anacreontic; but it also made use of the short anapaestic, Ionic, iambic and other lyrical measures, as well as the hexameter and pentameter. Its principal authors were Methodius, bishop of Olympus, who died about a.d. 311, Synesius, who became bishop of Ptolemais in Cyrenaica in 410, and Gregory Nazianzen, for a short time (380-381) patriarch of Constantinople. The merits of these writers have been perhaps too much depreciated by the admirers of the later Greek "Melodists." They have found an able English translator in the Rev. Allen Chatfield (Songs and Hymns of Earliest Greek Christian Poets, London, 1876). Among the most striking of their works are $\mu \nu \omega \varepsilon$ 人 X
 үદvદ́бӨ人เ ("O soul of mine, repining"), by Gregory; also äv $\omega \theta \varepsilon \nu ~ \pi \alpha \rho \theta \varepsilon ́ v o ı ~(" T h e ~ B r i d e g r o o m ~$ cometh"), by Methodius. There continued to be Greek metrical hymn-writers, in a similar style, till a much later date. Sophronius, patriarch of Jerusalem in the 7th century, wrote seven Anacreontic hymns; and St John Damascene, one of the most copious of the second school of "Melodists," was also the author of some long compositions in trimeter iambics.

An important development of hymnody at Constantinople arose out of the Arian controversy. Early in the 4th century Athanasius had rebuked, not only the doctrine of Arius, but the light character of certain hymns by which he endeavoured to make that doctrine

Period of Arian controversy. popular. When, towards the close of that century (398), St John Chrysostom was raised to the metropolitan see, the Arians, who were still numerous at Constantinople, had no places of worship within the walls; but they were in the habit of coming into the city at sunset on Saturdays, Sundays and the greater festivals, and congregating in the porticoes and other places of public resort, where they sung, all night through, antiphonal songs, with "acroteleutia" (closing strains, or refrains), expressive of Arian doctrine, often accompanied by taunts and insults to the orthodox. Chrysostom was
apprehensive that this music might draw some of the simpler church people to the Arian side; he therefore organized, in opposition to it, under the patronage and at the cost of Eudoxia, the empress of Arcadius (then his friend), a system of nightly processional hymn-singing, with silver crosses, wax-lights and other circumstances of ceremonial pomp. Riots followed, with bloodshed on both sides, and with some personal injury to the empress's chief eunuch, who seems to have officiated as conductor or director of the church musicians. This led to the suppression, by an imperial edict, of all public Arian singing; while in the church the practice of nocturnal hymn-singing on certain solemn occasions, thus first introduced, remained an established institution.

It is not improbable that some rudiments of the peculiar system of hymnody which now prevails throughout the Greek communion, and whose affinities are rather to the Hebrew and Syriac than to the classical forms, may have existed in the church of

## Greek system

 of hymnody. than that he wrote hymns still extant, and lived towards the end of that century. The importance of that system in the services of the Greek church may be understood from the fact that $\operatorname{Dr}$ J. M. Neale computed four-fifths of the whole space (about 5000 pages) contained in the different service-books of that church to be occupied by hymnody, all in a language or dialect which has ceased to be anywhere spoken.The system has a peculiar technical terminology, in which the words "troparion," "ode," "canon" and "hirmus" ( $\varepsilon$ Ĩ $\rho \mu \circ \varsigma$ ) chiefly require explanation.
The troparion is the unit of the system, being a strophe or stanza, seen, when analysed, to be divisible into verses or clauses, with regulated caesuras, but printed in the books as a single prose sentence, without marking any divisions. The following (turned into English, from a "canon" by John Mauropus) may be taken as an example: "The never-sleeping Guardian, | the patron of my soul, | the guide of my life, | allotted me by God, | I hymn thee, Divine Angel | of Almighty God." Dr Neale and most other writers regard all these "troparia" as rhythmical or modulated prose. Cardinal J. B. Pitra, on the other hand, who in 1867 and 1876 published two learned works on this subject, maintains that they are really metrical, and governed by definite rules of prosody, of which he lays down sixteen. According to him, each "troparion" contains from three to thirty-three verses; each verse varies from two to thirteen syllables, often in a continuous series, uniform, alternate or reciprocal, the metre being always syllabic, and depending, not on the quantity of vowels or the position of consonants, but on an harmonic series of accents.

In various parts of the services solitary troparia are sung, under various names, "contacion," "oecos," "cathisma," \&c., which mark distinctions either in their character or in their use.

An ode is a song or hymn compounded of several similar "troparia,"-usually three, four or five. To these is always prefixed a typical or standard "troparion," called the hirmus, by which the syllabic measure, the periodic series of accents, and in fact the whole structure and rhythm of the stanzas which follow it are regulated. Each succeeding "troparion" in the same "ode" contains the same number of verses, and of syllables in each verse, and similar accents on the same or equivalent syllables. The "hirmus" may either form the first stanza of the "ode" itself, or (as is more frequently the case) may be taken from some other piece; and, when so taken, it is often indicated by initial words only, without being printed at length. It is generally printed within commas, after the proper rubric of the "ode." A hymn in irregular "stichera" or stanzas, without a "hirmus," is called "idiomelon." A system of three or four odes is "triodion" or "tetraodion."

A canon is a system of eight (theoretically nine) connected odes, the second being always suppressed. Various pauses, relieved by the interposition of other short chants or readings, occur during the singing of a whole "canon." The final "troparion" in each ode of the series is not unfrequently detached in sense (like the "ephymnia" of Ephraem Syrus), particularly when it is in the (very common) form of a "theotokion," or ascription of praise to the mother of our Lord, and when it is a recurring refrain or burden.

There were two principal periods of Greek hymnography constructed on these principles-the first that of Romanos and his followers, extending over the 6th and 7th centuries, the second that of the schools which arose during the Iconoclastic controversy in the 8th century, and which continued for some centuries afterwards, until the art itself died out.

The works of the writers of the former period were collected in Tropologia, or church hymnbooks, which were held in high esteem till the 10th century, when they ceased to be regarded as church-books, and so fell into neglect. They are now preserved only in a very small number of manuscripts. From three of these, belonging to public
Romanos. libraries at Moscow, Turin and Rome, Cardinal Pitra has printed, in his Analecta, a number of interesting examples, the existence of which appears to
have been unknown to Dr Neale, and which, in the cardinal's estimation, are in many respects superior to the "canons," \&c., of the modern Greek service-books, from which all Neale's translations (except some from Anatolius) are taken. Cardinal Pitra's selections include twentynine works by Romanos, and some by Sergius, and nine other known, as well as some unknown, authors. He describes them as having generally a more dramatic character than the "melodies" of the later period, and a much more animated style; and he supposes that they may have been originally sung with dramatic accompaniments, by way of substitution for the theatrical performances of Pagan times. As an instance of their peculiar character, he mentions a Christmas or Epiphany hymn by Romanos, in twenty-five long strophes, in which there is, first, an account of the Nativity and its accompanying wonders, and then a dialogue between the wise men, the Virgin mother and Joseph. The magi arrive, are admitted, describe the moral and religious condition of Persia and the East, and the cause and adventures of their journey, and then offer their gifts. The Virgin intercedes for them with her Son, instructs them in some parts of Jewish history, and ends with a prayer for the salvation of the world.

The controversies and persecutions of the 8th and succeeding centuries turned the thoughts of the "melodists" of the great monasteries of the Studium at Constantinople and St Saba in Palestine and their followers, and those of the adherents of the Greek rite in Melodists. Sicily and South Italy (who suffered much from the Saracens and the Normans), into a less picturesque but more strictly theological course; and the influence of those controversies, in which the final success of the cause of "Icons" was largely due to the hymns, as well as to the courage and sufferings, of these confessors, was probably the cause of their supplanting, as they did, the works of the older school. Cardinal Pitra gives them the praise of having discovered a graver and more solemn style of chant, and of having done much to fix the dogmatic theology of their church upon its present lines of near approach to the Roman.

Among the "melodists" of this latter Greek school there were many saints of the Greek church, several patriarchs and two emperors-Leo the Philosopher, and Constantine Porphyrogenitus, his son. Their greatest poets were Theodore and Joseph of the Studium, and Cosmas and John (called Damascene) of St Saba. Neale translated into English verse several selected portions, or centoes, from the works of these and others, together with four selections from earlier works by Anatolius. Some of his translations-particularly "The day is past and over," from Anatolius, and "Christian, dost thou see them," from Andrew of Crete-have been adopted into hymn-books used in many English churches; and the hymn "Art thou weary," which is rather founded upon than translated from one by Stephen the Sabaite, has obtained still more general popularity.
4. Western Church Hymnody.-It was not till the 4th century that Greek hymnody was imitated in the West, where its introduction was due to two great lights of the Latin Church-St Hilary of Poitiers and St Ambrose of Milan.

Hilary was banished from his see of Poitiers in 356, and was absent from it for about four years, which he spent in Asia Minor, taking part during that time in one of the councils of the Eastern Church. He thus had full opportunity of becoming acquainted with the Greek church music of that day; and he wrote (as St Jerome, who was thirty years old when Hilary died, and who was well acquainted with his acts and writings, and spent some time in or near his diocese, informs us) a "book of hymns," to one of which Jerome particularly refers, in the preface to the second book of his own commentary on the epistle to the Galatians. Isidore, archbishop of Seville, who presided over the fourth council of Toledo, in his book on the offices of the church, speaks of Hilary as the first Latin hymn-writer; that council itself, in its 13th canon, and the prologue to the Mozarabic hymnary (which is little more than a versification of the canon), associate his name, in this respect, with that of Ambrose. A tradition, ancient and widely spread, ascribed to him the authorship of the remarkable "Hymnum dicat turba fratrum, hymnum cantus personet" ("Band of brethren, raise the hymn, let your song the hymn resound"), which is a succinct narrative, in hymnal form, of the whole gospel history; and is perhaps the earliest example of a strictly didactic hymn. Both Bede and Hincmar much admired this composition, though the former does not mention, in connexion with it, the name of Hilary. The private use of hymns of such a character by Christians in the West may probably have preceded their ecclesiastical use; for Jerome says that in his day those who went into the fields might hear "the ploughman at his hallelujahs, the mower at his hymns, and the vine-dresser singing David's psalms." Besides this, seven shorter metrical hymns attributed to Hilary are still extant.

Of the part taken by Ambrose, not long after Hilary's death, in bringing the use of hymns into the church of Milan, we have a contemporary account from his convert, St Augustine. Justina, mother of the emperor Valentinian, favoured the Arians, and desired to remove

## Ambrose.

 Ambrose from his see. The "devout people," of whom Augustine's mother, Monica, was one, combined to protect him, and kept guard in the church."Then," says Augustine, "it was first appointed that, after the manner of the Eastern churches, hymns and psalms should be sung, lest the people should grow weary and faint through sorrow; which custom has ever since been retained, and has been followed by almost all congregations in other parts of the world." He describes himself as moved to tears by the sweetness of these "hymns and canticles":-"The voices flowed into my ears; the truth distilled into my heart; I overflowed with devout affections, and was happy." To this time, according to an uncertain but not improbable tradition which ascribed the composition of the "Te Deum" to Ambrose, and connected it with the conversion of Augustine, is to be referred the commencement of the use in the church of that sublime unmetrical hymn.

It is not, however, to be assumed that the hymnody thus introduced by Ambrose was from the first used according to the precise order and method of the later Western ritual. To bring it into (substantially) that order and method appears to have been the work of St Benedict. Walafrid Strabo, the earliest ecclesiastical writer on this subject (who lived at the beginning of the 9th century), says that Benedict, on the constitution of the religious order known by his name (about 530), appointed the Ambrosian hymns to be regularly sung in his offices for the canonical hours. Hence probably originated the practice of the Italian churches, and of others which followed their example, to sing certain hymns (Ambrosian, or by the early successors of the Ambrosian school) daily throughout the week, at "Vespers," "Lauds" and "Nocturns," and on some days at "Compline" also-varying them with the different ecclesiastical seasons and festivals, commemorations of saints and martyrs and other special offices. Different dioceses and religious houses had their own peculiarities of ritual, including such hymns as were approved by their several bishops or ecclesiastical superiors, varying in detail, but all following the same general method. The national rituals, which were first reduced into a form substantially like that which has since prevailed, were probably those of Lombardy and of Spain, now known as the "Ambrosian" and the "Mozarabic." The age and origin of the Spanish ritual are uncertain, but it is mentioned in the 7th century by Isidore, bishop of Seville. It contained a copious hymnary, the original form of which may be regarded as canonically approved by the fourth council of Toledo (633). By the 13th canon of that council, an opinion (which even then found advocates) against the use in churches of any hymns not taken from the Scriptures-apparently the same opinion which had been held by Paul of Samosata-was censured; and it was ordered that such hymns should be used in the Spanish as well as in the Gallican churches, the penalty of excommunication being denounced against all who might presume to reject them.

The hymns of which the use was thus established and authorized were those which entered into the daily and other offices of the church, afterwards collected in the "Breviaries"; in which the hymns "proper" for "the week," and for "the season," continued for many centuries, with very few exceptions, to be derived from the earliest epoch of Latin Church poetry-reckoning that epoch as extending from Hilary and Ambrose to the end of the pontificate of Gregory the Great. The "Ambrosian" music, to which those hymns were generally sung down to the time of Gregory, was more popular and congregational than the "Gregorian," which then came into use, and afterwards prevailed. In the service of the mass it was not the general practice, before the invention of sequences in the 9th century, to sing any hymns, except some from the Scriptures esteemed canonical, such as the "Song of the Three Children" ("Benedicite omnia opera"). But to this rule there were, according to Walafrid Strabo, some occasional exceptions; particularly in the case of Paulinus, patriarch of Aquileia under Charlemagne, himself a hymnwriter, who frequently used hymns, composed by himself or others, in the eucharistic office, especially in private masses.

Some of the hymns called "Ambrosian" (nearly 100 in number) are beyond all question by Ambrose himself, and the rest probably belong to his time or to the following century. Four, those beginning "Aeterne rerum conditor" ("Dread Framer of the earth and sky"), "Deus Creator omnium" ("Maker of all things, glorious God"), "Veni Redemptor Gentium" ("Redeemer of the nations, come") and "Jam surgit hora tertia" ("Christ at this hour was crucified"), are quoted as works of Ambrose by Augustine. These, and others by the hand of the same master, have the qualities most valuable in hymns intended for congregational use. They are short and complete in themselves; easy, and at the same time elevated in their expression and rhythm; terse and masculine in thought and language; and (though sometimes criticized as deficient in theological precision) simple, pure and not technical in their rendering of the great facts and doctrines of Christianity, which they present in an objective and not a subjective manner. They have exercised a powerful influence, direct or indirect, upon many of the best works of the same kind in all succeeding generations. With the Ambrosian hymns are properly classed those of Hilary, and the contemporary works of Pope Damasus I. (who wrote two hymns in commemoration of saints), and of Prudentius, from whose Cathemerina ("Daily Devotions") and Peristephana ("Crown-songs for Martyrs"), all poems of considerable, some of great lengthabout twenty-eight hymns, found in various Breviaries, were derived. Prudentius was a layman, a native of Saragossa, and it was in the Spanish ritual that his hymns were most largely used.

In the Mozarabic Breviary almost the whole of one of his finest poems (from which most churches took one part only, beginning "Corde natus ex parentis") was appointed to be sung between Easter and Ascension-Day, being divided into eight or nine hymns; and on some of the commemorations of Spanish saints long poems from his Peristephana were recited or sung at large. He is entitled to a high rank among Christian poets, many of the hymns taken from his works being full of fervour and sweetness, and by no means deficient in dignity or strength.

These writers were followed in the 5th and early in the 6th century by the priest Sedulius, whose reputation perhaps exceeded his merit; Elpis, a noble Roman lady (considered, by an erroneous tradition, to have been the wife of the philosophic statesman

## 5th and 6th centuries.

 Boetius); Pope Gelasius I.; and Ennodius, bishop of Pavia. Sedulius and Elpis wrote very little from which hymns could be extracted; but the small number taken from their compositions obtained wide popularity, and have since held their ground. Gelasius was of no great account as a hymn-writer; and the works of Ennodius appear to have been known only in Italy and Spain. The latter part of the 6th century produced Pope Gregory the Great and Venantius Fortunatus, an Italian poet, the friend of Gregory, and the favourite of Radegunda, queen of the Franks, who died (609) bishop of Poitiers. Eleven hymns of Gregory, and twelve or thirteen (mostly taken from longer poems) by Fortunatus, came into general use in the Italian, Gallican and British churches. Those of Gregory are in a style hardly distinguishable from the Ambrosian; those of Fortunatus are graceful, and sometimes vigorous. He does not, however, deserve the praise given to him by Dr Neale, of having struck out a new path in Latin hymnody. On the contrary, he may more justly be described as a disciple of the school of Prudentius, and as having affected the classical style, at least as much as any of his predecessors.The poets of this primitive epoch, which closed with the 6th century, wrote in the old classical metres, and made use of a considerable variety of them-anapaestic, anacreontic, hendecasyllabic, asclepiad, hexameters and pentameters and others. Gregory and some of the Ambrosian authors occasionally wrote in sapphics; but the most frequent measure was the iambic dimeter, and, next to that, the trochaic. The full alcaic stanza does not appear to have been used for church purposes before the 16th century, though some of its elements were. In the greater number of these works, a general intention to conform to the rules of Roman prosody is manifest; but even those writers (like Prudentius) in whom that conformity was most decided allowed themselves much liberty of deviation from it. Other works, including some of the very earliest, and some of conspicuous merit, were of the kind described by Bede as not metrical but "rhythmical"-i.e. (as he explains the term "rhythm"), "modulated to the ear in imitation of different metres." It would be more correct to call them metrical-(e.g. still trochaic or iambic, \&c., but, according to new laws of syllabic quantity, depending entirely on accent, and not on the power of vowels or the position of consonants)-laws by which the future prosody of all modern European nations was to be governed. There are also, in the hymns of the primitive period (even in those of Ambrose), anticipations-irregular indeed and inconstant, but certainly not accidental-of another great innovation, destined to receive important developments, that of assonance or rhyme, in the final letters or syllables of verses. Archbishop Trench, in the introduction to his Sacred Latin Poetry, has traced the whole course of the transition from the ancient to the modern forms of versification, ascribing it to natural and necessary causes, which made such changes needful for the due development of the new forms of spiritual and intellectual life, consequent upon the conversion of the Latin-speaking nations to Christianity.
From the 6th century downwards we see this transformation making continual progress, each nation of Western Christendom adding, from time to time, to the earlier hymns in its servicebooks others of more recent and frequently of local origin. For these additions,
6th century
downwards. the commemorations of saints, \&c., as to which the devotion of one place often differed from that of another, offered especial opportunities. This process, while it promoted the development of a medieval as distinct from the primitive style, led also to much deterioration in the quality of hymns, of which, perhaps, some of the strongest examples may be found in a volume published in 1865 by the Irish Archaeological Society from a manuscript in the library of Trinity College, Dublin. It contains a number of hymns by Irish saints of the 6th, 7th and 8th centuries-in several instances fully rhymed, and in one mixing Erse and Latin barbarously together, as was not uncommon, at a much later date, in semi-vernacular hymns of other countries. The Mozarabic Breviary, and the collection of hymns used in the Anglo-Saxon churches, published in 1851 by the Surtees Society (chiefly from a Benedictine MS. In the college library of Durham, supplemented by other MSS. in the British Museum), supply many further illustrations of the same decline of taste:-such Sapphics, e.g., as the "Festum insigne prodiit coruscum" of Isidore, and the "O veneranda Trinitas laudanda" of the Anglo-Saxon books. The early medieval period, however, from the time of Gregory the Great to that of Hildebrand, was far from deficient in the production of good hymns, wherever learning flourished. Bede in England, and Paul "the Deacon"-the author of a fairly classical sapphic ode on St John the Baptist-in Italy, were successful
followers of the Ambrosian and Gregorian styles. Eleven metrical hymns are attributed to Bede by Cassander; and there are also in one of Bede's works (Collectanea et flores) two rhythmical hymns of considerable length on the Day of Judgment, with the refrains "In tremendo die" and "Attende homo," both irregularly rhymed, and, in parts, not unworthy of comparison with the "Dies Irae." Paulinus, patriarch of Aquileia, contemporary with Paul, wrote rhythmical trimeter iambics in a manner peculiar to himself. Theodulph, bishop of Orleans (793-835), author of the famous processional hymn for Palm Sunday in hexameters and pentameters, "Gloria, laus, et honor tibi sit, Rex Christe Redemptor" ("Glory and honour and laud be to Thee, King Christ the Redeemer"), and Hrabanus Maurus, archbishop of Mainz, the pupil of Alcuin, and the most learned theologian of his day, enriched the church with some excellent works. Among the anonymous hymns of the same period there are three of great beauty, of which the influence may be traced in most, if not all, of the "New Jerusalem" hymns of later generations, including those of Germany and Great Britain:-"Urbs beata Hierusalem" ("Blessed city, heavenly Salem"); "Alleluia piis edite laudibus" ("Alleluias sound ye in strains of holy praise"-called, from its burden, "Alleluia perenne"); and "Alleluia dulce carmen" ("Alleluia, song of sweetness"), which, being found in Anglo-Saxon hymnaries certainly older than the Conquest, cannot be of the late date assigned to it, in his Mediaeval Hymns and Sequences, by Neale. These were followed by the "Chorus novae Hierusalem" ("Ye Choirs of New Jerusalem") of Fulbert, bishop of Chartres. This group of hymns is remarkable for an attractive union of melody, imagination, poetical colouring and faith. It represents, perhaps, the best and highest type of the middle school, between the severe Ambrosian simplicity and the florid luxuriance of later times.

Another celebrated hymn, which belongs to the first medieval period, is the "Veni Creator Spiritus" ("Come, Holy Ghost, our souls inspire"). The earliest recorded occasion of its use is that of a translation (898) of the relics of St Marcellus, mentioned in the

## Veni Creator.

 Annals of the Benedictine order. It has since been constantly sung throughout Western Christendom (as versions of it still are in the Church of England), as part of the appointed offices for the coronation of kings, the consecration and ordination of bishops and priests, the assembling of synods and other great ecclesiastical solemnities. It has been attributed-probably in consequence of certain corruptions in the text of Ekkehard's Life of Notker (a work of the 13th century)-to Charlemagne. Ekkehard wrote in the Benedictine monastery of St Gall, to which Notker belonged, with full access to its records; Notker. and an ignorant interpolator, regardless of chronology, added, at some later date, the word "Great" to the name of "the emperor Charles," wherever it was mentioned in that work. The biographer relates that Notker-a man of a gentle, contemplative nature, observant of all around him, and accustomed to find spiritual and poetical suggestions in common sights and sounds-was moved by the sound of a mill-wheel to compose his "sequence" on the Holy Spirit, "Sancti Spiritus adsit nobis gratia" ("Present with us ever be the Holy Spirit's grace"); and that, when finished, he sent it as a present to "the emperor Charles," who in return sent him back, "by the same messenger," the hymn "Veni Creator," which (says Ekkehard) the same "Spirit had inspired him to write" ("Sibi idem Spiritus inspiraverat"). If this story is to be credited-and, from its circumstantial and almost dramatic character, it has an air of truth-the author of "Veni Creator" was not Charlemagne, but his grandson the emperor Charles the Bald. Notker himself long survived that emperor, and died in 912.The invention of "sequences" by Notker may be regarded as the beginning of the later medieval epoch of Latin hymnody. In the eucharistic service, in which (as has been stated) hymns were not generally used, it had been the practice, except at certain

Sequences. to fill up what would otherwise have been a long pause, by extending the cadence upon the two final vowels of the "Alleluia" into a protracted strain of music. It occurred to Notker that, while preserving the spirit of that part of the service, the monotony of the interval might be relieved by introducing at that point a chant of praise specially composed for the purpose. With that view he produced the peculiar species of rhythmical composition which obtained the name of "sequentia" (probably from following after the close of the "Alleluia"), and also that of "prosa," because its structure was originally irregular and unmetrical, resembling in this respect the Greek "troparia," and the "Te Deum," "Benedicite" and canticles. That it was in some measure suggested by the forms of the later Greek hymnody seems probable, both from the intercourse (at that time frequent) between the Eastern and Western churches, and from the application by Ekkehard, in his biography and elsewhere (e.g. in Lyndwood's Provinciale), of some technical terms, borrowed from the Greek terminology, to works of Notker and his school and to books containing them.

[^0]Nicholas I., who authorized their use, and that of others composed after the same model by other brethren of St Gall, in all churches of the West.

Although the sequences of Notker and his school, which then rapidly passed into most German, French and British Missals, were not metrical, the art of "assonance" was much practised in them. Many of those in the Sarum and French Missals have every verse, and even every clause or division of a verse, ending with the same vowel "a"-perhaps with some reference to the terminal letter of "Alleluia." Artifices such as these naturally led the way to the adaptation of the same kind of composition to regular metre and fully developed rhyme. Neale's full and large collection, and the second volume of Daniel's Thesaurus, contain numerous examples, both of the "proses," properly so called, of the Notkerian type, and of those of the later school, which (from the religious house to which its chief writer belonged) has been called "Victorine." Most Missals appear to have contained some of both kinds. In the majority of those from which Neale's specimens are taken, the metrical kind largely prevailed; but in some (e.g. those of Sarum and Liége) the greater number were Notkerian.

Of the sequence on the Holy Ghost, sent by Notker (according to Ekkehard) to Charles the Bald, Neale says that it "was in use all over Europe, even in those countries, like Italy and Spain, which usually rejected sequences"; and that, "in the Missal of Palencia, the priest was ordered to hold a white dove in his hands, while intoning the first syllables, and then to let it go." Another of the most remarkable of Notker's sequences, beginning "Media in vita" ("In the midst of life we are in death"), is said to have been suggested to him while observing some workmen engaged in the construction of a bridge over a torrent near his monastery. Catherine Winkworth (Christian Singers of Germany, 1869) states that this was long used as a battlesong, until the custom was forbidden, on account of its being supposed to exercise a magical influence. A translation of it ("Mitten wir im Leben sind") is one of Luther's funeral hymns; and all but the opening sentence of that part of the burial service of the Church of England which is directed to be "said or sung" at the grave, "while the corpse is made ready to be laid into the earth," is taken from it.

The "Golden Sequence," "Veni, sancte Spiritus" ("Holy Spirit, Lord of Light"), is an early example of the transition of sequences from a simply rhythmical to a metrical form. Archbishop Trench, who esteemed it "the loveliest of all the hymns in the whole circle of Latin sacred poetry," inclined to give credit to a tradition which ascribes its authorship to Robert II., king of France, son of Hugh Capet. Others have assigned to it a later date-some attributing it to Pope Innocent III., and some to Stephen Langton, archbishop of Canterbury. Many translations, in German, English and other languages, attest its merit. Berengarius of Tours, St Bernard of Clairvaux and Abelard, in the 11th century and early in the 12th, followed in the same track; and the art of the Victorine school was carried to its greatest perfection by Adam of St Victor (who died between 1173 and 1194)-"the most fertile, and" (in the concurrent judgment of Archbishop Trench and Neale) "the greatest of the Latin hymnographers of the Middle Ages." The archbishop's selection contains many excellent specimens of his works.

But the two most widely celebrated of all this class of compositions-works which have exercised the talents of the greatest musical composers, and of innumerable translators in almost all languages-are the "Dies Irae" ("That day of wrath, that dreadful

## Dies Irae.

Stabat Mater. Aquinas. day"), by Thomas of Celano, the companion and biographer of St Francis of Assisi, and the "Stabat Mater dolorosa" ("By the cross sad vigil keeping") of Jacopone, or Jacobus de Benedictis, a Franciscan humorist and reformer, who was persecuted by Pope Boniface VIII. for his satires on the prelacy of the time, and died in 1306. Besides these, the 13th century produced the famous sequence "Lauda Sion salvatorem" ("Sion, lift thy voice and sing"), and the four other well-known sacramental hymns of St Thomas Aquinas, viz. "Pange lingua gloriosi corporis mysterium" ("Sing, my tongue, the Saviour's glory"), "Verbum supernum prodiens" ("The Word, descending from above"-not to be confounded with the Ambrosian hymn from which it borrowed the first line), "Sacris solemniis juncta sint gaudia" ("Let us with hearts renewed our grateful homage pay"), and "Adoro Te devote, latens Deitas" ("O Godhead hid, devoutly I adore Thee")-a group of remarkable compositions, written by him for the then new festival of Corpus Christi, of which he induced Pope Urban IV. (1261-1265) to decree the observance. In these (of which all but "Adoro Te devote" passed rapidly into breviaries and missals) the doctrine of transubstantiation is set forth with a wonderful degree of scholastic precision; and they exercised, probably, a not unimportant influence upon the general reception of that dogma. They are undoubtedly works of genius, powerful in thought, feeling and expression.

These and other medieval hymn-writers of the 12th and 13th centuries may be described, generally, as poet-schoolmen. Their tone is contemplative, didactic, theological; they are especially fertile and ingenious in the field of mystical interpretation. Two great monasteries in the East had, in the 8th and 9th centuries, been the hymns. principal centres of Greek hymnology; and, in the West, three monasteries-St

Gall, near Constance (which was long the especial seat of German religious literature), Cluny in Burgundy and St Victor, near Paris-obtained a similar distinction. St Gall produced, besides Notker, several distinguished sequence writers, probably his pupils-Hartmann, Hermann and Gottschalk-to the last of whom Neale ascribes the "Alleluiatic Sequence" ("Cantemus cuncti melodum nunc Alleluia"), well known in England through his translation, "The strain upraise of joy and praise." The chief poets of Cluny were two of its abbots, Odo and Peter the Venerable (1122-1156), and one of Peter's monks, Bernard of Morlaix, who wrote the remarkable poem on "Contempt of the World" in about 3000 long rolling "leonine-dactylic" verses, from parts of which Neale's popular hymns, "Jerusalem the golden," \&c., are taken. The abbey of St Victor, besides Adam and his follower Pistor, was destined afterwards to produce the most popular church poet of the 17th century.

There were other distinguished Latin hymn-writers of the later medieval period besides those already mentioned. The name of St Bernard of Clairvaux cannot be passed over with the mere mention of the fact that he was the author of some metrical sequences. He

## Bernard of Clairvaux.

 was, in truth, the father, in Latin hymnody, of that warm and passionate form of devotion which some may consider to apply too freely to Divine Objects the language of human affection, but which has, nevertheless, been popular with many devout persons, in Protestant as well as Roman Catholic churches. F. von Spee, "Angelus Silesius," Madame Guyon, Bishop Ken, Count Zinzendorf and Frederick William Faber may be regarded as disciples in this school. Many hymns, in various languages, have been founded upon St Bernard's "Jesu dulcis memoria" ("Jesu, the very thought of Thee"), "Jesu dulcedo cordium" ("Jesu, Thou joy of loving hearts") and "Jesu Rex admirabilis" ("O Jesu, King most wonderful")-three portions of one poem, nearly 200 lines long. Pietro Damiani, the friend of Pope Gregory VII, Marbode, bishop of Rennes, in the 11th, Hildebert, archbishop of Tours, in the 12 th, and St Bonaventura in the 13th centuries, are other eminent men who added poetical fame as hymnographers to high public distinction.Before the time of the Reformation, the multiplication of sequences (often as unedifying in matter as unpoetical in style) had done much to degrade the common conception of hymnody. In some parts of France, Portugal, Sardinia and Bohemia, their use in the vernacular language had been allowed. In Germany also there were vernacular sequences as early as the 12th century, specimens of which may be seen in the third chapter of C. Winkworth's Christian Singers of Germany. Scoffing parodies upon sequences are said to have been among the means used in Scotland to discredit the old church services. After the 15th century they were discouraged at Rome. They retained for a time some of their old popularity among German Protestants, and were only gradually relinquished in France. A new "prose," in honour of St Maxentia, is among the compositions of Jean Baptiste Santeul; and Dr Daniel's second volume closes with one written in 1855 upon the dogma of the Immaculate Conception.

The taste of the Renaissance was offended by all deviations from classical prosody and Latinity. Pope Leo X. directed the whole body of the hymns in use at Rome to be reformed; and the Hymni novi ecclesiastici juxta veram metri et Latinitatis normam,

## Roman revision of hymns.

 prepared by Zacharie Ferreri (1479-1530), a Benedictine of Monte Cassino, afterwards a Carthusian and bishop of Guardia, to whom Leo had committed that task, appeared at Rome in 1525, with the sanction of a later pope, Clement VII. The next step was to revise the whole Roman Breviary. That undertaking, after passing through several stages under different popes (particularly Pius V. and Clement VIII.), was at last brought to a conclusion by Urban VIII., in 1631. From this revised Breviary a large number of medieval hymns, both of the earlier and the later periods, were excluded; and in their places many new hymns, including some by Pope Urban himself, and some by Cardinal Bellarmine and another cardinal (Silvius Antonianus) were introduced. The hymns of the primitive epoch, from Hilary to Gregory the Great, for the most part retained their places (especially in the offices for every day of the week); and there remained altogether from seventy to eighty of earlier date than the 11th century. Those, however, which were so retained were freely altered, and by no means generally improved. The revisers appointed by Pope Urban (three learned Jesuits-Strada, Gallucci and Petrucci) professed to have made "as few changes as possible" in the works of Ambrose, Gregory, Prudentius, Sedulius, Fortunatus and other "poets of great name." But some changes, even in those works, were made with considerable boldness; and the pope, in the "constitution" by which his new book was promulgated, boasted that, "with the exception of a very small number ('perpaucis'), which were either prose or merely rhythmical, all the hymns had been made conformable to the laws of prosody and Latinity, those which could not be corrected by any milder method being entirely rewritten." The latter fate befel, among others, the beautiful "Urbs beata Hierusalem," which now assumed the form (to many, perhaps, better known), of "Caelestis urbs Jerusalem." Of the "very few" which were spared, the chief were "Ave maris stella" ("Gentle star of ocean"), "Dies Irae," "Stabat Mater dolorosa," the hymns of Thomas Aquinas, two of St Bernard and one Ambrosian hymn, "Jesu nostra Redemptio" ("O Jesu, our Redemption"), which approaches nearer than others to the tone of St Bernard. A then recent hymn of St Francis Xavier, withscarcely enough merit of any kind to atone for its neglect of prosody, "O Deus, ego amo Te" ("O God, I love Thee, not because"), was at the same time introduced without change. This hymnary of Pope Urban VIII. is now in general use throughout the Roman Communion.

The Parisian hymnary underwent three revisions-the first in 1527, when a new "Psaltery with hymns" was issued. In this such changes only were made as the revisers thought justifiable upon the principle of correcting supposed corruptions of the original text. Of

## Parisian revisions.

 these, the transposition, "Urbs Jerusalem beata," instead of "Urbs beata Hierusalem," may be taken as a typical example. The next revision was in 1670-1680, under Cardinal Péréfixe, preceptor of Louis XIV., and Francis Harlay, successively archbishops of Paris, who employed for this purpose Claude Santeul, of the monastery of St Magloire, and, through him, obtained the assistance of other French scholars, including his more celebrated brother, Jean Baptiste Santeul, of the abbey of St Victor-better known as "Santolius Victorinus." The third and final revision was completed in 1735, under the primacy of Cardinal Archbishop de Vintimille, who engaged for it the services of Charles Coffin, then rector of the university of Paris. Many old hymns were omitted in Archbishop Harlay's Breviary, and a large number of new compositions, by the Santeuls and others, was introduced. It still, however, retained in their old places (without further changes than had been made in 1527) about seventy of earlier date than the 11th century-including thirty-one Ambrosian, one by Hilary, eight by Prudentius, seven by Fortunatus, three by Paul the Deacon, two each by Sedulius, Elpis, Gregory and Hrabanus Maurus, "Veni Creator" and "Urbs Jerusalem beata." Most of these disappeared in 1735, although Cardinal Vintimille, in his preface, professed to have still admitted the old hymns, except when the new were better-("veteribus hymnis locus datus est, nisi quibus, ob sententiarum vim, elegantiam verborum, et teneriores pietatis sensus, recentiores anteponi satius visum est"). The number of the new was, at the same time, very largely increased. Only twenty-one more ancient than the 16 th century remained, of which those belonging to the primitive epoch were but eight, viz. four Ambrosian, two by Fortunatus and one each by Prudentius and Gregory. The number of Jean Baptiste Santeul's hymns rose to eighty-nine; those by Coffin-including some old hymns, e.g. "Jam lucis orto sidere" ("Once more the sun is beaming bright"), which he substantially re-wrote-were eighty-three; those of other modern French writers, ninety-seven. Whatever opinion may be entertained of the principles on which these Roman and Parisian revisions proceeded, it would be unjust to deny very high praise as hymn-writers to several of their poets, especially to Coffin and Jean Baptiste Santeul. The noble hymn by Coffin, beginning-> "O luce qui mortalibus Lates inaccessa, Deus, Praesente quo sancti tremunt Nubuntque vultus angeli,"
and several others of his works, breathe the true Ambrosian spirit; and though Santeul (generally esteemed the better poet of the two) delighted in alcaics, and did not greatly affect the primitive manner, there can be no question as to the excellence of such hymns as his "Fumant Sabaeis templa vaporibus" ("Sweet incense breathes around"), "Stupete gentes, fit Deus hostia" ("Tremble, ye Gentile lands"), "Hymnis dum resonat curia caelitum" ("Ye in the house of heavenly morn"), and "Templi sacratas pande, Sion, fores" ("O Sion, open wide thy gates"). It is a striking testimony to the merits of those writers that such accomplished translators as the Rev. Isaac Williams and the Rev. John Chandler appear (from the title-page of the latter, and the prefaces of both) to have supposed their hymns to be "ancient" and "primitive." Among the other authors associated with them, perhaps the first place is due to the Abbé Besnault, of Sens, who contributed to the book of 1735 the "Urbs beata vera pacis Visio Jerusalem," in the opinion of Neale "much superior" to the "Caelestis urbs Jerusalem" of the Roman Breviary. This stood side by side with the "Urbs Jerusalem beata" of 1527 (in the office for the dedication of churches) till 1822, when the older form was at last finally excluded by Archbishop de Quelen.

The Parisian Breviary of 1735 remained in use till the national French service-books were superseded (as they have lately been, generally, if not universally) by the Roman. Almost all French dioceses followed, not indeed the Breviary, but the example, of Paris; and before the end of the 18th century the ancient Latin hymnody was all but banished from France.
In some parts of Germany, after the Reformation, Latin hymns continued to be used even by Protestants. This was the case at Halberstadt until quite a recent date. In England, a few are still occasionally used in the older universities and colleges. Some, also, have

## Modern Latin hymns.

 been composed in both countries since the Reformation. The "Carmina lyrica" of Johann Jakob Balde, a native of Alsace, and a Jesuit priest in Bavaria, have received high commendation from very eminent German critics, particularly Herder and Augustus Schlegel. Some of the Latin hymns of William Alard (1572-1645), a Protestant refugee from Belgium, and pastor in Holstein, have been thought worthy of a place in Archbishop Trench's selection. Two by W. Petersen (printed at the end of Haberkorn'ssupplement to Jacobi's Psalmodia Germanica) are good in different ways-one, "Jesu dulcis amor meus" ("Jesus, Thee my soul doth love"), being a gentle melody of spiritual devotion, and the other, entitled Spes Sionis, violently controversial against Rome. An English hymn of the 17th century, in the Ambrosian style, "Te Deum Patrem colimus" ("Almighty Father, just and good"), is sung on every May-Day morning by the choristers of Magdalen College, Oxford, from the top of the tower of their chapel; and another in the style of the Renaissance, of about the same date, "Te de profundis, summe Rex" ("Thee from the depths, Almighty King"), long formed part of a grace formerly sung by the scholars of Winchester College.
5. German Hymnody.-Luther was a proficient in and a lover of music. He desired (as he says in the preface to his hymn-book of 1545) that this "beautiful ornament" might "in a right manner serve the great Creator and His Christian people." The persecuted
Luther. Bohemian or Hussite Church, then settled on the borders of Moravia under the name of "United Brethren," had sent to him, on a mission in 1522, Michael Weiss, who not long afterwards published a number of German translations from old Bohemian hymns (known as those of the "Bohemian Brethren"), with some of his own. These Luther highly approved and recommended. He himself, in 1522, published a small volume of eight hymns, which was enlarged to 63 in 1527, and to 125 in 1545 . He had formed what he called a "house choir" of musical friends, to select such old and popular tunes (whether secular or ecclesiastical) as might be found suitable, and to compose new melodies, for church use. His fellow labourers in this field (besides Weiss) were Justus Jonas, his own especial colleague; Paul Eber, the disciple and friend of Melanchthon; John Walther, choirmaster successively to several German princes, and professor of arts, \&c., at Wittenberg; Nicholas Decius, who from a monk became a Protestant teacher in Brunswick, and translated the "Gloria in Excelsis," \&c.; and Paul Speratus, chaplain to Duke Albert of Prussia in 1525. Some of their works are still popular in Germany. Weiss's "Funeral Hymn," "Nun lasst uns den Leib begraben" ("Now lay we calmly in the grave"); Eber's "Herr Jesu Christ, wahr Mensch und Gott" ("Lord Jesus Christ, true Man and God"), and "Wenn wir in höchsten Nöthen sein" ("When in the hour of utmost need"); Walther's "New Heavens and new Earth" ("Now fain my joyous heart would sing"); Decius's "To God on high be thanks and praise"; and Speratus's "Salvation now has come for all," are among those which at the time produced the greatest effect, and are still best remembered.

Luther's own hymns, thirty-seven in number (of which about twelve are translations or adaptations from Latin originals), are for the principal Christian seasons; on the sacraments, the church, grace, death, \&c.; and paraphrases of seven psalms, of a passage in Isaiah, and of the Lord's Prayer, Ten Commandments, Creed, Litany and "Te Deum." There is also a very touching and stirring song on the martyrdom of two youths by fire at Brussels, in 1523-1524. Homely and sometimes rugged in form, and for the most part objective in tone, they are full of fire, manly simplicity and strong faith. Three rise above the rest. One for Christmas, "Vom Himmel hoch da komm ich her" ("From Heaven above to earth I come"), has a reverent tenderness, the influence of which may be traced in many later productions on the same subject. That on salvation through Christ, of a didactic character, "Nun freuet euch, lieben Christen g'mein" ("Dear Christian people, now rejoice"), is said to have made many conversions, and to have been once taken up by a large congregation to silence a Roman Catholic preacher in the cathedral of Frankfort. Pre-eminent above all is the celebrated paraphrase of the 46th Psalm: "Ein' feste Burg ist unser Gott" ("A sure stronghold our God is He")-"the production" (as Ranke says) "of the moment in which Luther, engaged in a conflict with a world of foes, sought strength in the consciousness that he was defending a divine cause which could never perish." Carlyle compares it to "a sound of Alpine avalanches, or the first murmur of earthquakes." Heine called it "the Marseillaise of the Reformation."
Luther spent several years in teaching his people at Wittenberg to sing these hymns, which soon spread over Germany. Without adopting the hyperbolical saying of Coleridge, that "Luther did as much for the Reformation by his hymns as by his translation of the Bible," it may truly be affirmed, that, among the secondary means by which the success of the Reformation was promoted, none was more powerful. They were sung everywhere-in the streets and fields as well as the churches, in the workshop and the palace, "by children in the cottage and by martyrs on the scaffold." It was by them that a congregational character was given to the new Protestant worship. This success they owed partly to their metrical structure, which, though sometimes complex, was recommended to the people by its ease and variety; and partly to the tunes and melodies (many of them already well known and popular) to which they were set. They were used as direct instruments of teaching, and were therefore, in a large measure, didactic and theological; and it may be partly owing to this cause that German hymnody came to deviate, so soon and so generally as it did, from the simple idea expressed in the ancient Augustinian definition, and to comprehend large classes of compositions which, in most other countries, would be thought hardly suitable for church use.
The principal hymn-writers of the Lutheran school, in the latter part of the 16 th century, were Nikolaus Selnecker, Herman and Hans Sachs, the shoemaker of Nuremberg, also known

## Followers of Luther

in other branches of literature. All these wrote some good hymns. They were succeeded by men of another sort, to whom F. A. Cunz gives the name of "master-singers," as having raised both the poetical and the musical standard of German hymnody:-Bartholomäus Ringwaldt, Ludwig Helmbold, Johannes Pappus, Martin Schalling, Rutilius and Sigismund Weingartner. The principal topics of their hymns (as if with some foretaste of the calamities which were soon to follow) were the vanity of earthly things, resignation to the Divine will, and preparation for death and judgment. The wellknown English hymn, "Great God, what do I see and hear," is founded upon one by Ringwaldt. Of a quite different character were two of great beauty and universal popularity, composed by Philip Nicolai, a Westphalian pastor, during a pestilence in 1597, and published by him, with fine chorales, two years afterwards. One of these (the "Sleepers wake! a voice is calling," of Mendelssohn's oratorio, St Paul) belongs to the family of Advent or New Jerusalem hymns. The other, a "Song of the believing soul concerning the Heavenly Bridegroom" ("Wie schön leucht't uns der Morgenstern"-"O morning Star, how fair and bright"), became the favourite marriage hymn of Germany.

The hymns produced during the Thirty Years' War are characteristic of that unhappy time, which (as Miss Winkworth says) "caused religious men to look away from this world," and made their songs more and more expressive of personal feelings. In point of

Period of Thirty Years' War. refinement and graces of style, the hymn-writers of this period excelled their predecessors. Their taste was chiefly formed by the influence of Martin Opitz, the founder of what has been called the "first Silesian school" of German poetry, who died comparatively young in 1639, and who, though not of any great original genius, exercised much power as a critic. Some of the best of these works were by men who wrote little. In the famous battle-song of Gustavus Adolphus, published (1631) after the victory of Breitenfeld, for the use of his army, "Verzage nicht du Häuflein klein" ("Fear not, O little flock, the foe"), we have almost certainly a composition of the hero-king himself, the versification corrected by his chaplain Jakob Fabricius (1593-1654) and the music composed by Michael Altenburg, whose name has been given to the hymn. This, with Luther's paraphrase of the 67th Psalm, was sung by Gustavus and his soldiers before the battle of Lützen in 1632. Two very fine hymns, one of prayer for deliverance and peace, the other of trust in God under calamities, were written about the same time by Matthäus Löwenstern, a saddler's son, poet, musician and statesman, who was ennobled after the peace by the emperor Ferdinand III. Martin Rinckhart, in 1636, wrote the "Chorus of God's faithful children" ("Nun danket alle Gott"-"Now thank we all our God"), introduced by Mendelssohn in his "Lobgesang," which has been called the "Te Deum" of Germany, being usually sung on occasions of public thanksgiving. Weissel, in 1635, composed a beautiful Advent hymn ("Lift up your heads, ye mighty gates"), and J. M. Meyfart, professor of theology at Erfurt, in 1642, a fine adaptation of the ancient "Urbs beata Hierusalem." The hymn of trust in Providence by George Neumark, librarian to that duke of Weimar ("Wer nur den lieben Gott lässt walten"-"Leave God to order all thy ways"), is scarcely, if at all, inferior to that of Paul Gerhardt on the same theme. Paul Flemming, a great traveller and lover of nature, who died in 1639, also wrote excellent compositions, coloured by the same tone of feeling; and some, of great merit, were composed, soon after the close of the war, by Louisa Henrietta, electress of Brandenburg, granddaughter of the famous admiral Coligny, and mother of the first king of Prussia. With these may be classed (though of later date) a few striking hymns of faith and prayer under mental anxiety, by Anton Ulrich, duke of Brunswick.

The most copious, and in their day most esteemed, hymn-writers of the first half of the 17th century, were Johann Heermann and Johann Rist. Heermann, a pastor in Silesia, the theatre (in a peculiar degree) of war and persecution, experienced in his own person a very large share of the miseries of the time, and several times narrowly escaped a violent death. His Devoti musica cordis, published in 1630, reflects

## Rist.

 the feelings natural under such circumstances. With a correct style and good versification, his tone is subjective, and the burden of his hymns is not praise, but prayer. Among his works (which enter largely into most German hymn-books), two of the best are the "Song of Tears" and the "Song of Comfort," translated by Miss Winkworth in her Christian Singers of Germany. Rist published about 600 hymns, "pressed out of him," as he said, "by the cross." He was a pastor, and son of a pastor, in Holstein, and lived after the peace to enjoy many years of prosperity, being appointed poet-laureate to the emperor and finally ennobled. The bulk of his hymns, like those of other copious writers, are of inferior quality; but some, particularly those for Advent, Epiphany, Easter Eve and on Angels, are very good. They are more objective than those of Heermann, and written, upon the whole, in a more manly spirit. NextDach. to Heermann and Rist in fertility of production, and above them in poetical genius, was Simon Dach, professor of poetry at Königsberg, who died in 1659. Miss Winkworth ranks him high among German poets, "for the sweetness of form and depth of tender contemplative emotion to be found in his verses."

The fame of all these writers was eclipsed in the latter part of the same century by three of the greatest hymnographers whom Germany has produced-Paul Gerhardt (1604-1676), Johann Franck (1618-1677) and Johann Scheffler (1624-1677), the founder of the Gerhardt. "second Silesian school," who assumed the name of "Angelus Silesius." Gerhardt is by universal consent the prince of Lutheran poets. His compositions, which may be compared, in many respects, to those of the Christian Year, are lyric poems, of considerable length, rather than hymns, though many hymns have been taken from them. They are, with few exceptions, subjective, and speak the language of individual experience. They occupy a middle ground between the masculine simplicity of the old Lutheran style and the highly wrought religious emotion of the later pietists, towards whom they on the whole incline. Being nearly all excellent, it is not easy to distinguish among the 123 those which are entitled to the highest praise. Two, which were written one during the war and the other after the conclusion of peace, "Zeuch ein zu deinen Thoren" ("Come to Thy temple here on earth"), and "Gottlob, nun ist erschollen" ("Thank God, it hath resounded"), are historically interesting. Of the rest, one is well known and highly appreciated in English through Wesley's translation, "Commit thou all thy ways"; and the evening and spring-tide hymns ("Now all the woods are sleeping" and "Go forth, my heart, and seek delight") show an exquisite feeling for nature; while nothing can be more tender and pathetic than "Du bist zwar mein und bleibest mein" ("Thou'rt mine, yes, still thou art mine own"), on the death of his son.

## Franck.

 Franck, who was burgomaster of Guben in Lusatia, has been considered by some second only to Gerhardt. If so, it is with a great distance between them. His approach to the later pietists is closer than that of Gerhardt. His hymns were published, under the title of Geistliche und weltliche Gedichte, in 1674, some of them being founded on Ambrosian and other Latin originals. Miss Winkworth gives them the praise of a condensed and polished style and fervid and impassioned thought. It was after his conversion to Roman Catholicism that Scheffler adopted the name of "Angelus Silesius," and Scheffler. published in 1657 his hymns, under a fantastic title, and with a still more fantastic preface. Their keynote is divine love; they are enthusiastic, intense, exuberant in their sweetness, like those of St Bernard among medieval poets. An adaptation of one of them, by Wesley, "Thee will I love, my Strength, my Tower," is familiar to English readers. Those for the first Sunday after Epiphany, for Sexagesima Sunday and for Trinity Sunday, in Lyra Germanica, are good examples of his excellences, with few of his defects. His hymns are generally so free from the expression, or even the indirect suggestion, of Roman Catholic doctrine, that it has been supposed they were written before his conversion, though published afterwards. The evangelical churches of Germany found no difficulty in admitting them to that prominent place in their services which they have ever since retained.Towards the end of the 17 th century, a new religious school arose, to which the name of "Pietists" was given, and of which Philipp Jakob Spener was esteemed the founder. He and his pupils and successors, August Hermann Francke and Anastasius
Pietists. Freylinghausen, all wrote hymns. Spener's hymns are not remarkable, and Francke's are not numerous. Freylinghausen was their chief singer; his rhythm is lively, his music florid; but, though his book attained extraordinary popularity, he was surpassed in solid merit by other less fertile writers of the same school. The "Auf hinauf zu deiner Freude" ("Up, yes, upward to thy gladness") of Schade may recall to an English reader a hymn by Seagrave, and more than one by Lyte; the "Malabarian hymn" (as it was called by Jacobi) of Johann Schütz, "All glory to the Sovereign Good," has been popular in England as well as Germany; and one of the most exquisite strains of pious resignation ever written is "Whate'er my God ordains is right," by Samuel Rodigast.

Joachim Neander, a schoolmaster at Düsseldorf, and a friend of Spener and Schütz (who died before the full development of the "Pietistic" school), was the first man of eminence in the "Reformed" or Calvinistic Church who imitated Lutheran hymnody. This he

## Neander.

 did, while suffering persecution from the elders of his own church for some other religious practices, which he had also learnt from Spener's example. As a poet, he is sometimes deficient in art; but there is feeling, warmth and sweetness in many of his "Bundeslieder" or "Songs of the Covenant," and they obtained general favour, both in the Reformed and in Lutheran congregations. The Summer Hymn ("O Thou true God alone") and that on the glory of God in creation ("Lo, heaven and earth and sea and air") are instances of his best style.With the "Pietists" may be classed Benjamin Schmolke and Dessler, representatives of the "Orthodox" division of Spener's school; Philipp Friedrich Hiller, their leading poet in South Germany; Gottfried Arnold and Gerhard Tersteegen, who were practically
Schmolke. independent of ecclesiastical organization, though connected, one with the "Orthodox" and the other with the "Reformed" churches; and Nikolaus Ludwig, Graf von Zinzendorf. Schmolke, a pastor in Silesia, called the Silesian Rist (16721737), was perhaps the most voluminous of all German hymn-writers. He wrote 1188 religious
poems and hymns, a large proportion of which do not rise above mediocrity. His style, if less refined, is also less subjective and more simple than that of most of his contemporaries. Among his best and most attractive works, which indeed, it would be difficult to praise too highly, are the "Hosianna David's Sohn," for Palm Sunday-much resembling a shorter hymn by Jeremy Taylor; and the Ascension, Whitsuntide and Sabbath hymns-"Heavenward doth our journey tend," "Come deck our feast to-day," and "Light of light, enlighten me."

## Dessler.

 Hiller. Dessler was a greater poet than Schmolke. Few hymns, of the subjective kind, are better than his "I will not let Thee go, Thou Help in time of need," "O Friend of souls, how well is me," and "Now, the pearly gates unfold." Hiller (1699-1769), was a pastor in Württemberg who, falling into ill-health during the latter part of his ministry, published a Geistliche Liederhöstlein in a didactic vein, with more taste than power, but (as Miss Winkworth says) in a tone of "deep, thoughtful, practical piety." They were so well adapted to the wants of his people that to this day Hiller's Casket is prized, next to their Bibles, by the peasantry of Württemberg; and the numerous emigrants from that part of Germany to America and other foreign countries generally take it with them
## Arnold.

 wherever they go. Arnold, a professor at Giessen, and afterwards a pastor in Brandenburg, was a man of strong will, uncompromising character and austere views of life, intolerant and controversial towards those whose doctrine or practice he disapproved, and more indifferent to separatism and sectarianism than the "orthodox" generally thought right. His hymns, like those of Augustus M. Toplady, whom in these respects he resembled, unite with considerable strength more gentleness and breadth of sympathy than might be expected from a man of such a character. Tersteegen (1697-1769),
## Tersteegen.

 who never formally separated himself from the "Reformed" communion, in which he was brought up, but whose sympathies were with the Moravians and with Zinzendorf, was, of all the more copious German hymn-writers after Luther, perhaps the most remarkable man. Pietist, mystic and missionary, he was also a great religious poet. His 111 hymns were published In 1731, in a volume called Geistlicher Blumengärtlein inniger Seelen. They are intensely individual, meditative and subjective. Wesley's adaptations of two -"Lo! God is here; let us adore," and "Thou hidden Love of God, whose source"-are well known. Among those translated by Miss Winkworth, "O God, O Spirit, Light of all that live," and "Come, brethren, let us go," are specimens which exhibit favourably his manner and power. Miss Cox speaks of him as "a gentle heaven-inspired soul, whose hymns are the reflection of a heavenly, happy life, his mind being full of a child-like simplicity"; and his own poem on the child-character, which Miss Winkworth has appropriately connected with Innocents' day ("Dear Soul, couldst thou become a child")-one of his best compositions, exquisitely conceived and expressed-shows that this was in truth the ideal which he sought to realize. The hymns of Zinzendorf are often disfigured by excess in the application of the language
## Zinzendorf.

 and imagery of human affections to divine objects; and this blemish is also found in many later Moravian hymns. But one hymn, at least, of Zinzendorf may be mentioned with unqualified praise, as uniting the merits of force, simplicity and brevity -"Jesu, geh voran" ("Jesus, lead the way"), which is taught to most children of religious parents in Germany. Wesley's "Jesus, Thy blood and righteousness" is a translation from Zinzendorf.The transition from Tersteegen and Zinzendorf to Gellert and Klopstock marks strongly the reaction against Pietism which took place towards the middle of the 18th century. The Geistlichen Oden und Lieder of Christian F. Gellert were published in 1757,

## Gellert.

 and are said to have been received with an enthusiasm almost like that which "greeted Luther's hymns on their first appearance." It is a proof of the moderation both of the author and of his times that they were largely used, not only by Protestant congregations, but in those German Roman Catholic churches in which vernacular services had been established through the influence of the emperor Joseph II. They became the model which was followed by most succeeding hymn-writers, and exceeded all others in popularity till the close of the century, when a new wave of thought was generated by the movement which produced the French Revolution. Since that time they have been, perhaps, too much depreciated. They are, indeed, cold and didactic, as compared with Scheffler or Tersteegen; but there is nevertheless in them a spirit of genuine practical piety; and, if not marked by genius, they are pure in taste, and often terse, vigorous and graceful.Klopstock, the author of the Messiah, cannot be considered great as a hymn-writer, though his "Sabbath Hymn" (of which there is a version in Hymns from the Land of
Klopstock. Luther) is simple and good. Generally his hymns (ten of which are translated in Sheppard's Foreign Sacred Lyre) are artificial and much too elaborate.

Of the "romantic" school, which came in with the French Revolution, the two leading writers are Friedrich Leopold von Hardenberg, called "Novalis," and Friedrich de la Motte Fouqué, the celebrated author of Undine and Sintram-both romance-writers, as well as poets. The genius of Novalis was early lost to the world; he died in 1801, not thirty years old. Some of his hymns
are very beautiful; but even in such works as "Though all to Thee were faithless," and "If only He is mine," there is a feeling of insulation and of despondency as to good in the actual world, which was perhaps inseparable from his ecclesiastical idealism. Fouqué

## Fouqué.

 survived till 1843. In his hymns there is the same deep flow of feeling, richness of imagery and charm of expression which distinguishes his prose works. The two missionary hymns-"Thou, solemn Ocean, rollest to the strand," and "In our sails all soft and sweetly"-and the exquisite composition which finds its motive in the gospel narrative of blind Bartimeus, "Was du vor tausend Jahren" (finely translated both by Miss Winkworth and by Miss Cox), are among the best examples.The later German hymn-writers of the 19th century belong, generally, to the revived "Pietistic" school. Some of the best, Johann Baptist von Albertini, Friedrich Adolf Krummacher, and especially Karl Johann Philipp Spitta (1801-1859) have produced works
Spitta. not unworthy of the fame of their nation. Mr Massie, the able translator of Spitta's Psalter und Harfe (Leipzig, 1833), speaks of it as having "obtained for him in Germany a popularity only second to that of Paul Gerhardt." In Spitta's poems (for such they generally are, rather than hymns) the subjective and meditative tone is tempered, not ungracefully, with a didactic element; and they are not disfigured by exaggerated sentiment, or by a too florid and rhetorical style.
6. British Hymnody.-After the Reformation, the development of hymnody was retarded, in both parts of Great Britain, by the example and influence of Geneva. Archbishop Cranmer appears at one time to have been disposed to follow Luther's course, and to present to the people, in an English dress, some at least of the hymns of the ancient church. In a letter to King Henry VIII. (October 7, 1544), among some new "processions" which he had himself translated, into English, he mentions the Easter hymn, "Salve, festa dies, toto memorabilis aevo" ("Hail, glad day, to be joyfully kept through all generations"), of Fortunatus. In the "Primer" of 1535 (by Marshall) and the one of 1539 (by Bishop Hilsey of Rochester, published by order of the vicar-general Cromwell) there had been several rude English hymns, none of them taken from ancient sources. King Henry's "Primer" of 1545 (commanded by his injunction of the 6th of May 1545 to be used throughout his dominions) was formed on the model of the daily offices of the Breviary; and it contains English metrical translations from some of the best-known Ambrosian and other early hymns. But in the succeeding reign different views prevailed. A new direction had been given to the taste of the "Reformed" congregations in France and Switzerland by the French metrical translation of the Old Testament Psalms, which appeared about 1540. This was the joint work of Clement Marot, valet or groom of the chamber to Francis I., and Theodore Beza, then a mere youth, fresh from his studies at Orleans.

Marot's psalms were dedicated to the French king and the ladies of France, and, being set to popular airs, became fashionable. They were sung by Francis himself, the queen, the princesses and the courtiers, upon all sorts of secular occasions, and also, more seriously

## Marot's

 Psalms. and religiously, by the citizens and the common people. They were soon perceived to be a power on the side of the Reformation. Calvin, who had settled at Geneva in the year of Marot's return to Paris, was then organizing his ecclesiastical system. He rejected the hymnody of the breviaries and missals, and fell back upon the idea, anciently held by Paul of Samosata, and condemned by the fourth council of Toledo, that whatever was sung in churches ought to be taken out of the Scriptures. Marot's Psalter, appearing thus opportunely, was introduced into his new system of worship, and appended to his catechism. On the other hand, it was interdicted by the Roman Catholic priesthood. Thus it became a badge to the one party of the "reformed" profession, and to the other of heresy.The example thus set produced in England the translation commonly known as the "Old Version" of the Psalms. It was begun by Thomas Sternhold, whose position in the household of Henry VIII., and afterwards of Edward VI., was similar to that of Marot with

## Sternhold and Hopkins.

 Francis I., and whose services to the former of those kings were rewarded by a substantial legacy under his will. Sternhold published versions of nineteen Psalms, with a dedication to King Edward, and died soon afterwards. A second edition appeared in 1551, with eighteen more Psalms added, of Sternhold's translating, and seven others by John Hopkins, a Suffolk clergyman. The work was continued during Queen Mary's reign by British refugees at Geneva, the chief of whom were William Whittingham, afterwards dean of Durham, who succeeded John Knox as minister of the English congregation there, and William Kethe or Keith, said by Strype to have been a Scotsman. They published at Geneva in 1556 a service-book, containing fifty-one English metrical psalms, which number was increased, in later editions, to eighty-seven. On the accession of Queen Elizabeth, this Genevan Psalmody was at once brought into use in England-first (according to a letter of Bishop Jewell to Peter Martyr, dated 5th March 1560) in one London church, from which it quickly spread to others both in London and in other cities. Jewell describes the effect produced by largecongregations, of as many as 6000 persons, young and old, women and children, singing it after the sermons at St Paul's Cross-adding, "Id sacrificos et diabolum aegre habet; vident enim sacras conciones hoc pacto profundius descendere in hominum animos." The first edition of the completed "Old Version" (containing forty Psalms by Sternhold, sixty-seven by Hopkins, fifteen by Whittingham, six by Kethe and the rest by Thomas Norton the dramatist, Robert Wisdom, John Marckant and Thomas Churchyard) appeared in 1562.

In the meantime, the Books of Common Prayer, of 1549, 1552 and 1559, had been successively established as law by the acts of uniformity of Edward VI. and Queen Elizabeth. In these no provision was made for the use of any metrical psalm or hymn on any occasion whatever, except at the consecration of bishops and the ordination of priests, in which offices (first added in 1552) an English version of "Veni Creator" (the longer of the two now in use) was appointed to be "said or sung." The canticles, "Te Deum," "Benedicite," the Nicene and Athanasian Creeds, the "Gloria in Excelsis," and some other parts of the communion and other special offices were also directed to be "said or sung"; and, by general rubrics, the chanting of the whole service was allowed.

The silence, however, of the rubrics in these books as to any other singing was not meant to exclude the use of psalms not expressly appointed, when they could be used without interfering with the prescribed order of any service. It was expressly provided by King Edward's first act of uniformity (by later acts made applicable to the later books) that it should be lawful "for all men, as well in churches, chapels, oratories or other places, to use openly any psalms or prayers taken out of the Bible, at any due time, not letting or omitting thereby the service, or any part thereof, mentioned in the book." And Queen Elizabeth, by one of the injunctions issued in the first year of her reign, declared her desire that the provision made, "in divers collegiate and also some parish churches, for singing in the church, so as to promote the laudable service of music," should continue. After allowing the use of "a modest and distinct song in all parts of the common prayers of the church, so that the same may be as plainly understanded as if it were read without singing," the injunction proceeded thus-"And yet, nevertheless, for the comforting of such that delight in music, it may be permitted that in the beginning or in the end of the Common Prayer, either at morning or evening, there may be sung an hymn, or such like song to the praise of Almighty God, in the best sort of melody and music that may be conveniently devised, having respect that the sentence" (i.e. sense) "of hymn may be understanded and perceived."

The "Old Version," when published (by John Daye, for the Stationers' Company, "cum gratia et privilegio Regiae Majestatis"), bore upon the face of it that it was "newly set forth, and allowed to be sung of the people in churches, before and after morning and evening prayer, as also before and after the sermon." The question of its authority has been at different times much debated, chiefly by Peter Heylyn and Thomas Warton on one side (both of whom disliked and disparaged it), and by William Beveridge, bishop of St Asaph, and the Rev. H. J. Todd on the other. Heylyn says, it was "permitted rather than allowed," which seems to be a distinction without much difference. "Allowance," which is all that the book claimed for itself, is authorization by way of permission, not of commandment. Its publication in that form could hardly have been licensed, nor could it have passed into use as it did without question, throughout the churches of England, unless it had been "allowed" by some authority then esteemed to be sufficient. Whether that authority was royal or ecclesiastical does not appear, nor (considering the proviso in King Edward's act of uniformity, and Queen Elizabeth's injunctions) is it very important. No inference can justly be drawn from the inability of inquirers, in Heylyn's time or since, to discover any public record bearing upon this subject, many public documents of that period having been lost.
In this book, as published in 1562, and for many years afterwards, there were (besides the versified Psalms) eleven metrical versions of the "Te Deum," canticles, Lord's Prayer (the best of which is that of the "Benedicite"); and also "Da pacem, Domine," a hymn suitable to the times, rendered into English from Luther; two original hymns of praise, to be sung before morning and evening prayer; two penitential hymns (one of them the "humble lamentation of a sinner"); and a hymn of faith, beginning, "Lord, in Thee is all my trust." In these respects, and also in the tunes which accompanied the words (stated by Dr Charles Burney, in his History of Music, to be German, and not French), there was a departure from the Genevan platform. Some of these hymns, and some of the psalms also (e.g. those by Robert Wisdom, being alternative versions), were omitted at a later period; and many alterations and supposed amendments were from time to time made by unknown hands in the psalms which remained, so that the text, as now printed, is in many places different from that of 1562.

In Scotland, the General Assembly of the kirk caused to be printed at Edinburgh in 1564, and enjoined the use of, a book entitled The Form of Prayers and Ministry of the

Scotch
Psalms. Sacraments used in the English Church at Geneva, approved and received by the Church of Scotland; whereto, besides that was in the former books, are also added sundry other prayers, with the whole Psalms of David in English
by Whittingham, twenty-six by Kethe and thirty-five by Hopkins. Of the remainder two were by John Pulleyn (one of the Genevan refugees, who became archdeacon of Colchester); six by Robert Pont, Knox's son-in-law, who was a minister of the kirk, and also a lord of session; and fourteen signed with the initials I. C., supposed to be John Craig; one was anonymous, eight were attributed to N., two to M. and one to T. N. respectively.

So matters continued in both churches until the Civil War. During the interval, King James I. conceived the project of himself making a new version of the Psalms, and appears to have translated thirty-one of them-the correction of which, together with the translation of the rest, he entrusted to Sir William Alexander, afterwards earl of Stirling. Sir William having completed his task, King Charles I. had it examined and approved by several archbishops and bishops of England, Scotland and Ireland, and caused it to be printed in 1631 at the Oxford University Press, as the work of King James; and, by an order under the royal sign manual, recommended its use in all churches of his dominions. In 1634 he enjoined the Privy Council of Scotland not to suffer any other psalms, "of any edition whatever," to be printed in or imported into that kingdom. In 1636 it was republished, and was attached to the famous Scottish service-book, with which the troubles began in 1637. It need hardly be added that the king did not succeed in bringing this Psalter into use in either kingdom.

When the Long Parliament undertook, in 1642, the task of altering the liturgy, its attention was at the same time directed to psalmody. It had to judge between two rival translations of the Psalms-one by Francis Rouse, a member of the House of Commons, afterwards one of Cromwell's councillors and finally provost of Eton; the other by William Barton, a clergyman of Leicester. The House of Lords favoured Barton, the House of Commons Rouse, who had made much use of the labours of Sir William Alexander. Both versions were printed by order of parliament, and were referred for consideration to the Westminster Assembly. They decided in favour of Rouse. His version, as finally amended, was published in 1646, under an order of the House of Commons dated 14th November 1645. In the following year it was recommended by the parliament to the General Assembly at Edinburgh, who appointed a committee, with large powers, to prepare a revised Psalter, recommending to their consideration not only Rouse's book but that of 1564, and two other versions (by Zachary Boyd and Sir William Mure of Rowallan), then lately executed in Scotland. The result of the labours of this committee was the "Paraphrase" of the Psalms, which, in 1649-1650, by the concurrent authority of the General Assembly and the committee of estates, was ordered to be exclusively used throughout the church of Scotland. Some use was made in the preparation of this book of the versions to which the attention of the revisers had been directed, and also of Barton's; but its basis was that of Rouse. It was received in Scotland with great favour, which it has ever since retained; and it is fairly entitled to the praise of striking a tolerable medium between the rude homeliness of the "Old," and the artificial modernism of the "New" English versions-perhaps as great a success as was possible for such an undertaking. Sir Walter Scott is said to have dissuaded any attempt to alter it, and to have pronounced it, "with all its acknowledged occasional harshness, so beautiful, that any alterations must eventually prove only so many blemishes." No further step towards any authorized hymnody was taken by the kirk of Scotland till the following century.

In England, two changes bearing on church hymnody were made upon the revision of the prayer-book after the Restoration, in 1661-1662. One was the addition, in the offices for consecrating bishops and ordaining priests, of the shorter version of "Veni Creator" ("Come, Holy Ghost, our souls inspire"), as an alternative form. The other, and more important, was the insertion of the rubric after the third collect, at morning and evening prayer: "In quires and places where they sing, here followeth the anthem." By this rubric synodical and parliamentary authority was given for the interruption, at that point, of the prescribed order of the service by singing an anthem, the choice of which was left to the discretion of the minister. Those actually used, under this authority, were for some time only unmetrical passages of scripture, set to music by Blow, Purcell and other composers, of the same kind with the anthems still generally sung in cathedral and collegiate churches. But the word "anthem" had no technical signification which could be an obstacle to the use under this rubric of metrical hymns.

The "New Version" of the Psalms, by Dr Nicholas Brady and the poet-laureate Nahum Tate (both Irishmen), appeared in 1696, under the sanction of an order in council of William III., "allowing and permitting" its use "in all such churches, chapels and Tate and congregations as should think fit to receive it." Dr Compton, bishop of London, Brady. recommended it to his diocese. No hymns were then appended to it; but the authors added a "supplement" in 1703, which received an exactly similar sanction from an order in council of Queen Anne. In that supplement there were several new versions of the canticles, and of the "Veni Creator"; a variation of the old "humble lamentation of a sinner"; six hymns for Christmas, Easter and Holy Communion (all versions or paraphrases of scripture), which are still usually printed at the end of the prayer-books containing the new version; and a hymn "on the divine use of music"-all accompanied by tunes. The authors also
reprinted, with very good taste, the excellent version of the "Benedicite" which appeared in the book of 1562 . Of the hymns in this "supplement," one ("While shepherds watched their flocks by night") greatly exceeded the rest in merit. It has been ascribed to Tate, but it has a character of simplicity unlike the rest of his works.

The relative merits of the "Old" and "New" versions have been very variously estimated. Competent judges have given the old the praise, which certainly cannot be accorded to the new, of fidelity to the Hebrew. In both, it must be admitted, that those parts

## Old and new versions compared.

 which have poetical merit are few and far between; but a reverent taste is likely to be more offended by the frequent sacrifice, in the new, of depth of tone and accuracy of sense to a fluent commonplace correctness of versification and diction, than by any excessive homeliness in the old. In both, however, some psalms, or portions of psalms, are well enough rendered to entitle them to a permanent place in the hymn-books-especially the 8th, and parts of the 18th Psalm, by Sternhold; the 57th, 84th and 100th, by Hopkins; the 23rd, 34th and 36th, and part of the 148th, by Tate and Brady.The judgment which a fastidious critic might be disposed to pass upon both these books may perhaps be considerably mitigated by comparing them with the works of other labourers in the same field, of whom Holland, in his interesting volumes entitled Psalmists of Great Britain, enumerates above 150. Some of them have been real poets-the celebrated earl of Surrey, Sir Philip Sidney and his sister the countess of Pembroke, George Sandys, George Wither, John Milton and John Keble. In their versions, as might be expected, there are occasional gleams of power and beauty, exceeding anything to be found in Sternhold and Hopkins, or Tate and Brady; but even in the best these are rare, and chiefly occur where the strict idea of translation has been most widely departed from. In all of them, as a rule, the life and spirit, which in prose versions of the psalms are so wonderfully preserved, have disappeared. The conclusion practically suggested by so many failures is that the difficulties of metrical translation, always great, are in this case insuperable; and that, while the psalms like other parts of scripture are abundantly suggestive of motive and material for hymnographers, it is by assimilation and adaptation, and not by any attempt to transform their exact sense into modern poetry, that they may be best used for this purpose.

The order in council of 1703 is the latest act of any public authority by which an express sanction has been given to the use of psalms or hymns in the Church of England. At the end, indeed, of many Prayer-books, till about the middle of the 19th century, there were commonly found, besides some of the hymns sanctioned by that order in council, or of those contained in the book of 1562, a sacramental and a Christmas hymn by Doddridge; a Christmas hymn (varied by Martin Madan) from Charles Wesley; an Easter hymn of the 18th century, beginning "Jesus Christ has risen to-day"; and abridgments Bishop Ken's Morning and Evening Hymns. These additions first began to be made in or about 1791, in London editions of the Prayer-book and Psalter, at the mere will and pleasure (so far as appears) of the printers. They had no sort of authority.

In the state of authority, opinion and practice disclosed by the preceding narrative may be found the true explanation of the fact that, in the country of Chaucer, Spenser, Shakespeare and Milton, and notwithstanding the example of Germany, no native English congregational hymnody worthy of the name arose till after the congregational commencement of the 18th century. Yet there was no want of appreciation of hymnody. the power and value of congregational church music. Milton could write, before 1645:-
"There let the pealing organ blow
To the full-voiced quire below
In service high, and anthems clear,
As may with sweetness through mine ear
Dissolve me into ecstasies,
And bring all Heaven before mine eyes."

Thomas Mace, in his Music's Monument (1676), thus described the effect of psalm-singing before sermons by the congregation in York Minster on Sundays, during the siege of 1644: "When that vast concording unity of the whole congregational chorus came thundering in, even so as it made the very ground shake under us, oh, the unutterable ravishing soul's delight! in the which I was so transported and wrapt up in high contemplations that there was no room left in my whole man, body, soul and spirit, for anything below divine and heavenly raptures; nor could there possibly be anything to which that very singing might be truly compared, except the right apprehension or conceiving of that glorious and miraculous quire, recorded in the scriptures at the dedication of the temple." Nor was there any want of men well qualified, and by the turn of their minds predisposed, to shine in this branch of literature. Some (like

Sandys, Boyd and Barton) devoted themselves altogether to paraphrases of other scriptures as well as the psalms. Others (like George Herbert, and Francis and John Quarles) moralized, meditated, soliloquized and allegorized in verse. Without reckoning these, there were a few, even before the Restoration, who came very near to the ideal of hymnody.

First in time is the Scottish poet John Wedderburn, who translated several of Luther's hymns, and in his Compendious Book of Godly and Spiritual Songs added others of his own (or his brothers') composition. Some of these poems, published before 1560, are of
Wedderburn. uncommon excellence, uniting ease and melody of rhythm, and structural skill, with grace of expression, and simplicity, warmth and reality of religious feeling. Those entitled "Give me thy heart," "Go, heart," and "Leave me not," which will be found in a collection of 1860 called Sacred Songs of Scotland, require little, beyond the change of some archaisms of language, to adapt them for church or domestic use at the present day.

Next come the two hymns of "The new Jerusalem," by an English Roman Catholic priest signing himself F. B. P. (supposed to be "Francis Baker, Presbyter"), and by another Scottish poet, David Dickson, of which the history is given by Dr Bonar in his edition of

## Dickson.

 Dickson's work. This (Dickson's), which begins "O mother dear, Jerusalem," and has long been popular in Scotland, is a variation and amplification by the addition of a large number of new stanzas of the English original, beginning "Jerusalem, my happy home," written in Queen Elizabeth's time, and printed (as appears by a copy in the British Museum) about 1616, when Dickson was still young. Both have an easy natural flow, and a simple happy rendering of the beautiful scriptural imagery upon the subject, with a spirit of primitive devotion uncorrupted by medieval peculiarities. The English hymn of which some stanzas are now often sung in churches is the true parent of the several shorter forms,-all of more than common merit,-which, in modern hymn-books, begin with the same first line, but afterwards deviate from the original. Kindred to these is the very fine and faithful translation, by Dickson's contemporary Drummond of Hawthornden of the ancient "Urbs beata Hierusalem" ("Jerusalem, that place divine"). Other ancient hymns (two of Thomas Aquinas, and the "Dies Irae") were also well translated, in 1646, by Richard Crashaw, after he had become a Roman Catholic and had been deprived by the parliament of his fellowship at Cambridge.Conspicuous among the sacred poets of the first two Stuart reigns in England was George Wither. His Hymnes and Songs of the Church appeared in 1622-1623, under a patent of King James I., by which they were declared "worthy and profitable to be inserted, in

## Wither.

 convenient manner and due place, into every English Psalm-book to metre." His Hallelujah (in which some of the former Hymnes and Songs were repeated) followed in 1641. Some of the Hymnes and Songs were set to music by Orlando Gibbons, and those in both books were written to be sung, though there is no evidence that the author contemplated the use of any of them in churches. They included hymns for every day in the week (founded, as those contributed nearly a century afterwards by Charles Coffin to the Parisian Breviary also were, upon the successive works of the days of creation); hymns for all the church seasons and festivals, including saints' days; hymns for various public occasions; and hymns of prayer, meditation and instruction, for all sorts and conditions of men, under a great variety of circumstances-being at once a "Christian Year" and a manual of practical piety. Many of them rise to a very high point of excellence,-particularly the "general invitation to praise God" ("Come, O come, in pious lays"), with which Hallelujah opens; the thanksgivings for peace and for victory, the Coronation Hymn, a Christmas, an Epiphany, and an Easter Hymn, and one for St Bartholomew's day (Hymns 1, 74, 75, and 84 in part i., and 26, 29, 36 and 54 in part ii. of Hallelujah).John Cosin, afterwards bishop of Durham, published in 1627 a volume of "Private Devotions," for the canonical hours and other occasions. In this there are seven or eight
Cosin. hymns of considerable merit,-among them a very good version of the Ambrosian "Jam lucis orto sidere," and the shorter version of the "Veni Creator," which was introduced after the Restoration into the consecration and ordination services of the Church of England.

The hymns of Milton (on the Nativity, Passion, Circumcision and "at a
Milton. Solemn Music"), written about 1629, in his early manhood, were probably not intended for singing; but they are odes full of characteristic beauty and power.

During the Commonwealth, in 1654, Jeremy Taylor published at the end of his Golden Grove, twenty-one hymns, described by himself as "celebrating the mysteries and chief festivals of the year, according to the manner of the ancient church, fitted to the fancy and

Jeremy
Taylor. devotion of the younger and pious persons, apt for memory, and to be joined, to their other prayers." Of these, his accomplished editor, Bishop Heber, justly says:-
metre, indeed, which is that species of spurious Pindaric which was fashionable with his contemporaries, is an obstacle, and must always have been one, to their introduction into public or private psalmody; and the mixture of that alloy of conceits and quibbles which was an equally frequent and still greater defilement of some of the finest poetry of the 17th century will materially diminish their effect as devotional or descriptive odes. Yet, with all these faults, they are powerful, affecting, and often harmonious; there are many passages of which Cowley need not have been ashamed, and some which remind us, not disadvantageously, of the corresponding productions of Milton."

He mentions particularly the advent hymn ("Lord, come away"), part of the hymn "On heaven," and (as "more regular in metre, and in words more applicable to public devotion") the "Prayer for Charity" ("Full of mercy, full of love").

The epoch of the Restoration produced in 1664 Samuel Crossman's Young Man's Calling, with a few "Divine Meditations" in verse attached to it; in 1668 John Austin's

## Restoration period.

 Devotions in the ancient way of offices, with psalms, hymns and prayers for every day in the week and every holyday in the year, and in 1681 Richard Baxter's Poetical Fragments. In these books there are altogether seven or eight hymns, the whole or parts of which are extremely good: Crossman's "New Jerusalem" ("Sweet place, sweet place alone"), one of the best of that class, and "My life's a shade, my days"; Austin's "Hark, my soul, how everything," "Fain would my thoughts fly up to Thee," "Lord, now the time returns," "Wake all my hopes, lift up your eyes"; and Baxter's "My whole, though broken heart, O Lord," and "Ye holy angels bright." Austin's Offices (he was a Roman Catholic) seem to have attracted much attention. Theophilus Dorrington, in 1686, published variations of them under the title of Reformed Devotions; George Hickes, the non-juror, wrote one of his numerous recommendatory prefaces to S. Hopton's edition; and the Wesleys, in their earliest hymn-book, adopted hymns from them, with little alteration. These writers were followed by John Mason in 1683, and Thomas Shepherd in 1692,-the former, a country clergyman, much esteemed by Baxter and other Nonconformists; the latter himself a Nonconformist, who finally emigrated to America. Between these two men there was a close alliance, Shepherd's Penitential Cries being published as an addition to the Spiritual Songs of Mason. Their hymns came into early use in several Nonconformist congregations; but, with the exception of one by Mason ("There is a stream which issues forth"), they are not suitable for public singing. In those of Mason there is often a very fine vein of poetry; and later authors have, by extracts or centoes from different parts of his works (where they were not disfigured by his general quaintness), constructed several hymns of more than average excellence.Three other eminent names of the 17th century remain to be mentioned, John Dryden, Bishop Ken and Bishop Simon Patrick; with which may be associated that of Addison, though he wrote in the 18th century.

Dryden's translation of "Veni Creator" a cold and laboured performance, is to be met with in many hymn-books. Abridgments of Ken's morning and evening hymns are in all. These, with the midnight hymn, which is not inferior to them, first appeared In 1697, appended to the third edition of the author's Manual of Prayers for Winchester Scholars. Between
Dryden, Ken. these and a large number of other hymns (on the attributes of God, and for the festivals of the church) published by Bishop Ken after 1703 the contrast is remarkable. The universal acceptance of the morning and evening hymns is due to their transparent simplicity, warm but not overstrained devotion, and extremely popular style. Those afterwards published have no such qualities. They are mystical, florid, stiff,
Patrick

Addison. didactic and seldom poetical, and deserve the neglect into which they have fallen. Bishop Patrick's hymns were chiefly translations from the Latin, most of them from Prudentius. The best is a version of "Alleluia dulce carmen." Of the five attributed to Addison, not more than three are adapted to public singing; one ("The spacious firmament on high") is a very perfect and finished composition, taking rank among the best hymns in the English language. ${ }^{3}$

From the preface to Simon Browne's hymns, published in 1720, we learn that down to the time of Dr Watts the only hymns known to be "in common use, either in private families or in Christian assemblies," were those of Barton, Mason and Shepherd, together with "an attempt to turn some of George Herbert's poems into common metre," and a few sacramental hymns by authors now forgotten, named Joseph Boyse (1660-1728) and Joseph Stennett. Of the 1410 authors of original British hymns enumerated in Daniel Sedgwick's catalogue, published in 1863,1213 are of later date than 1707; and, if any correct enumeration could be made of the total number of hymns of all kinds published in Great Britain before and after that date, the proportion subsequent to 1707 would be very much larger.

The English Independents, as represented by Dr Isaac Watts, have a just claim to be considered the real founders of modern English hymnody. Watts was the first to understand the
nature of the want, and, by the publication of his Hymns in 1707-1709, and Psalms (not translations, but hymns founded on psalms) in 1709, he led the way in providing for it. His immediate followers were Simon Browne and Philip Doddridge. Later in the 18th century, Joseph Hart, Thomas Gibbons, Miss Anne Steele, Samuel Medley, Samuel Stennett, John Ryland, Benjamin Beddome and Joseph Swain succeeded to them.

Among these writers, most of whom produced some hymns of merit, and several are extremely voluminous, Isaac Watts and Philip Doddridge are pre-eminent. It has been the fashion with some to disparage Watts, as if he had never risen above the level

## Watts.

 of his Hymns for Little Children. No doubt his taste is often faulty, and his style very unequal, but, looking to the good, and disregarding the large quantity of inferior matter, it is probable that more hymns which approach to a very high standard of excellence, and are at the same time suitable for congregational use, may be found in his works than in those of any other English writer. Such are "When I survey the wondrous cross," "Jesus shall reign where'er the sun" (and also another adaptation of the same 72nd Psalm), "Before Jehovah's awful throne" (first line of which, however, is not his, but Wesley's), "Joy to the world, the Lord is come," "My soul, repeat His praise," "Why do we mourn departing friends," "There is a land of pure delight," "Our God, our help in ages past," "Up to the hills I lift mine eyes," and many more. It is true that in some of these cases dross is found in the original poems mixed with gold; but the process of separation, by selection without change, is not difficult. As long as pure nervous English, unaffected fervour, strong simplicity and liquid yet manly sweetness are admitted to be characteristics of a good hymn, works such as these must command admiration.Doddridge is, generally, much more laboured and artificial; but his place also as a hymnwriter ought to be determined, not by his failures, but by his successes, of which the number is not inconsiderable. In his better works he is distinguished by a graceful and

## Doddridge.

 pointed, sometimes even a noble style. His "Hark, the glad sound, the Saviour comes" (which is, indeed, his masterpiece), is as sweet, vigorous and perfect a composition as can anywhere be found. Two other hymns, "How gentle God's commands," and that which, in a form slightly varied, became the "O God of Bethel, by whose hand," of the Scottish "Paraphrases," well represent his softer manner.Of the other followers in the school of Watts, Miss Anne Steele (1717-1778) is the most popular and perhaps the best. Her hymn beginning "Far from these narrow scenes of night" deserves high praise, even by the side of other good performances on the same subject.

The influence of Watts was felt in Scotland, and among the first whom it reached there was Ralph Erskine. This seems to have been after the publication of Erskine's Gospel Sonnets, which appeared in 1732, five years before he joined his brother Ebenezer in the Secession Church. The Gospel Sonnets became, as some have said, a "people's classic"; but there is in them very little which belongs to the category of hymnody. More than nineteen-twentieths of this very curious book are occupied with what are, in fact, theological treatises and catechisms, mystical meditations on Christ as a bridegroom or husband, and spiritual enigmas, paradoxes, and antithetical conceits, versified, it is true, but of a quality of which such lines as-
"Faith's certain by fiducial arts,
Sense by its evidential facts,"
may be taken as a sample. The grains of poetry scattered through this large mass of Calvinistic divinity are very few; yet in one short passage of seven stanzas ("O send me down a draught of love"), the fire burns with a brightness so remarkable as to justify a strong feeling of regret that the gift which this writer evidently had in him was not more often cultivated. Another passage, not so well sustained, but of considerable beauty (part of the last piece under the title "The believer's soliloquy"), became afterwards, in the hands of John Berridge, the foundation of a very striking hymn ("O happy saints, who walk in light").

After his secession, Ralph Erskine published two paraphrases of the "Song of Solomon," and a number of other "Scripture songs," paraphrased, in like manner, from the Old and New Testaments. In these the influence of Watts became very apparent, not only by a change in the writer's general style, but by the direct appropriation of no small quantity of matter from Dr Watts's hymns, with variations which were not always improvements. His paraphrases of I Cor. i. 24 ; Gal. vi. 14 ; Heb. vi. 17-19; Rev. v. 11, 12, vii. 10-17, and xii. $7-12$ are little else than Watts transformed. One of these (Rev. vii. 10-17) is interesting as a variation and improvement, intermediate between the original and the form which it ultimately assumed as the 66th "Paraphrase" of the Church of Scotland, of Watts's "What happy men or angels these," and "These glorious minds, how bright they shine." No one can compare it with its ultimate product, "How bright these glorious spirits shine," without perceiving that William Cameron followed Erskine, and only added finish and grace to his work,-both excelling Watts, in this
instance, in simplicity as well as in conciseness.
Of the contributions to the authorized "Paraphrases" (with the settlement of which committees of the General Assembly of the Church of Scotland were occupied from 1745, or earlier, till 1781), the most noteworthy, besides the two already mentioned,

## Scottish paraphrases.

 were those of John Morrison and those claimed for Michael Bruce. The obligations of these "Paraphrases" to English hymnody, already traced in some instances (to which may be added the adoption from Addison of three out of the five "hymns" appended to them), are perceptible in the vividness and force with which these writers, while adhering with a severe simplicity to the sense of the passages of Scripture which they undertook to render, fulfilled the conception of a good original hymn. Morrison's "The race that long in darkness pined" and "Come, let us to the Lord our God," and Bruce's "Where high the heavenly temple stands" (if this was really his), are well entitled to that praise. The advocates of Bruce in the controversy, not yet closed, as to the poems said to have been entrusted by him to John Logan, and published by Logan in his own name, also claim for him the credit of having varied the paraphrase "Behold, the mountain of the Lord," from its original form, as printed by the committee of the General Assembly in 1745, by some excellent touches.Attention must now be directed to the hymns produced by the "Methodist" movement, which began about 1738, and which afterwards became divided, between those esteemed Arminian, under John Wesley, those who adhered to the Moravians, when the original

## Methodist

 hymns. alliance between that body and the founders of Methodism was dissolved, and the Calvinists, of whom Whitfield was the leader, and Selina, countess of Huntingdon, the patroness. Each of these sections had its own hymn-writers, some of whom did, and others did not, secede from the Church of England. The Wesleyans had Charles Wesley, Robert Seagrave and Thomas Olivers; the Moravians, John Cennick, with whom, perhaps, may be classed John Byrom, who imbibed the mystical ideas of some of the German schools; the Calvinists, Augustus Montague Toplady, John Berridge, William Williams, Martin Madan, Thomas Haweis, Rowland Hill, John Newton and William Cowper.Among all these writers, the palm undoubtedly belongs to Charles Wesley. In the first volume of hymns published by the two brothers are several good translations from the German, believed to be by John Wesley, who, although he translated and adapted, is not

## Charles <br> Wesley.

 supposed to have written any original hymns; and the influence of German hymnody, particularly of the works of Paul Gerhardt, Scheffler, Tersteegen and Zinzendorf, may be traced in a large proportion of Charles Wesley's works. He is more subjective and meditative than Watts and his school; there is a didactic turn, even in his most objective pieces, as, for example, in his Christmas and Easter hymns; most of his works are supplicatory, and his faults are connected with the same habit of mind. He is apt to repeat the same thoughts, and to lose force by redundancy-he runs sometimes even to a tedious length; his hymns are not always symmetrically constructed, or well balanced and finished off. But he has great truth, depth and variety of feeling; his diction is manly and always to the point; never florid, though sometimes passionate and not free from exaggeration; often vivid and picturesque. Of his spirited style there are few better examples than "O for a thousand tongues to sing," "Blow ye the trumpet, blow," "Rejoice, the Lord is King" and "Come, let us join our friends above"; of his more tender vein, "Happy soul, thy days are ended"; and of his fervid contemplative style (without going beyond hymns fit for general use), "O Thou who earnest from above," "Forth in Thy name, O Lord, I go" and "Eternal beam of light divine." With those whose taste is for hymns in which warm religious feelings are warmly and demonstratively expressed, "Jesus, lover of my soul," is as popular as any of these.Of the other Wesleyan hymn-writers, Olivers, originally a Welsh shoemaker

## Olivers.

 and afterwards a preacher, is the most remarkable. He is the author of only two works, both odes, in a stately metre, and from their length unfit for congregational singing, but one of them, "The God of Abraham praise," an ode of singular power and beauty.The Moravian Methodists produced few hymns now available for general use. The best are Cennick's "Children of the heavenly King" and Hammond's "Awake and sing the song of Moses and the Lamb," the former of which (abridged), and the latter as varied by

Cennick,
Hammond, Byrom. Madan, are found in many hymn-books, and are deservedly esteemed. John Byrom, whose name we have thought it convenient to connect with these, though he did not belong to the Moravian community, was the author of a Christmas hymn ("Christians awake, salute the happy morn") which enjoys great popularity; and also of a short subjective hymn, very fine both in feeling and in expression, "My spirit longeth for Thee within my troubled breast."

The contributions of the Calvinistic Methodists to English hymnody are of greater extent and value. Few writers of hymns had higher gifts than Toplady, author of "Rock of ages," by some
esteemed the finest in the English language. He was a man of ardent temperament, enthusiastic zeal, strong convictions and great energy of character. "He had," says one of his biographers, "the courage of a lion, but his frame was brittle as glass." Between him and John Wesley there was a violent opposition of opinion, and much acrimonious controversy; but the same fervour and zeal which made him an intemperate theologian gave warmth, richness and spirituality to his hymns. In some of them, particularly those which, like "Deathless principle, arise," are meditations after the German manner, and not without direct obligation to German originals, the setting is somewhat too artificial; but his art is never inconsistent with a genuine flow of real feeling. Others (e.g. "When languor and disease invade" and "Your harps, ye trembling saints") fail to sustain to the end the beauty with which they began, and would have been better for abridgment. But in all these, and in most of his other works, there is great force and sweetness, both of thought and language, and an easy and harmonious versification.

Berridge, William Williams (1717-1791) and Rowland Hill, all men remarkable for eccentricity, activity and the devotion of their lives to the special work of missionary preaching, though not the authors of many good hymns, composed, or adapted from

## Berridge, Williams and R. Hill.

 earlier compositions, some of great merit. One of Berridge, adapted from Erskine, has been already mentioned; another, adapted from Watts, is "Jesus, cast a look on me." Williams, a Welshman, who wrote "Guide me, O Thou great Jehovah," was especially an apostle of Calvinistic Methodism in his own country, and his hymns are still much used in the principality. Rowland Hill wrote the popular hymn beginning "Exalted high at God's right hand."If, however, the number as well as the quality of good hymns available for general use is to be regarded, the authors of the Olney Hymns are entitled to be placed at the head of all the writers of this Calvinistic school. The greater number of the Olney Hymns are,

Cowper and Newton. no doubt, homely and didactic; but to the best of them, and they are no inconsiderable proportion, the tenderness of Cowper and the manliness of John Newton (1725-1807) give the interest of contrast, as well as that of sustained reality. If Newton carried to some excess the sound principle laid down by him, that "perspicuity, simplicity and ease should be chiefly attended to, and the imagery and colouring of poetry, if admitted at all, should be indulged very sparingly and with great judgment," if he is often dry and colloquial, he rises at other times into "soul-animating strains," such as "Glorious things of thee are spoken, Zion, city of our God"; and sometimes (as in "Approach, my soul, the mercy seat") rivals Cowper himself in depth of feeling. Cowper's hymns in this book are, almost without exception, worthy of his name. Among them are "Hark, my soul, it is the Lord," "There is a fountain filled with blood," "Far from the world, O Lord, I flee," "God moves in a mysterious way" and "Sometimes a light surprises." Some, perhaps, even of these, and others of equal excellence (such as "O for a closer walk with God"), speak the language of a special experience, which, in Cowper's case, was only too real, but which could not, without a degree of unreality not desirable in exercises of public worship, be applied to themselves by all ordinary Christians.

During the first quarter of the 19th century there were not many indications of the tendency, which afterwards became manifest, to enlarge the boundaries of British hymnody. The Remains of Henry Kirke White, published by Southey in 1807, contained a series of

19th-century
hymns.
R. Grant.

Bowdler. hymns, some of which are still in use; and a few of Bishop Heber's hymns and those of Sir Robert Grant, which, though offending rather too much against John Newton's canon, are well known and popular, appeared between 1811 and 1816, in the Christian Observer. In John Bowdler's Remains, published soon after his death in 1815, there are a few more of the same, perhaps too scholarlike, character. But the chief hymn-writers of that period were two clergymen of the Established Church—one in Ireland, Thomas Kelly, and the other in England, William Hurnwho both became Nonconformists, and the Moravian poet, James Montgomery (1771-1854), a native of Scotland.

Kelly was the son of an Irish judge, and in 1804 published a small volume of ninety-six hymns, which grew in successive editions till, in the last before his death in 1854, they amounted to 765. There is, as might be expected, in this great number a large

Kelly. preponderance of the didactic and commonplace. But not a few very excellent hymns may be gathered from them. Simple and natural, without the vivacity and terseness of Watts or the severity of Newton, Kelly has some points in common with both those writers, and he is less subjective than most of the "Methodist" school. His hymns beginning "Lo! He comes, let all adore Him," and "Through the day Thy love hath spared us," have a rich, melodious movement; and another, "We sing the praise of Him who died," is distinguished by a calm, subdued power, rising gradually from a rather low to a very high key.

There is little in them which deserves to be saved from oblivion; but one at least, "There is a river deep and broad," may bear comparison with the best of those which have been produced upon the same, and it is rather a favourite, theme.

The Psalms and Hymns of James Montgomery were published in 1822 and 1825, though written earlier. More cultivated and artistic than Kelly, he is less simple and

## Montgomery.

 natural. His "Hail to the Lord's Anointed," "Songs of praise the angels sang" and "Mercy alone can meet my case" are among his most successful efforts.During this period, the collections of miscellaneous hymns for congregational use, of which the example was set by the Wesleys, Whitfield, Toplady and Lady Huntingdon, had greatly multiplied; and with them the practice (for which, indeed, too many precedents

## Collections of hymns.

 existed in the history of Latin and German hymnody) of every collector altering the compositions of other men without scruple, to suit his own doctrine or taste; with the effect, too generally, of patching and disfiguring, spoiling and emasculating the works so altered, substituting neutral tints for natural colouring, and a dead for a living sense. In the Church of England the use of these collections had become frequent in churches and chapels, principally in cities and towns, where the sentiments of the clergy approximated to those of the Nonconformists. In rural parishes, when the clergy were not of the "Evangelical" school, they were generally held in disfavour; for which, even if doctrinal prepossessions had not entered into the question, the great want of taste and judgment often manifested in their compilation, and perhaps also the prevailing mediocrity of the bulk of the original compositions from which most of them were derived, would be enough to account. In addition to this, the idea that no hymns ought to be used in any services of the Church of England, except prose anthems after the third collect, without express royal or ecclesiastical authority, continued down to that time largely to prevail among high churchmen.Two publications, which appeared almost simultaneously in 1827-Bishop Heber's Hymns, with a few added by Dean Milman, and John Keble's Christian Year (not a hymn-book, but one from which several admirable hymns have been taken, and the well-spring of

Heber, Milman, Keble. many streams of thought and feeling by which good hymns have since been produced)-introduced a new epoch, breaking down the barrier as to hymnody which had till then existed between the different theological schools of the Church of England. In this movement Richard Mant, bishop of Down, was also one of the first to co-operate. It soon received a great additional impulse from the increased attention which, about the same time, began to be paid to ancient hymnody, and from the publication in 1833 of Bunsen's Gesangbuch. Among its earliest fruits was the Lyra apostolica, containing hymns, sonnets and other devotional poems, most of them originally contributed by some of the leading authors of the Tracts for the Times to the British Magazine; the finest of which is the pathetic "Lead, kindly Light, amid th' encircling gloom," by Cardinal Newman-well known, and universally admired. From that time hymns and hymn-writers rapidly multiplied in the Church of

## Newman.

 England, and in Scotland also. Nearly 600 authors whose publications were later than 1827 are enumerated in Sedgwick's catalogue of 1863, and about half a million hymns are now in existence. Works, critical and historical, upon the subject of hymns, have also multiplied; and collections for church use have become innumerable-several of the various religious denominations, and many of the leading ecclesiastical and religious societies, having issued hymn-books of their own, in addition to those compiled for particular dioceses, churches and chapels, and to books (like Hymns Ancient and Modern, published 1861, supplemented 1889, revised edition, 1905) which have become popular without any sanction from authority. To mention all the authors of good hymns since the commencement of this new epoch would be impossible; but probably no names could be chosen more fairly representative of its characteristic merits, and perhaps also of some of its defects, than those of Josiah Conder and James Edmeston among English Nonconformists; Henry Francis Lyte and Charlotte Elliott among evangelicals in the Church of England; John Mason Neale and Christopher Wordsworth, bishop of Lincoln, among English churchmen of the higher school; Arthur Penrhyn Stanley, Edward H. Plumptre, Frances Ridley Havergal; and in Scotland, Dr Horatius Bonar, Dr Norman Macleod and Dr George Matheson. American hymn-writers belong to the same schools, and have been affected by the same influences. Some of them have enjoyed a just reputation on both sides of the Atlantic. Among those best known are John Greenleaf Whittier, Bishop Doane, Dr W. A. Muhlenberg and Thomas Hastings; and it is difficult to praise too highly such works as the Christmas hymn, "It came upon the midnight clear," by Edmund H. Sears; the Ascension hymn, "Thou, who didst stoop below," by Mrs S. E. Miles; two by Dr Ray Palmer, "My faith looks up to Thee, Thou Lamb of Calvary," and "Jesus, Thou joy of loving hearts," the latter of which is the best among several good English versions of "Jesu, dulcedo, cordium"; and "Lord of all being, throned afar," by Oliver Wendell Holmes.The more modern "Moody and Sankey" hymns (see Moody, D. L.) popularized a new

Evangelical type, and the Salvation Army has carried this still farther.
7. Conclusion.-The object aimed at in this article has been to trace the general history of the principal schools of ancient and modern hymnody, and especially the history of its use in the Christian church. For this purpose it has not been thought necessary to give any account of the hymns of Racine, Madame Guyon and others, who can hardly be classed with any school, nor of the works of Caesar Malan of Geneva (1787-1864) and other quite modern hymn-writers of the Reformed churches in Switzerland and France.

On a general view of the whole subject, hymnody is seen to have been a not inconsiderable factor in religious worship. It has been sometimes employed to disseminate and popularize particular views, but its spirit and influence has been, on the whole, catholic. It has embodied the faith, trust and hope, and no small part of the inward experience, of generation after generation of men, in many different countries and climates, of many different nations, and in many varieties of circumstances and condition. Coloured, indeed, by these differences, and also by the various modes in which the same truths have been apprehended by different minds and sometimes reflecting partial and imperfect conceptions of them, and errors with which they have been associated in particular churches, times and places, its testimony is, nevertheless, generally the same. It has upon it a stamp of genuineness which cannot be mistaken. It bears witness to the force of a central attraction more powerful than all causes of difference, which binds together times ancient and modern, nations of various race and language, churchmen and nonconformists, churches reformed and unreformed; to a true fundamental unity among good Christians; and to a substantial identity in their moral and spiritual experience.

The regular practice of hymnody in English musical history dates from the beginning of the 16th century. Luther's verses were adapted sometimes to ancient church melodies, sometimes to tunes of secular songs, and sometimes had music composed for them by himself and others. Many rhyming Latin hymns are of earlier date whose tunes are identified with them, some of which tunes, with the subject of their Latin text, are among the Reformer's appropriations; but it was he who put the words of praise and prayer into the popular mouth, associated with rhythmical music which aided to imprint the words upon the memory and to enforce their enunciation. In conjunction with his friend Johann Walther, Luther issued a collection of poems for choral singing in 1524, which was followed by many others in North Germany. The English versions of the Psalms by Sternhold and Hopkins and their predecessors, and the French version by Clement Marot and Theodore Beza, were written with the same purpose of fitting sacred minstrelsy to the voice of the multitude. Goudimel in 1566 and Claudin le Jeune in 1607 printed harmonizations of tunes that had then become standard for the Psalms, and in England several such publications appeared, culminating in Thomas Ravenscroft's famous collection, The Whole Book of Psalms (1621); in all of these the arrangements of the tunes were by various masters. The English practice of hymn-singing was much strengthened on the return of the exiled reformers from Frankfort and Geneva, when it became so general that, according to Bishop Jewell, thousands of the populace who assembled at Paul's Cross to hear the preaching would join in the singing of psalms before and after the sermon.

The placing of the choral song of the church within the lips of the people had great religious and moral influence; it has had also its great effect upon art, shown in the productions of the North German musicians ever since the first days of the Reformation, which abound in exercises of scholarship and imagination wrought upon the tunes of established acceptance. Some of these are accompaniments to the tunes with interludes between the several strains, and some are compositions for the organ or for orchestral instruments that consist of such elaboration of the themes as is displayed in accompaniments to voices, but of far more complicated and extended character. A special art-form that was developed to a very high degree, but has passed into comparative disuse, was the structure of all varieties of counterpoint extemporaneously upon the known hymn-tunes (chorals), and several masters acquired great fame by success in its practice, of whom J. A. Reinken (1623-1722), Johann Pachelbel (1653-1706), Georg Boehm and the great J. S. Bach are specially memorable. The hymnody of North Germany has for artistic treatment a strong advantage which is unpossessed by that of England, in that for the most part the same verses are associated with the same tunes, so that, whenever the text or the music is heard, either prompts recollection of the other, whereas in England tunes were always and are now often composed to metres and not to poems; any tune in a given metre is available for every poem in the same, and hence there are various tunes to one poem, and various poems to one tune. ${ }^{4}$ In England a tune is named generally after some place-as "York," "Windsor," "Dundee,"-or by some other unsignifying word; in North Germany a tune is mostly named by the initial words of the verses to which it is allied, and consequently, whenever it is heard, whether with words or without, it necessarily suggests to the hearer the whole subject of that hymn of which it is the musical moiety undivorceable from the literary half. Manifold as they are, knowledge of the choral tunes is included in the earliest schooling of every Lutheran and every Calvinist in Germany, which thus enables all to take part in performance of the tunes, and hence expressly the definition of "choral." Compositions grounded on the standard tune are then not merely school exercises,
but works of art which link the sympathies of the writer and the listener, and aim at expressing the feeling prompted by the hymn under treatment.

Bibliography: I. Ancient.-George Cassander, Hymni ecclesiastici (Cologne, 1556); Georgius Fabricius, Poëtarum veterum ecclesiasticorum (Frankfort, 1578); Cardinal J. M. Thomasius, Hymnarium in Opera, ii. 351 seq. (Rome, 1747); A. J. Rambach, Anthologie christlicher Gesänge (Altona, 1817); H. A. Daniel, Thesaurus hymnologicus (Leipzig, 5 vols., 1841-1856); J. M. Neale, Hymni ecclesiae et sequentiae (London, 1851-1852); and Hymns of the Eastern Church (1863). The dissertation prefixed to the second volume of the Acta sanctorum of the Bollandists; Cardinal J. B. Pitra, Hymnographie de l'église grecque (1867), Analecta sacra (1876); W. Christ and M. Paranikas, Anthologia Graeca carminum Christianorum (Leipzig, 1871); F. A. March, Latin Hymns with English Notes (New York, 1875); R. C. Trench, Sacred Latin Poetry (London, 4th ed., 1874); J. Pauly, Hymni breviarii Romani (Aix-la-Chapelle, 3 vols., 1868-1870); Pimont, Les Hymnes du bréviaire romain (vols. 1-3, 1874-1884, unfinished); A. W. F. Fischer, Kirchenlieder-Lexicon (Gotha, 1878-1879); J. Kayser, Beiträge zur Geschichte der ältesten Kirchenhymnen (1881); M. Manitius, Geschichte der christlichen lateinischen Poesie (Stuttgart, 1891); John Julian, Dictionary of Hymnology (1892, new ed. 1907). For criticisms of metre, see also Huemer, Untersuchungen über die ältesten christlichen Rhythmen (1879); E. Bouvy, Poètes et mélodes (Nîmes, 1886); C. Krumbacher, Geschichte der byzantinischen Literatur (Munich, 1897, p. 700 seq.); J. M. Neale, Latin dissertation prefixed to Daniel's Thesaurus, vol. 5; and D. J. Donahoe, Early Christian Hymns (London, 1909).
II. Medieval.-Walafrid Strabo's treatise, ch. 25, De hymnis, \&c.; Radulph of Tongres, De psaltario observando (14th century); Clichtavaens, Elucidatorium ecclesiasticum (Paris, 1556); Faustinus Arevalus, Hymnodia Hispanica (Rome, 1786); E. du Méril, Poésies populaires latines antérieures au XIIIe siècle (Paris, 1843); J. Stevenson, Latin Hymns of the Anglo-Saxon Church (Surtees Society, Durham, 1851); Norman, Hymnarium Sarisburiense (London, 1851); J. D. Chambers, Psalter, \&c., according to the Sarum use (1852); F. J. Mone, Lateinische Hymnen des Mittelalters (Freiburg, 3 vols., 1853-1855); Ph. Wackernagel, Das deutsche Kirchenlied von der ältesten Zeit bis zum Anfang des 17. Jahrhunderts, vol. i. (Leipzig, 1864); E. Dümmler, Poëtae latini aevi Carolini (1881-1890); the Hymnologische Beiträge: Quellen und Forschungen zur Geschichte der lateinischen Hymnendichtung, edited by C. Blume and G. M. Dreves (Leipzig, 1897); G. C. F. Mohnike, Hymnologische Forschungen; Klemming, Hymni et sequentiae in regno Sueciae (Stockholm, 4 vols., 1885-1887); Das katholische deutsche Kirchenlied (vol. i. by K. Severin Meister, 1862, vol. ii. by W. Baumker, 1883); the "Hymnodia Hiberica," Spanische Hymnen des Mittelalters, vol. xvi. (1894); the "Hymnodia Gotica," Mozarabische Hymnen des altspanischen Ritus, vol. xxvii. (1897); J. Dankó, Vetus hymnarium ecclesiasticae Hungariae (Budapest, 1893); J. H. Bernard and R. Atkinson, The Irish Liber Hymnorum (2 vols., London, 1898); C. A. J. Chevalier, Poésie liturgique du moyen âge (Paris, 1893).
III. Modern.-J. C. Jacobi, Psalmodia Germanica (1722-1725 and 1732, with supplement added by J. Haberkorn, 1765); F. A. Cunz, Geschichte des deutschen Kirchenliedes (Leipzig, 1855); Baron von Bunsen, Versuch eines allgemeinen Gesang- und Gebetbuches (1833) and Allgemeines evangelisches Gesang- und Gebetbuch (1846); Catherine Winkworth, Christian Singers of Germany (1869) and Lyra Germanica (1855); Catherine H. Dunn, Hymns from the German (1857); Frances E. Cox, Sacred Hymns from the German (London, 1841); Massie, Lyra domestica (1860); Appendix on Scottish Psalmody in D. Laing's edition of Baillie's Letters and Journals (1841-1842); J. and C. Wesley, Collection of Psalms and Hymns (1741); Josiah Miller, Our Hymns, their Authors and Origin (1866); John Gadsby, Memoirs of the Principal Hymnwriters (3rd ed., 1861); L. C. Biggs, Annotations to Hymns Ancient and Modern (1867); Daniel Sedgwick, Comprehensive Index of Names of Original Authors of Hymns (2nd ed., 1863); R. E. Prothero, The Psalms in Human Life (1907); C. J. Brandt and L. Helweg, Den danske Psalmedigtning (Copenhagen, 1846-1847); J. N. Skaar, Norsk Salmehistorie (Bergen, 18791880); H. Schück, Svensk Literaturhistoria (Stockholm, 1890); Rudolf Wolkan, Geschichte der deutschen Literatur in Böhmen, 246-256, and Das deutsche Kirchenlied der böhm. Brüder (Prague, 1891); Zahn, Die geistlichen Lieder der Brüder in Böhmen, Mähren u. Polen (Nuremberg, 1875); and J. Müller, "Bohemian Brethren's Hymnody," in J. Julian's Dictionary of Hymnology.

For account of hymn-tunes, \&c., see W. Cowan and James Love, Music of the Church Hymnody and the Psalter in Metre (London, 1901); and Dickinson, Music in the History of the Western Church (New York, 1902); S. Kümmerle, Encyklopädie der evangelischen Kirchenmusik (4 vols., 1888-1895); Chr. Palmer, Evangelische Hymnologie (Stuttgart, 1865); and P. Urto Kornmüller, Lexikon der kirchlichen Tonkunst (1891). earlier; Assyria and Egypt have left specimens, while India has the Vedic hymns, and Confucius collected "praise songs" in China.

The authorship of this and of one other, "When all thy mercies, O my God," has been made a subject of controversy, -being claimed for Andrew Marvell (who died in 1678), in the preface to Captain E. Thompson's edition (1776) of Marvell's Works. But this claim does not appear to be substantiated. The editor did not give his readers the means of judging as to the real age, character or value of a manuscript to which he referred; he did not say that these portions of it were in Marvell's handwriting; he did not even himself include them among Marvell's poems, as published in the body of his edition; and he advanced a like claim on like grounds to two other poems, in very different styles, which had been published as their own by Tickell and Mallet. It is certain that all the five hymns were first made public in 1712, in papers contributed by Addison to the Spectator (Nos. $441,453,465,489,513$ ), in which they were introduced in a way which might have been expected if they were by the hand which wrote those papers, but which would have been improbable, and unworthy of Addison, if they were unpublished works of a writer of so much genius, and such note in his day, as Marvell. They are all printed as Addison's in Dr Johnson's British Poets.

4 The old tune for the 100th Psalm and Croft's tune for the 104th are almost the only exceptions, unless "God save the King" may be classed under "hymnody." In Scotland also the tune for the 124th Psalm is associated with its proper text.
 the Greek term quoted by Vitruvius (iii. 2) for the opening in the middle of the roof of decastyle temples, of which "there was no example in Rome, but one in Athens in the temple of Jupiter Olympius, which is octastyle." But at the time he wrote (c. 25 в.c.) the cella of this temple was unroofed, because the columns which had been provided to carry, at all events, part of the ceiling and roof had been taken away by Sulla in 80 в.с. The decastyle temple of Apollo Didymaeus near Miletus was, according to Strabo (c. 50 в.c.), unroofed, on account of the vastness of its cella, in which precious groves of laurel bushes were planted. Apart from these two examples, the references in various writers to an opening of some kind in the roofs of temples dedicated to particular deities, and the statement of Vitruvius, which was doubtless based on the writings of Greek authors, that in decastyle or large temples the centre was open to the sky and without a roof (medium autem sub divo est sine tecto), render the existence of the hypaethros probable in some cases; and therefore C. R. Cockerell's discovery in the temple at Aegina of two fragments of a coping-stone, in which there were sinkings on one side to receive the tiles and covering tiles, has been of great importance in the discussion of this subject. In the conjectural restoration of the opaion or opening in the roof shown in Cockerell's drawing, it has been made needlessly large, having an area of about one quarter of the superficial area of the cella between the columns, and since in the Pantheon at Rome the relative proportions of the central opening in the dome and the area of the Rotunda are 1: 22, and the light there is ample, in the clearer atmosphere of Greece it might have been less. The larger the opening the more conspicuous would be the notch in the roof which is so greatly objected to; in this respect T. J. Hittorff would seem to be nearer the truth when, in his conjectural restoration of Temple R. at Selinus, he shows an opaion about half the relative size shown in Cockerell's of that at Aegina, the coping on the side elevation being much less noticeable. The problem was apparently solved in another way at Bassae, where, in the excavations of the temple of Apollo by Cockerell and Baron Haller von Hallerstein, three marble tiles were found with pierced openings in them about 18 in . by 10 in .; five of these pierced tiles on either side would have amply lighted the interior of the cella, and the amount of rain passing through (a serious element to be considered in a country where torrential rains occasionally fall) would not be very great or more than could be retained to dry up in the cella sunk pavement. In favour of both these methods of lighting the interior of the cella, the sarcophagus tomb at Cyrene, about 20 ft . long, carved in imitation of a temple, has been adduced, because, on the top of the roof and in its centre, there is a raised coping, and a similar feature is found on a tomb found near Delos; an example from Crete now in the British Museum shows a pierced tile on each side of the roof, and a large number of pierced tiles have been found in Pompeii, some of them surrounded with a rim identical with that of the marble tiles at Bassae. On the other hand, there are many authorities, among them Dr W. Dörpfeld, who have adhered to their original opinion that it was only through the open doorway that light was ever admitted into the cella, and with the clear atmosphere of Greece and the reflections from the marble pavement such lighting would be quite sufficient. There remains still another source of light to be considered, that passing through the Parian marble tiles of the roof; the superior translucency of Parian to any other marble may have suggested its employment for the roofs of temples, and if, in the framed ceilings carried over the cella, openings were left, some light from the Parian tile roof might have been obtained. It is possibly to this that Plutarch refers when describing the ceiling and roof of the temple of Demeter at Eleusis, where the columns in
the interior of the temple carried a ceiling, probably constructed of timbers crossing one another at right angles, and one or more of the spaces was left open, which Xenocles surmounted by a roof formed of tiles.


#### Abstract

James Fergusson put forward many years ago a conjectural restoration in which he adopted a clerestory above the superimposed columns inside the cella; in order to provide the light for these windows he indicated two trenches in the roof, one on each side, and pointed out that the great Hall of Columns at Karnak was lighted in this way with clerestory windows; but in the first place the light in the latter was obtained over the flat roofs covering lower portions of the hall, and in the second place, as it rarely rains in Thebes, there could be no difficulty about the drainage, while in Greece, with the torrential rains and snow, these trenches would be deluged with water, and with all the appliances of the present day it would be impossible to keep these clerestory windows watertight. There is, however, still another objection to Fergusson's theory; the water collecting in these trenches on the roof would have to be discharged, for which Fergusson's suggestions are quite inadequate, and the gargoyles shown in the cella wall would make the peristyle insupportable just at the time when it was required for shelter. No drainage otherwise of any kind has ever been found in any Greek temple, which is fatal to Fergusson's view. Nor is it in accordance with the definition "open to the sky." English cathedrals and churches are all lighted by clerestory windows, but no one has described them as open to the sky, and although Vitruvius's statements are sometimes confusing, his description is far too clear to leave any misunderstanding as to the lighting of temples (where it was necessary on account of great length) through an opening in the roof.


There is one other theory which has been put forward, but which can only apply to nonperistylar temples,-that light and air was admitted through the metopes, the apertures between the beams crossing the cella,-and it has been assumed that because Orestes was advised in one of the Greek plays to climb up and look through the metopes of the temple, these were left open; but if Orestes could look in, so could the birds, and the statue of the god would be defiled. The metopes were probably filled in with shutters of some kind which Orestes knew how to open.

HYPALLAGE (Gr. $\dot{\sim} \pi \alpha \lambda \lambda \alpha \gamma \eta$, interchange or exchange), a rhetorical figure, in which the proper relation between two words according to the rules of syntax are inverted. The stock instance is that in Virgil, Aen. iii. 61, where dare classibus austros, to give winds to the fleet, is put for dare classes austris, to give the fleet to the winds. The term is also loosely applied to figures of speech properly known as "metonymy" and, generally, to any striking turn of expression.

HYPATIA ('Yп $\alpha \tau_{i ́ \alpha}$ ) (c. A.D. 370-415) mathematician and philosopher, born in Alexandria, was the daughter of Theon, also a mathematician and philosopher, author of scholia on Euclid and a commentary on the Almagest, in which it is suggested that he was assisted by Hypatia (on the 3rd book). After lecturing in her native city, Hypatia ultimately became the recognized head of the Neoplatonic school there (c. 400). Her great eloquence and rare modesty and beauty, combined with her remarkable intellectual gifts, attracted to her class-room a large number of pupils. Among these was Synesius, afterwards (c. 410) bishop of Ptolemaïs, several of whose letters to her, full of chivalrous admiration and reverence, are still extant. Suidas, misled by an incomplete excerpt in Photius from the life of Isidorus (the Neoplatonist) by Damascius, states that Hypatia was the wife of Isidorus; but this is chronologically impossible, since Isidorus could not have been born before 434 (see Hoche in Philologus). Shortly after the accession of Cyril to the patriarchate of Alexandria in 412, owing to her intimacy with Orestes, the pagan prefect of the city, Hypatia was barbarously murdered by the Nitrian monks and the fanatical Christian mob (March 415). Socrates has related how she was torn from her chariot, dragged to the Caesareum (then a Christian church), stripped naked, done to death with oyster-shells (ȯбтра́коıऽ ả้ะモ̃̃入ov perhaps "cut her throat") and finally burnt piecemeal. Most prominent among the actual perpetrators of the crime was one Peter, a reader; but there seems little reason to doubt Cyril's complicity (see Cyril of Alexandria).

Hypatia, according to Suidas, was the author of commentaries on the Arithmetica of

Diophantus of Alexandria, on the Conics of Apollonius of Perga and on the astronomical canon (of Ptolemy). These works are lost; but their titles, combined with expressions in the letters of Synesius, who consulted her about the construction of an astrolabe and a hydroscope, indicate that she devoted herself specially to astronomy and mathematics. Little is known of her philosophical opinions, but she appears to have embraced the intellectual rather than the mystical side of Neoplatonism, and to have been a follower of Plotinus rather than of Porphyry and Iamblichus. Zeller, however, in his Outlines of Greek Philosophy (1886, Eng. trans. p. 347), states that "she appears to have taught the Neoplatonic doctrine in the form in which Iamblichus had stated it." A Latin letter to Cyril on behalf of Nestorius, printed in the Collectio nova conciliorum, i. (1623), by Stephanus Baluzius (Étienne Baluze, q.v.), and sometimes attributed to her, is undoubtedly spurious. The story of Hypatia appears in a considerably disguised yet still recognizable form in the legend of St Catherine as recorded in the Roman Breviary (November 25), and still more fully in the Martyrologies (see A. B. Jameson, Sacred and Legendary Art (1867) ii. 467.)

The chief source for the little we know about Hypatia is the account given by Socrates (Hist. ecclesiastica, vii. 15). She is the subject of an epigram by Palladas in the Greek Anthology (ix. 400). See Fabricius, Bibliotheca Graeca (ed. Harles), ix. 187; John Toland, Tetradymus (1720); R. Hoche in Philologus (1860), xv. 435; monographs by Stephan Wolf (Czernowitz, 1879), H. Ligier (Dijon, 1880) and W. A. Meyer (Heidelberg, 1885), who devotes attention to the relation of Hypatia to the chief representatives of Neoplatonism; J. B. Bury, Hist. of the Later Roman Empire (1889), i. 208,317; A. Güldenpenning, Geschichte des oströmischen Reiches unter Arcadius und Theodosius II. (Halle, 1885), p. 230; Wetzer and Welte, Kirchenlexikon, vi. (1889), from a Catholic standpoint. The story of Hypatia also forms the basis of the well-known historical romance by Charles Kingsley (1853).

HYPERBATON (Gr. ט́né $\beta \alpha \tau$, a stepping over), the name of a figure of speech, consisting of a transposition of words from their natural order, such as the placing of the object before instead of after the verb. It is a common method of securing emphasis.

HYPERBOLA, a conic section, consisting of two open branches, each extending to infinity. It may be defined in several ways. The in solido definition as the section of a cone by a plane at a less inclination to the axis than the generator brings out the existence of the two infinite branches if we imagine the cone to be double and to extend to infinity. The in plano definition, i.e. as the conic having an eccentricity greater than unity, is a convenient starting-point for the Euclidian investigation. In projective geometry it may be defined as the conic which intersects the line at infinity in two real points, or to which it is possible to draw two real tangents from the centre. Analytically, it is defined by an equation of the second degree, of which the highest terms have real roots (see Conic Section).

While resembling the parabola in extending to infinity, the curve has closest affinities to the ellipse. Thus it has a real centre, two foci, two directrices and two vertices; the transverse axis, joining the vertices, corresponds to the major axis of the ellipse, and the line through the centre and perpendicular to this axis is called the conjugate axis, and corresponds to the minor axis of the ellipse; about these axes the curve is symmetrical. The curve does not appear to intersect the conjugate axis, but the introduction of imaginaries permits us to regard it as cutting this axis in two unreal points. Calling the foci $S, S^{\prime}$, the real vertices $A, A^{\prime}$, the extremities of the conjugate axis $B, B^{\prime}$ and the centre $C$, the positions of $B, B^{\prime}$ are given by $A B=A B^{\prime}=C S$. If a rectangle be constructed about $\mathrm{AA}^{\prime}$ and $\mathrm{BB}^{\prime}$, the diagonals of this figure are the "asymptotes" of the curve; they are the tangents from the centre, and hence touch the curve at infinity. These two lines may be pictured in the in solido definition as the section of a cone by a plane through its vertex and parallel to the plane generating the hyperbola. If the asymptotes be perpendicular, or, in other words, the principal axes be equal, the curve is called the rectangular hyperbola. The hyperbola which has for its transverse and conjugate axes the transverse and conjugate axes of another hyperbola is said to be the conjugate hyperbola.

Some properties of the curve will be briefly stated: If PN be the ordinate of the point P on the curve, $\mathrm{AA}^{\prime}$ the vertices, X the meet of the directrix and axis and C the centre, then $\mathrm{PN}^{2}: \mathrm{AN}^{\prime} \cdot \mathrm{NA}^{\prime}$ : : $\mathrm{SX}^{2}$ : $\mathrm{AX} \cdot \mathrm{A}^{\prime} \mathrm{X}$, i.e. $\mathrm{PN}^{2}$ is to $\mathrm{AN} \cdot \mathrm{NA}^{\prime}$ in a constant ratio. The circle on $\mathrm{AA}^{\prime}$ as diameter is called
the auxiliarly circle; obviously $\mathrm{AN} \cdot \mathrm{NA}^{\prime}$ equals the square of the tangent to this circle from N , and hence the ratio of $P N$ to the tangent to the auxiliarly circle from $N$ equals the ratio of the conjugate axis to the transverse. We may observe that the asymptotes intersect this circle in the same points as the directrices. An important property is: the difference of the focal distances of any point on the curve equals the transverse axis. The tangent at any point bisects the angle between the focal distances of the point, and the normal is equally inclined to the focal distances. Also the auxiliarly circle is the locus of the feet of the perpendiculars from the foci on any tangent. Two tangents from any point are equally inclined to the focal distance of the point. If the tangent at $P$ meet the conjugate axis in t , and the transverse in N , then Ct . PN $=\mathrm{BC}^{2}$; similarly if g and G be the corresponding intersections of the normal, $\mathrm{PG}: \mathrm{Pg}:: \mathrm{BC}^{2}$ : $\mathrm{AC}^{2}$. A diameter is a line through the centre and terminated by the curve: it bisects all chords parallel to the tangents at its extremities; the diameter parallel to these chords is its conjugate diameter. Any diameter is a mean proportional between the transverse axis and the focal chord parallel to the diameter. Any line cuts off equal distances between the curve and the asymptotes. If the tangent at $P$ meets the asymptotes in $R, R^{\prime}$, then $C R \cdot C R^{\prime}=C S^{2}$. The geometry of the rectangular hyperbola is simplified by the fact that its principal axes are equal.
Analytically the hyperbola is given by $\mathrm{ax}^{2}+2 \mathrm{hxy}+\mathrm{by}^{2}+2 \mathrm{gx}+2 \mathrm{fy}+\mathrm{c}=0$ wherein $\mathrm{ab}>\mathrm{h}^{2}$. Referred to the centre this becomes $\mathrm{Ax}^{2}+2 \mathrm{Hxy}+\mathrm{By}^{2}+\mathrm{C}=0$; and if the axes of coordinates be the principal axes of the curve, the equation is further simplified to $\mathrm{Ax}^{2}-\mathrm{By}^{2}=\mathrm{C}$, or if the semi-transverse axis be $a$, and the semi-conjugate $b, x^{2} / a^{2}-y^{2} / b^{2}=1$. This is the most commonly used form. In the rectangular hyperbola $a=b$; hence its equation is $x^{2}-y^{2}=0$. The equations to the asymptotes are $\mathrm{x} / \mathrm{a}= \pm \mathrm{y} / \mathrm{b}$ and $\mathrm{x}= \pm \mathrm{y}$ respectively. Referred to the asymptotes as axes the general equation becomes $x y=k^{2}$; obviously the axes are oblique in the general hyperbola and rectangular in the rectangular hyperbola. The values of the constant $\mathrm{k}^{2}$ are $1 / 2\left(\mathrm{a}^{2}\right.$ $+\mathrm{b}^{2}$ ) and $1 / 2 \mathrm{a}^{2}$ respectively. (See Geometry: Analytical; Projective.)

HYPERBOLE (from Gr. Úmع $\beta$ 人́́ $\lambda \lambda \varepsilon ו \nu$, to throw beyond), a figure of rhetoric whereby the speaker expresses more than the truth, in order to produce a vivid impression; hence, an exaggeration.

HYPERBOREANS ('Yாєрßó $\rho \varepsilon о \iota, ~ ' Y \pi \varepsilon \rho \beta o ́ \rho \varepsilon เ o t), ~ a ~ m y t h i c a l ~ p e o p l e ~ i n t i m a t e l y ~ c o n n e c t e d ~ w i t h ~$ the worship of Apollo. Their name does not occur in the Iliad or the Odyssey, but Herodotus (iv. 32) states that they were mentioned in Hesiod and in the Epigoni, an epic of the Theban cycle. According to Herodotus, two maidens, Opis and Arge, and later two others, Hyperoche and Laodice, escorted by five men, called by the Delians Perphereës, were sent by the Hyperboreans with certain offerings to Delos. Finding that their messengers did not return, the Hyperboreans adopted the plan of wrapping the offerings in wheat-straw and requested their neighbours to hand them on to the next nation, and so on, till they finally reached Delos. The theory of H. L. Ahrens, that Hyperboreans and Perphereës are identical, is now widely accepted. In some of the dialects of northern Greece (especially Macedonia and Delphi) $\varphi$ had a
 who carry over"), which becoming ún $\rho \beta \circ \rho o t ~ g a v e ~ r i s e ~ t o ~ t h e ~ p o p u l a r ~ d e r i v a t i o n ~ f r o m ~ \beta o \rho \varepsilon ́ \alpha \varsigma ~$ ("dwellers beyond the north wind"). The Hyperboreans were thus the bearers of the sacrificial gifts to Apollo over land and sea, irrespective of their home, the name being given to Delphians, Thessalians, Athenians and Delians. It is objected by O. Schröder that the form Пع $\varnothing \varphi \rho \varepsilon \varepsilon \varsigma$ requires a passive meaning, "those who are carried round the altar," perhaps dancers like the whirling dervishes; distinguishing them from the Hyperboreans, he explains the latter as those who live "above the mountains," that is, in heaven. Under the influence of the derivation from $\beta$ ор́ $\alpha \varsigma$, the home of the Hyperboreans was placed in a region beyond the north wind, a paradise like the Elysian plains, inaccessible by land or sea, whither Apollo could remove those mortals who had lived a life of piety. It was a land of perpetual sunshine and great fertility; its inhabitants were free from disease and war. The duration of their life was 1000 years, but if any desired to shorten it, he decked himself with garlands and threw himself from a rock into the sea. The close connexion of the Hyperboreans with the cult of Apollo may be seen by comparing the Hyperborean myths, the characters of which by their names mostly recall Apollo or Artemis (Agyieus, Opis, Hecaergos, Loxo), with the ceremonial of the Apolline worship. No
meat was eaten at the Pyanepsia; the Hyperboreans were vegetarians. At the festival of Apollo at Leucas a victim flung himself from a rock into the sea, like the Hyperborean who was tired of life. According to an Athenian decree ( 380 в.c.) asses were sacrificed to Apollo at Delphi, and Pindar (Pythia, x. 33) speaks of "hecatombs of asses" being offered to him by the Hyperboreans. As the latter conveyed sacrificial gifts to Delos hidden in wheat-straw, so at the Thargelia a sheaf of corn was carried round in procession, concealing a symbol of the god (for other resemblances see Crusius's article). Although the Hyperborean legends are mainly connected with Delphi and Delos, traces of them are found in Argos (the stories of Heracles, Perseus, Io), Attica, Macedonia, Thrace, Sicily and Italy (which Niebuhr indeed considers their original home). In modern times the name has been applied to a group of races, which includes the Chukchis, Koryaks, Yukaghirs, Ainus, Gilyaks and Kamchadales, inhabiting the arctic regions of Asia and America. But if ever ethnically one, the Asiatic and American branches are now as far apart from each other as they both are from the Mongolo-Tatar stock.

See O. Crusius in Roscher's Lexikon der Mythologie; O. Schröder in Archiv für Religionswissenschaft (1904), viii. 69; W. Mannhardt, Wald- und Feldkulte (1905); L. R. Farnell, Cults of the Greek States (1907), iv. 100.

HYPEREIDES (c. 390-322 в.с.), one of the ten Attic orators, was the son of Glaucippus, of the deme of Collytus. Having studied under Isocrates, he began life as a writer of speeches for the courts, and in 360 he prosecuted Autocles, a general charged with treason in Thrace (frags. 5565 , Blass). At the time of the so-called "Social War" (358-355) he accused Aristophon, then one of the most influential men at Athens, of malpractices (frags. 40-44, Blass), and impeached Philocrates (343) for high treason. From the peace of 346 to 324 Hypereides supported Demosthenes in the struggle against Macedon; but in the affair of Harpalus he was one of the ten public prosecutors of Demosthenes, and on the exile of his former leader he became the head of the patriotic party (324). After the death of Alexander, he was the chief promoter of the Lamian war against Antipater and Craterus. After the decisive defeat at Crannon (322), Hypereides and the other orators, whose surrender was demanded by Antipater, were condemned to death by the Athenian partisans of Macedonia. Hypereides fled to Aegina, but Antipater's emissaries dragged him from the temple of Aeacus, where he had taken refuge, and put him to death; according to others, he was taken before Antipater at Athens or Cleonae. His body was afterwards removed to Athens for burial.

Hypereides was an ardent pursuer of "the beautiful," which in his time generally meant pleasure and luxury. His temper was easy-going and humorous; and hence, though in his development of the periodic sentence he followed Isocrates, the essential tendencies of his style are those of Lysias, whom he surpassed, however, in the richness of his vocabulary and in the variety of his powers. His diction was plain and forcible, though he occasionally indulged in long compound words probably borrowed from the Middle Comedy, with which, and with the everyday life of his time, he was in full sympathy. His composition was simple. He was specially distinguished for subtlety of expression, grace and wit, as well as for tact in approaching his case and handling his subject matter. Sir R. C. Jebb sums up the criticism of pseudo-Longinus (De sublimitate, 34) in the phrase-"Hypereides was the Sheridan of Athens."

Seventy-seven speeches were attributed to Hypereides, of which twenty-five were regarded as spurious even by ancient critics. It is said that a MS. of most of the speeches was in existence in the 16th century in the library of Matthias Corvinus, king of Hungary, at Ofen, but was destroyed at the capture of the city by the Turks in 1526. Only a few fragments were known until comparatively recent times. In 1847 large fragments of his speeches Against Demosthenes (see above) and For Lycophron (incidentally interesting as elucidating the order of marriage processions and other details of Athenian life, and the Athenian government of Lemnos), and the whole of the For Euxenippus (c. 330, a locus classicus on عiбaүүع入íal or state prosecutions), were found in a tomb at Thebes in Egypt, and in 1856 a considerable portion of a $\lambda o ́ y o s ~ \pi \alpha \rho \alpha v o ́ \mu \omega \nu$, a Funeral Oration over Leosthenes and his comrades who had fallen in the Lamian war, the best extant specimen of epideictic oratory (see Babington, Churchill). Towards the end of the century further discoveries were made of the conclusion of the speech Against Philippides (dealing with a $\gamma \rho \alpha \varphi \grave{\eta}$ п $\alpha \rho \alpha \nu o ́ \mu \omega \nu$, or indictment for the proposal of an unconstitutional measure, arising out of the disputes of the Macedonian and anti-Macedonian parties at Athens), and of the whole of the Against Athenogenes (a perfumer accused of fraud in the sale of his business). These have been edited by F. G. Kenyon (1893). An important speech that is lost is the Deliacus (frags. 67-75, Blass) on the presidency of the Delian temple claimed by both Athens and Delos, which was adjudged by the Amphictyons to Athens.

On Hypereides generally see pseudo-Plutarch, Decem oratorum vitae; F. Blass, Attische Beredsamkeit, iii.; R. C. Jebb, Attic Orators, ii. 381. A full list of editions and articles is given in F. Blass, Hyperidis orationes sex cum ceterarum fragmentis (1894, Teubner series), to which may be added I. Bassi, Le Quattro Orazioni di Iperide (introduction and notes, 1888), and J. E. Sandys in Classical Review (January 1895) (a review of the editions of Kenyon and Blass). For the discourse against Athenogenes see H. Weil, Études sur l'antiquité grecque (1900).

HYPERION, in Greek mythology, one of the Titans, son of Uranus and Gaea and father of Helios, the sun-god (Hesiod, Theog. 134, 371; Apollodorus i. 1. 2). In the well-known passage in Shakespeare (Hamlet, i. 2: "Hyperion to a satyr," where as in other poets the vowel -i- though really long, is shortened for metrical reasons) Hyperion is used for Apollo as expressive of the idea of beauty. The name is often used as an epithet of Helios, who is himself sometimes called simply Hyperion. It is explained as (1) he who moves above ( $\dot{u} \pi \rho \rho-\iota \omega \nu$ ), but the quantity of the vowel is against this; (2) he who is above ( $\dot{(\prime \pi} \varepsilon \rho-\omega \nu$ ). Others take it to be a patronymic in form, like Kроvĩ $\omega v$, Moגĩ $\omega v$.

HYPERSTHENE, a rock-forming mineral belonging to the group of orthorhombic pyroxenes. It differs from the other members (enstatite [q.v.] and bronzite) of this group in containing a considerable amount of iron replacing magnesium: the chemical formula is $(\mathrm{Mg}, \mathrm{Fe}) \mathrm{SiO}_{3}$. Distinctly developed crystals are rare, the mineral being usually found as foliated masses embedded in those igneous rocks-norite, hypersthene-andesite, \&c.-of which it forms an essential constituent. The coarsely grained labradorite-hypersthene-rock (norite) of the island of St Paul off the coast of Labrador has furnished the most typical material; and for this reason the mineral has been known as "Labrador hornblende" or paulite. The colour is brownish-black, and the pleochrism strong; the hardness is 6 , and the specific gravity $3.4-3.5$. On certain surfaces it displays a brilliant copper-red metallic sheen or schiller, which has the same origin as the bronzy sheen of bronzite ( $q . v$. ), but is even more pronounced. Like bronzite, it is sometimes cut and polished for ornamental purposes.
(L. J. S.)

HYPERTROPHY (Gr. Úпદ́ $\rho$, over, and т $\rho \circ \varphi \eta$, nourishment), a term in medicine employed to designate an abnormal increase in bulk of one or more of the organs or component tissues of the body (see Pathology). In its strict sense this term can only be applied where the increase affects the natural textures of a part, and is not applicable where the enlargement is due to the presence of some extraneous morbid formation. Hypertrophy of a part may manifest itself either by simply an increase in the size of its constituents, or by this combined with an increase in their number (hyperplasia). In many instances both are associated.

The conditions giving rise to hypertrophy are the reverse of those described as producing Atrophy (q.v.). They are concisely stated by Sir James Paget as being chiefly or only three, namely: (1) the increased exercise of a part in its healthy functions; (2) an increased accumulation in the blood of the particular materials which a part appropriates to its nutrition or in secretion; and (3) an increased afflux of healthy blood.

Illustrations are furnished of the first of these conditions by the high development of muscular tissue under habitual active exercise; of the second in the case of obesity, which is an hypertrophy of the fatty tissues, the elements of which are furnished by the blood; and of the third in the occasional overgrowth of hair in the neighbourhood of parts which are the seat of inflammation. Obviously therefore, in many instances, hypertrophy cannot be regarded as a deviation from health, but rather on the contrary as indicative of a high degree of nutrition and physical power. Even in those cases where it is found associated with disease, it is often produced as a salutary effort of nature to compensate for obstructions or other difficulties which have arisen in the system, and thus to ward off evil consequences. No better example of
this can be seen than in the case of certain forms of heart disease, where from defect at some of the natural orifices of that organ the onward flow of the blood is interfered with, and would soon give rise to serious embarrassment to the circulation, were it not that behind the seat of obstruction the heart gradually becomes hypertrophied, and thus acquires greater propelling power to overcome the resistance in front. Again, it has been noticed, in the case of certain double organs such as the kidneys, that when one has been destroyed by disease the other has become hypertrophied to such a degree as enables it to discharge the functions of both.

Hypertrophy may, however, in certain circumstances constitute a disease, as in goitre and elephantiasis ( $q . v$. ), and also in the case of certain tumours and growths (such as cutaneous excrescences, fatty tumours, mucous polypi, \&c.), which are simply enlargements of normal textures. Hypertrophy does not in all cases involve an increase in bulk; for, just as in atrophy there may be no diminution in the size of the affected organ, so in hypertrophy there may be no increase. This is apt to be the case where certain only of the elements of an organ undergo increase, while the others remain unaffected or are actually atrophied by the pressure of the hypertrophied tissue, as is seen in the disease known as cirrhosis of the liver.

A spurious hypertrophy is observed in the rare disease to which G. B. Duchenne applied the name of pseudo-hypertrophic paralysis. This ailment, which appears to be confined to children, consists essentially of a progressive loss of power accompanied with a remarkable enlargement of certain muscles or groups of muscles, more rarely of the whole muscular system. This increase of bulk is, however, not a true hypertrophy, but rather an excessive development of connective tissue in the substance of the muscles, the proper texture of which tends in consequence to undergo atrophy or degeneration. The appearance presented by a child suffering from this disease is striking. The attitude and gait are remarkably altered, the child standing with shoulders thrown back, small of the back deeply curved inwards, and legs wide apart, while walking is accompanied with a peculiar swinging or rocking movement. The calves of the legs, the buttocks, the muscles of the back, and occasionally other muscles, are seen to be unduly enlarged, and contrast strangely with the general feebleness. The progress of the disease is marked by increasing failure of locomotory power, and ultimately by complete paralysis of the limbs. The malady is little amenable to treatment, and, although often prolonged for years, generally proves fatal before the period of maturity.

HYPNOTISM, a term now in general use as covering all that pertains to the art of inducing the hypnotic state, or hypnosis, and to the study of that state, its conditions, peculiarities and effects. Hypnosis is a condition, allied to normal sleep (Gr. ürvoc), which can be induced in a large majority of normal persons. Its most characteristic and constant symptom is the increased suggestibility of the subject (see Suggestion). Other symptoms are very varied and differ widely in different subjects and in the same subject at different times. There can be no doubt that the increased suggestibility and all the other symptoms of hypnosis imply some abnormal condition of the brain of a temporary and harmless nature. It would seem that in all ages and in almost all countries individuals have occasionally fallen into abnormal states of mind more or less closely resembling the hypnotic state, and have thereby excited the superstitious wonder of their fellows. In some cases the state has been deliberately induced, in others it has appeared spontaneously, generally under the influence of some emotional excitement. The most familiar of these allied states is the somnambulism or sleep-walking to which some persons seem to be hereditarily disposed. Of a rather different type are the states of ecstasy into which religious enthusiasts have occasionally fallen and which were especially frequent among the peoples of Europe during the middle ages. While in this condition individuals have appeared to be insensitive to all impressions made on their sense-organs, even to such as would excite acute pain in normal persons, have been capable of maintaining rigid postures for long periods of time, have experienced vivid hallucinations, and have produced, through the power of the imagination, extraordinary organic changes in the body, such as the bloody stigmata on the hands and feet in several well-attested instances. It has been proved in recent years that effects of all these kinds may be produced by hypnotic suggestion. Different again, but closely paralleled by some subjects in hypnosis, is the state of latah into which a certain proportion of persons of the Malay race are liable to fall. These persons, if their attention is suddenly and forcibly drawn to any other person, will begin to imitate his every action and attitude, and may do so in spite of their best efforts to restrain their imitative movements. Among the half-bred French-Canadians of the forest regions of Canada occur individuals, known as "jumpers," who are liable to fall suddenly into a similar state of abject imitativeness, and the same peculiar behaviour has been observed among some of the remote tribes of Siberia.

The deliberate induction of states identical with, or closely allied to, hypnosis is practised by many barbarous and savage peoples, generally for ceremonial purposes. Thus, certain dervishes of Algiers are said to induce in themselves, by the aid of the sound of drums, monotonous songs and movements, a state in which they are insensitive to pain, and a similar practice of religious devotees is reported from Tibet. Perhaps the most marvellous achievement among well-attested cases of this sort is that of certain yogis of Hindustan; by long training and practice they seem to acquire the power of arresting almost completely all their vital functions. An intense effort of abstraction from the impressions of the outer world, a prolonged fixation of the eyes upon the nose or in some other strained position and a power of greatly slowing the respiration, these seem to be important features of their procedure for the attainment of their abnormal states.

In spite of the wide distribution in time and space, and the not very infrequent occurrence, of these instances of states identical with or allied to hypnosis, some three centuries of enthusiastic investigation and of bitter controversy were required to establish the occurrence of the hypnotic state among the facts accepted by the world of European science. Scientific interest in them may be traced back at least as far as the end of the 16 th century. Paracelsus had founded the "sympathetic system" of medicine, according to which the stars and other bodies, especially magnets, influence men by means of a subtle emanation or fluid that pervades all space. J. B. van Helmont, a distinguished man of science of the latter part of the 16th century, extended this doctrine by teaching that a similar magnetic fluid radiates from men, and that it can be guided by their wills to influence directly the minds and bodies of others. In the middle of the 17th century there appeared in England several persons who claimed to have the power of curing diseases by stroking with the hand. Notable amongst these was Valentine Greatrakes, of Affane, in the county of Waterford, Ireland, who was born in February 1628, and who attracted great attention in England by his supposed power of curing the king's evil, or scrofula. Many of the most distinguished scientific and theological men of the day, such as Robert Boyle and R. Cudworth, witnessed and attested the cures supposed to be effected by Greatrakes, and thousands of sufferers crowded to him from all parts of the kingdom. About the middle of the 18th century John Joseph Gassner, a Roman Catholic priest in Swabia, took up the notion that the majority of diseases arose from demoniacal possession, and could only be cured by exorcism. His method was undoubtedly similar to that afterwards followed by Mesmer and others, and he had an extraordinary influence over the nervous systems of his patients. Gassner, however, believed his power to be altogether supernatural.

But it was not until the latter part of the 18th century that the doctrine of a magnetic fluid excited great popular interest and became the subject of fierce controversy in the scientific world. F. A. Mesmer (q.v.), a physician of Vienna, was largely instrumental in bringing the doctrine into prominence. He developed it by postulating a specialized variety of magnetic fluid which he called animal magnetism; and he claimed to be able to cure many diseases by means of this animal magnetism, teaching, also, that it may be imparted to and stored up in inert objects, which are thereby rendered potent to cure disease.

It would seem that Mesmer himself was not acquainted with the artificial somnambulism which for nearly a century was called mesmeric or magnetic sleep, and which is now familiar as hypnosis of a well-marked degree. It was observed and described about the year 1780 by the marquis de Puységur, a disciple of Mesmer, who showed that, while subjects were in this state, not only could some of their diseases be cured, but also their movements could be controlled by the "magnetizer," and that they usually remembered nothing of the events of the period of sleep when restored to normal consciousness. These are three of the most important features of hypnosis, and the modern study of hypnotism may therefore be said to have been initiated at this date by Puységur. For, though it is probable that this state had often been induced by the earlier magnetists, they had not recognized that the peculiar behaviour of their patients resulted from their being plunged into this artificial sleep, but had attributed all the symptoms they observed to the direct physical action of external agents upon the patients.

The success of Mesmer and his disciples, especially great in the fashionable world, led to the appointment in Paris of a royal commission for the investigation of their claims. The commission, which included men of great eminence, notably A. L. Lavoisier and Benjamin Franklin, reported in the year 1784 that it could not accept the evidence for the existence of the magnetic fluid; but it did not express an opinion as to the reality of the cures said to be effected by its means, nor as to the nature of the magnetic sleep. This report and the social upheavals of the following years seem to have abolished the public interest in "animal magnetism" for the space of one generation; after which Alexandre Bertrand, a Parisian physician, revived it by his acute investigations and interpretations of the phenomena. Bertrand was the first to give an explanation of the facts of the kind that is now generally accepted. He exhibited the affinity of the "magnetic sleep" to ordinary somnambulism, and he taught that the peculiar effects are to be regarded as due to the suggestions of the operator working
themselves out in the mind and body of the "magnetized" subject, i.e. he regarded the influence of the magnetizer as exerted in the first instance on the mind of the subject and only indirectly through the mind upon the body. Shortly after this revival of public interest, namely in the year 1831, a committee of the Academy of Medicine of Paris reported favourably upon "magnetism" as a therapeutic agency, and before many years had elapsed it was extensively practised by the physicians of all European countries, with few exceptions, of which England was the most notable. Most of the practitioners of this period adhered to the doctrine of the magnetic fluid emanating from the operator to his patient, and the acceptance of this doctrine was commonly combined with belief in phrenology, astrology and the influence of metals and magnets, externally applied, in curing disease and in producing a variety of strange sensations and other affections of the mind. These beliefs, claiming to rest upon carefully observed facts, were given a new elaboration and a more imposing claim to be scientifically established by the doctrine of odylic force propounded by Baron Karl von Reichenbach. In this mass of ill-based assertion and belief the valuable truths of "animal magnetism" and the psychological explanations of them given by Bertrand were swamped and well-nigh lost sight of. For it was this seemingly inseparable association between the facts of hypnotism and these bizarre practices and baseless beliefs that blinded the larger and more sober part of the scientific world, and led them persistently to assert that all this group of alleged phenomena was a mass of quackery, fraud and superstition. And the fact that magnetism was practised for pecuniary gain, often in a shameless manner, by exponents who claimed to cure by its means every conceivable ill, rendered this attitude on the part of the medical profession inevitable and perhaps excusable, though not justifiable. It was owing to this baleful association that John Elliotson, one of the leading London physicians of that time, who became an ardent advocate of "magnetism" and who founded and edited the Zoist in the interests of the subject, was driven out of the profession. This association may perhaps be held, also, to excuse the hostile attitude of the medical profession towards James Esdaile, a surgeon, who, practising in a government hospital in Calcutta among the natives of India, performed many major operations, such as the amputation of limbs, painlessly and with the most excellent results by aid of the "magnetic" sleep. For both Elliotson and Esdaile, though honourable practitioners, accepted the doctrine of the "magnetic" fluid and many of the erroneous beliefs that commonly were bound up with it.

In 1841 James Braid, a surgeon of Manchester, rediscovered independently Bertrand's physiological and psychological explanations of the facts, carried them further, and placed "hypnotism," as he named the study, on a sound basis. Braid showed that subjects in "magnetic" sleep, far from being in a profoundly insensitive condition, are often abnormally susceptible to impressions on the senses, and showed that many of the peculiarities of their behaviour were due to suggestions, made verbally or otherwise, but unintentionally, by the operator or by onlookers.

It seems, on looking back on the history of hypnotism, that at this time it was in a fair way to secure general recognition as a most interesting subject of psychological study and a valuable addition to the resources of the physician. But it was destined once more to be denied its rights by official science and to fall back into disrepute. This was due to the coincidence about the year 1848 of two events of some importance, namely-the discovery of the anaesthetic properties of chloroform and the sudden rise of modern spiritualism. The former afforded a very convenient substitute for the most obvious practical application of hypnotism, the production of anaesthesia during surgical operations; the latter involved it once more in a mass of fraud and superstition, and, for the popular mind, drove it back to the region of the marvellous, the supernatural and the dangerous, made it, in fact, once more a branch of the black art.

From this time onward there took place a gradual differentiation of the "animal magnetism" of the 18th century into two diverging branches, hypnotism and spiritualism, two branches which, however, are not yet entirely separated and, perhaps, never will be. At the same time the original system of "animal magnetism" has lived on in an enfeebled condition and is now very nearly, though not quite, extinct.

In the development of hypnotism since the time of Braid we may distinguish three lines, the physiological, the psychological and the pathological. The last may be dismissed in a few words. Its principal representative was J. M. Charcot, who taught at the Salpêtrière in Paris that hypnosis is essentially a symptom of a morbid condition of hysteria or hystero-epilepsy. This doctrine, which, owing to the great repute enjoyed by Charcot, has done much to retard the application of hypnotism, is now completely discredited. The workers of the physiological party attached special importance to the fixation of the eyes, or to other forms of long continued and monotonous, or violent, sensory stimulation in the induction of hypnosis. They believed that by acting on the senses in these ways they induced a peculiar condition of the nervous system, which consisted in the temporary abolition of the cerebral functions and the consequent reduction of the subject to machine-like unconscious automatism. The leading exponent of this
view was R. Heidenhain, professor of physiology at Breslau, whose experimental investigations played a large part in convincing the scientific world of the genuineness of the leading symptoms of hypnosis. The purely psychological doctrine of hypnosis puts aside all physical and physiological influences and effects as of but little or no importance, and seeks a psychological explanation of the induction of hypnosis and of all the phenomena. This dates from 1884, when H. Bernheim, professor of medicine at Nancy, published his work De la Suggestion (republished in 1887 with a second part on the therapeutics of hypnotism). Bernheim was led to the study of hypnotism by A. A. Liébeault, who for twenty years had used it very largely and successfully in his general practice among the poor of Nancy. Liébeault rediscovered independently, and Bernheim made known to the world the truths, twice previously discovered and twice lost sight of, that expectation is a most important factor in the induction of hypnosis, that increased suggestibility is its essential symptom, and that in general the operator works upon his patient by mental influences. Although they went too far in the direction of ignoring the peculiarity of the state of the brain in hypnosis and the predisposing effect of monotonous sensory stimulation, and in seeking to identify hypnosis with normal sleep, the views of the Nancy investigators have prevailed, and are now in the main generally accepted. Their methods of verbal suggestion have been adopted by leading physicians in almost all civilized countries and have been proved to be efficacious in the relief of many disorders; and as a method of psychological investigation hypnotism has proved, especially in the hands of the late Ed. Gurney, of Dr Pierre Janet and of other investigators, capable of throwing much light on the constitution of the mind, has opened up a number of problems of the deepest interest, and has done more than any other of the many branches of modern psychology to show the limitations and comparative barrenness of the old psychology that relied on introspection alone and figured as a department of general philosophy. In England, "always the last to enter into the general movement of the European mind," the prejudice, incredulity and ignorant misrepresentation with which hypnotism has everywhere been received have resisted its progress more stubbornly than elsewhere; but even in England its reality and its value as a therapeutic agent have at last been officially recognized. In 1892, just fifty years after Braid clearly demonstrated the facts and published explanations of them almost identical with those now accepted, a committee of the British Medical Association reported favourably upon hypnotism after a searching investigation; it is now regularly employed by a number of physicians of high standing, and the formation in 1907 of "The Medical Society for the Study of Suggestive Therapeutics" shows that the footing it has gained is likely to be made good.

Induction of Hypnosis.-It has now been abundantly proved that hypnosis can be induced in the great majority of normal persons, provided that they willingly submit themselves to the process. Several of the most experienced operators have succeeded in hypnotizing more than $90 \%$ of the cases they have attempted, and most of them are agreed that failure to induce hypnosis in any case is due either to lack of skill and tact on the part of the operator, or to some unfavourable mental condition of the subject. It has often been said that some races or peoples are by nature more readily hypnotizable than others; of the French people especially this has been maintained. But there is no sufficient ground for this statement. The differences that undoubtedly obtain between populations of different regions in respect to the ease or difficulty with which a large proportion of all persons can be hypnotized are sufficiently explained by the differences of the attitude of the public towards hypnotism; in France, e.g., and especially in Nancy, hypnotism has been made known to the public chiefly as a recognized auxiliary to the better known methods of medical treatment, whereas in England the medical profession has allowed the public to make acquaintance with hypnotism through the medium of disgusting stage-performances whose only object was to raise a laugh, and has, with few exceptions, joined in the general chorus of condemnation and mistrust. Hence in France patients submit themselves with confidence and goodwill to hypnotic treatment, whereas in England it is still necessary in most cases to remove an ill-based prejudice before the treatment can be undertaken with hope of success. For the confidence and goodwill of the patient are almost essential to success, and even after hypnosis has been induced on several occasions a patient may be so influenced by injudicious friends that he cannot again be hypnotized or, if hypnotized, is much less amenable to the power of suggestion. Various methods of hypnotization are current, but most practitioners combine the methods of Braid and of Bernheim. After asking the patient to resign himself passively into their hands, and after seating him in a comfortable arm-chair, they direct him to fix his eyes upon some small object held generally in such a position that some slight muscular strain is involved in maintaining the fixation; they then suggest to him verbally the idea or expectation of sleep and the sensations that normally accompany the oncoming of sleep, the heaviness of the eyes, the slackness of the limbs and so forth; and when the eyes show signs of fatigue, they either close them by gentle pressure or tell the subject to close them. Many also pass their hands slowly and regularly over the face, with or without contact. The old magnetizers attached great importance to such "passes," believing that by them the "magnetic fluid" was imparted to the patient; but it seems clear that, in so far as they contribute to induce hypnosis, it is in their character merely of
gentle, monotonous, sensory stimulations. A well-disposed subject soon falls into a drowsy state and tends to pass into natural sleep; but by speech, by passes, or by manipulating his limbs the operator keeps in touch with him, keeps his waning attention open to the impressions he himself makes. Most subjects then find it difficult or impossible to open their eyes or to make any other movement which is forbidden or said to be impossible by the operator, although they may be fully conscious of all that goes on about them and may have the conviction that if they did but make an effort they could break the spell. This is a light stage of hypnosis beyond which some subjects can hardly be induced to pass and beyond which few pass at the first attempt. But on successive occasions, or even on the first occasion, a favourable subject passes into deeper stages of hypnosis. Many attempts have been made to distinguish clearly marked and constantly occurring stages. But it seems now clear that the complex of symptoms displayed varies in all cases with the idiosyncrasies of the subject and with the methods adopted by the operator. In many subjects a waxy rigidity of the limbs appears spontaneously or can be induced by suggestion; the limbs then retain for long periods without fatigue any position given them by the operator. The most susceptible subjects pass into the stage known as artificial somnambulism. In this condition they continue to respond to all suggestions made by the operator, but seem as insensitive to all other impressions as a person in profound sleep or in coma; and on awaking from this condition they are usually oblivious of all that they have heard, said or done during the somnambulistic period. When in this last condition patients are usually more profoundly influenced by suggestions, especially post-hypnotic suggestions, than when in the lighter stages; but the lighter stages suffice for the production of many therapeutic effects. When a patient is completely hypnotized, his movements, his senses, his ideas and, to some extent, even the organic processes over which he has no voluntary control become more or less completely subject to the suggestions of the operator; and usually he is responsive to the operator alone (rapport) unless he is instructed by the latter to respond also to the suggestions of other persons. If left to himself the hypnotized subject will usually awake to his normal state after a period which is longer in proportion to the depth of hypnosis; and the deeper stages seem to pass over into normal sleep. The subject can in almost every case be brought quickly back to the normal state by the verbal command of the operator.

The Principal Effects produced by Suggestion during Hypnosis.-The subject may not only be rendered incapable of contracting any of the muscles of the voluntary system, but may also be made to use them with extraordinarily great or sustained force (though by no means in all cases). He can with difficulty refrain from performing any action commanded by the operator, and usually carries out any simple command without hesitation. Any one of the sense-organs, or any sensory region such as the skin or deep tissues of one limb may be rendered anaesthetic by verbal suggestion, aided perhaps by some gentle manipulation of the part. On this fact depends the surgical application of hypnotism. Sceptical observers are always inclined to doubt the genuineness of the anaesthesia produced by a mere word of command, but the number of surgical operations performed under hypnotic anaesthesia suffices to put its reality beyond all question. A convincing experiment may, however, be made on almost any good subject. Anaesthesia of one eye may be suggested and its reality tested in the following way. Anaesthesia of the left eye may be suggested, and the subject be instructed to fix his gaze on a distant point and to give some signal as soon as he sees the operator's finger in the peripheral field of view. The operator then brings his finger slowly from behind and to the right forwards towards the subject's line of sight. The subject signals as soon as it crosses the normal temporal boundary of the field of view of the right eye. The operator then brings his finger forward from a point behind and to the left of the subject's head. The subject allows it to cross the monocular field of the left eye and signals only when the finger enters the field of vision of the right eye across its nasal boundary. Since few persons, other than physiologists or medical men, are aware of the relations of the boundaries of the monocular and binocular fields of vision, the success of this experiment affords proof that the finger remains invisible to the subject during its passage across the monocular field of the left eye. The abolition of pain, especially of neuralgias, the pain of rheumatic and other inflammations, which is one of the most valuable applications of hypnotism, is an effect closely allied to the production of such anaesthesia.

It has often been stated that in hypnosis the senses may be rendered extraordinarily acute or hyperaesthetic, so that impressions too faint to affect the senses of the normal person may be perceived by the hypnotized subject; but in view of the fact that most observers are ignorant of the normal limits of sensitivity and discrimination, all such statements must be received with caution, until we have more convincing evidence than has yet been brought forward.
Positive and Negative Hallucinations are among the most striking effects of hypnotic suggestion. A good subject may be made to experience an hallucinatory perception of almost any object, the more easily the less unusual and out of harmony with the surroundings is the suggested object. He may, e.g., be given a blank card and asked if he thinks it a good photograph of himself. He may then assent and describe the photograph in some detail, and,
what is more astonishing, he may pick out the card as the one bearing the photograph, after it has been mixed with other similar blank cards. This seems to be due to the part played by points de repère, insignificant details of surface or texture, which serve as an objective basis around which the hallucinatory image is constructed by the pictorial imagination of the subject. A negative hallucination may be induced by telling the subject that a certain object or person is no longer present, when he ignores in every way that object or person. This is more puzzling than the positive hallucination and will be referred to again in discussing the theory of hypnosis. Both kinds of hallucination tend to be systematically and logically developed; if, e.g., the subject is told that a certain person is no longer visible, he may become insensitive to impressions made on any sense by that person.

Delusions, or false beliefs as to their present situation or past experiences may be induced in many subjects. On being assured that he is some other person, or that he is in some strange situation, the subject may accept the suggestion and adapt his behaviour with great histrionic skill to the induced delusion. It is probable that many, perhaps all, subjects are vaguely aware, as we sometimes are in dreams, that the delusions and hallucinations they experience are of an unreal nature. In the lighter stages of hypnosis a subject usually remembers the events of his waking life, but in the deeper stages he is apt, while remembering the events of previous hypnotic periods, to be incapable of recalling his normal life; but in this respect, as also in respect to the extent to which on awaking he remembers the events of the hypnotic period, the suggestions of the operator usually play a determining part.

Among the organic changes that have been produced by hypnotic suggestion are slowing or acceleration of the cardiac and respiratory rhythms; rise and fall of body-temperature through two or three degrees; local erythema and even inflammation of the skin with vesication or exudation of small drops of blood; evacuation of the bowel and vomiting; modifications of the secretory activity of glands, especially of the sweat-glands.

Post-hypnotic Effects.-Most subjects in whom any appreciable degree of hypnosis can be induced show some susceptibility to post-hypnotic suggestion, i.e. they may continue to be influenced, when restored to the fully waking state, by suggestions made during hypnosis, more especially if the operator suggests that this shall be the case; as a rule, the deeper the stage of hypnosis reached, the more effective are post-hypnotic suggestions. The therapeutic applications of hypnotism depend in the main upon this post-hypnotic continuance of the working of suggestions. If a subject is told that on awaking, or on a certain signal, or after the lapse of a given interval of time from the moment of awaking, he will perform a certain action, he usually feels some inclination to carry out the suggestion at the appropriate moment. If he remembers that the action has been suggested to him he may refuse to perform it, and if it is one repugnant to his moral nature, or merely one that would make him appear ridiculous, he may persist in his refusal. But if the action is of a simple and ordinary nature he will usually perform it, remarking that he cannot be comfortable till it is done. If the subject was deeply hypnotized and remembers nothing of the hypnotic period, he will carry out the post-hypnotic suggestion in almost every case, no matter how complicated or absurd it may be, so long as it is not one from which his normal self would be extremely averse; and he will respond appropriately to the suggested signals, although he is not conscious of their having been named; he will often perform the action in a very natural way, and will, if questioned, give some more or less adequate reason for it. Such actions, determined by post-hypnotic suggestions of which no conscious memory remains, may be carried out even after the lapse of many weeks or even months. Inhibitions of movement, anaesthesia, positive and negative hallucinations, and delusions may also be made to persist for brief periods after the termination of hypnosis; and organic effects, such as the action of the bowels, the oncoming of sleep and the cessation of pain, may be determined by post-hypnotic suggestion. In short, it may be said that in a good subject all the kinds of suggestion which will take effect during hypnosis will also be effective if given as post-hypnotic suggestions.

Theory of the Hypnotic State.-Very many so called theories of hypnosis have been propounded, but few of them demand serious consideration. One author ascribes all the symptoms to cerebral anaemia, another to cerebral congestion, a third to temporary suppression of the functions of the cerebrum, a fourth to abnormal cerebral excitability, a fifth to the independent functioning of one hemisphere. Another seeks to explain all the facts by saying that in hypnosis our normal consciousness disappears and is replaced by a dreamconsciousness; and yet another by the assumption that every human organism comprises two mental selves or personalities, a normal one and one which only comes into activity during sleep and hypnosis. Most of these "theories" would, even if true, carry us but a little way towards a complete understanding of the facts. There is, however, one theory or principle of explanation which is now gradually taking shape under the hands of a number of the more penetrating workers in this field, and which does seem to render intelligible many of the principle facts. This is the theory of mental dissociation.

It is clear that a theory of hypnosis must attempt to give some account of the peculiar condition of the brain which is undoubtedly present as an essential feature of the state. It is therefore not enough to say with Bernheim that hypnosis is a state of abnormally increased suggestibility produced by suggestion; nor is it enough, though it is partially true, to say that it is a state of mono-ideism or one of abnormally great concentration of attention. Any theory must be stated in terms of physiological psychology, it must take account of both the psychical and the nervous peculiarities of the hypnotic state; it must exhibit the physiological condition as in some degree similar to that obtaining in normal sleep; but principally it must account for that abnormally great receptivity for ideas, and that abnormally intense and effective operation of ideas so received, which constitute abnormally great suggestibility.

The theory of mental dissociation may be stated in purely mental terms, or primarily in terms of nervous structure and function, and the latter mode of statement is probably the more profitable at the present time. The increased effectiveness of ideas might be due to one of two conditions: (1) it might be that certain tracts of the brain or the whole brain were in a condition of abnormally great excitability; or (2) an idea might operate more effectively in the mind and on the body, not because it, or the underlying brain-process was more intense than normally, but because it worked out its effects free from the interference of contrary or irrelevant ideas that might weaken its force. It is along this second line that the theory of mental dissociation attempts to explain the increased suggestibility of hypnosis. To understand the theory we must bear in mind the nature of mental process in general and of its nervous concomitants. Mental process consists in the interplay, not merely of ideas, but rather of complex dispositions which are the more or less enduring conditions of the rise of ideas to consciousness. Each such disposition seems capable of remaining inactive or quiescent for long periods, and of being excited in various degrees, either by impressions made upon the sense-organs or by the spread of excitement from other dispositions. When its excitement rises above a certain pitch of intensity, the corresponding idea rises to the focus of consciousness. These dispositions are essential factors of all mental process, the essential conditions of all mental retention. They may be called simply mental dispositions, their nature being left undefined; but for our present purpose it is advantageous to regard them as neural dispositions, complex functional groups of nervous elements or neurones. The neurones of each such group must be conceived as being so intimately connected with one another that the excitement of any part of the group at once spreads through the whole group or disposition, so that it always functions as a unit. The whole cerebrum must be conceived as consisting of a great number of such dispositions, inextricably interwoven, but interconnected in orderly fashion with very various degrees of intimacy; groups of dispositions are very intimately connected to form neural systems, so that the excitement of any one member of such a system tends to spread in succession to all the other members. On the other hand, it is a peculiarity of the reciprocal relations of all such dispositions and systems that the excitement of any one to such a degree that the corresponding idea rises to consciousness prevents or inhibits the excitement of others, i.e. all of them are in relations of reciprocal inhibition with one another (see Muscle and Nerve). The excitement of dispositions associated together to form a system tends towards some end which, either immediately or remotely, is an action, a bodily movement, in many cases a movement of the organs of speech only. Now we know from many exact experiments that the neural dispositions act and react upon one another to some extent, even when they are excited only in so feeble a degree that the corresponding ideas do not rise to consciousness. In the normal state of the brain, then, when any idea is present to consciousness, the corresponding neural disposition is in a state of dominant excitement, but the intensity of that excitement is moderated, depressed or partially inhibited by the sub-excitement of many rival or competing dispositions of other systems with which it is connected. Suppose now that all the nervous connexions between the multitudinous dispositions of the cerebrum are by some means rendered less effective, that the associationpaths are partially blocked or functionally depressed; the result will be that, while the most intimate connexions, those between dispositions of any one system remain functional or permeable, the weaker less intimate connexions, those between dispositions belonging to different systems will be practically abolished for the time being; each system of dispositions will then function more or less as an isolated system, and its activity will no longer be subject to the depressing or inhibiting influence of other systems; therefore each system, on being excited in any way, will tend to its end with more than normal force, being freed from all interferences; that is to say, each idea or system of ideas will tend to work itself out and to realize itself in action immediately, without suffering the opposition of antagonistic ideas which, in the normal state of the brain, might altogether prevent its realization in action.

The theory of mental dissociation assumes that the abnormal state of the brain that obtains during hypnosis is of this kind, a temporary functional depression of all, or of many of the associations or nervous links between the neural dispositions; that is, it regards hypnosis as a state of relative dissociation. The lighter the stage of hypnosis the slighter is the degree of dissociation, the deeper the stage the more nearly complete is the dissociation.

It is not essential that the theory should explain in what change this stage of dissociation consists, but a view compatible with all that we know of the functions of the central nervous system may be suggested. The connexions between neural dispositions involve synapses or celljunctions, and these seem to be the places of variable resistance which demarcate the dispositions and systems; and there is good reason to think that their resistances vary with the state of the neurones which they connect, being lowered when these are excited and raised when their excitement ebbs. Now, in the waking state, the varied stimuli, which constantly rain upon all the sense-organs, maintain the whole cerebrum in a state of sub-excitement, keep all the cerebral neurones partially charged with free nervous energy. When the subject lies down to sleep or submits himself to the hypnotizer he arrests as far as possible the flow of his thoughts, and the sensory stimuli are diminished in number and intensity. Under these conditions the general cerebral activity tends to subside, the free energy with which the cerebral neurones are charged ebbs away, and the synaptic resistances rise proportionally; then the effect of sensory impressions tends to be confined to the lower nervous level, and the brain tends to come to rest. If this takes place the condition of normal sleep is realized. But in inducing hypnosis the operator, by means of his words and manipulations, keeps one system of ideas and the corresponding neural system in activity, namely, the ideas connected with himself; thus he keeps open one channel of entry to the brain and mind, and through this one open channel he can introduce whatever ideas he pleases; and the ideas so introduced then operate with abnormally great effect because they work in a free field, unchecked by rival ideas and tendencies.

This theory of relative dissociation has two great merits: in the first place it goes far towards enabling us to understand in some degree most of the phenomena of hypnosis; secondly, we have good evidence that dissociation really occurs in deep hypnosis and in some allied states. Any one may readily work out for himself the application of the theory to the explanation of the power of the operator's suggestions to control movement, to induce anaesthesia, hallucinations and delusions, and to exert on the organic processes an influence greater than can be exerted by mental processes in the normal state of the brain. But the positive evidence of the occurrence of dissociation is a matter of great psychological interest and its nature must be briefly indicated. The phenomena of automatic speech and writing afford the best evidence of cerebral dissociation. Many persons can, while in an apparently normal or but very slightly abnormal condition, produce automatic writing, i.e. intelligibly written sentences, in some cases long connected passages, of whose import they have no knowledge, their self-conscious intelligence being continuously directed to some other task. The carrying out of post-hypnotic suggestions affords in many cases similar evidence. Thus a subject may be told that after waking he will perform some action when a given signal, such as a cough, is repeated for the fifth time. In the post-hypnotic state he remains unaware of his instructions, is not conscious of noting the signals, and yet carries out the suggestion at the fifth signal, thereby proving that the signals have been in some sense noted and counted. Many interesting varieties of this experiment have been made, some of much greater complexity; but all agreeing in indicating that the suggested action is prepared for and determined by cerebral processes that do not affect the consciousness of the subject, but seem to occur as a system of processes detached from the main stream of cerebral activity; that is to say, they imply the operation of relatively dissociated neural systems.

Many authorities go further than this; they argue that, since actions of the kind described are determined by processes which involve operations, such as counting, that we are accustomed to regard as distinctly mental in character and that normally involve conscious activity, we must believe that in these cases also consciousness or psychical activity is involved, but that it remains as a separate system or stream of consciousness concurrent with the normal or personal consciousness.

In recent years the study of various abnormal mental states, especially the investigations by French physicians of severe forms of hysteria, have brought to light many facts which seem to justify this assumption of a secondary stream of consciousness, a co- or sub-consciousness coexistent with the personal consciousness; although, from the nature of the case, an absolute proof of such co-consciousness can hardly be obtained. The co-consciousness seems to vary in degree of complexity and coherence from a mere succession of fragmentary sensations to an organized stream of mental activity, which may rival in all respects the primary consciousness; and in cases of the latter type it is usual to speak of the presence of a secondary personality. The co-consciousness seems in the simpler cases, e.g. in cases of hysterical or hypnotic anaesthesia, to consist of elements split off from the normal primary consciousness, which remains correspondingly poorer; and the assumption is usually made that such a stream of coconsciousness is the psychical correlate of groups and systems of neurones dissociated from the main mass of cerebral neurones. If, in spite of serious objections, we entertain this conception, we find that it helps us to give some account of various hypnotic phenomena that otherwise remain quite inexplicable; some such conception seems to be required more
particularly by the facts of negative hallucination and the execution of post-hypnotic suggestions involving such operations as counting and exact discrimination without primary consciousness.

Supernormal Hypnotic Phenomena.-The facts hitherto considered, strange and perplexing as many of them are, do not seem to demand for their explanation any principles of action fundamentally different from those operative in the normal human mind. But much of the interest that has centred in hypnotism in recent years has been due to the fact that some of its manifestations seem to go beyond all such principles of explanation, and to suggest the reality of modes of influence and action that science has not hitherto recognized. Of these by far the best attested are the post-hypnotic unconscious reckoning of time and telepathy or "thoughttransference" (for the latter see Telepathy). The post-hypnotic reckoning and noting of the lapse of time seems in some instances to have been carried out, in the absence of all extraneous aids and with complete unconsciousness on the part of the normal personality, with such extreme precision that the achievement cannot be accounted for by any intensification of any faculty that we at present recognize or understand. Thus, Dr Milne Bramwell has reported the case of a patient who, when commanded in hypnosis to perform some simple action after the lapse of many thousands of minutes, would carry out the suggestion punctually to the minute, without any means of knowing the exact time of day at which the suggestion was given or the time of day at the moment its performance fell due; more recently a similar case, even more striking in some respects, has been carefully observed and described by Dr T. W. Mitchell. Other reported phenomena, such as telaesthesia or clairvoyance, and telekinesia, are hardly sufficiently well attested to demand serious consideration in this place.
Medical Applications of Hypnotism.-The study and practice of hypnotism is not yet, and probably never will be, regarded as a normal part of the work of the general practitioner. Its successful application demands so much time, tact, and special experience, that it will probably remain, as it is now, and as it is perhaps desirable that it should remain, a specialized branch of medical practice. In England it is only in recent years that it has been possible for a medical man to apply it in his practice without incurring professional odium and some risk of loss of reputation. That, in certain classes of cases, it may effect a cure or bring relief when all other modes of treatment are of no avail is now rapidly becoming recognized; but it is less generally recognized that it may be used with great advantage as a supplement to other modes of treatment in relieving symptoms that are accentuated by nervous irritability or mental disturbance. A third wide field of usefulness lies before it in the cure of undesirable habits of many kinds. Under the first heading may be put insomnia, neuralgia, neurasthenia, hysteria in almost all its many forms; under the second, inflammations such as that of chronic rheumatism, contractures and paralyses resulting from gross lesion of the brain, epilepsy, dyspepsia, menstrual irregularities, sea-sickness; under the third, inebriety, the morphia and other drug habits, nail-biting, enuresis nocturna, masturbation, constipation, facial and other twitchings. In pronounced mental diseases hypnotism seems to be almost useless; for in general terms it may be said that it can be applied most effectively where the brain, the instrument through which it works, is sound and vigorous. The widespread prejudice against the use of hypnotism is no doubt largely due to the marvellous and (to most minds) mysterious character of the effects producible by its means; and this prejudice may be expected to diminish as our insight into the mode of its operation deepens. The more purely bodily results achieved by hypnotic suggestion become in some degree intelligible if we regard it as a powerful means of diverting nervous energy from one channel or organ to others, so as to give physiological rest to an overworked organ or tissue, or so as to lead to the atrophy of one nervous habit and the replacement of it by a more desirable habit. And in the cure of those disorders which involve a large mental element the essential part played by it is to drive out some habitually recurrent idea and to replace it by some idea, expectation or conviction of healthy tendency.

It seems clear that the various systems of "mind-curing" in the hands of persons lacking all medical training, which are now so frequently the cause of distressing and needless disasters, owe their rapid spread to the fact that the medical profession has hitherto neglected to attach sufficient importance to the mental factor in the causation and cure of disease; and it seems clear, too, that a more general and more intelligent appreciation of the possibilities of hypnotic treatment would constitute the best means at the disposal of the profession for combating this growing evil.

The Dangers of Hypnotism.-Much has been written on this head of late years, and some of the enthusiastic advocates of hypnotic treatment have done harm to their cause by ignoring or denying in a too thoroughgoing manner the possibility of undesirable results of the spread of the knowledge and practice of hypnotism. Like all powerful agencies, chloroform or morphia, dynamite or strong electric currents, hypnotic suggestion can only be safely used by those who have special knowledge and experience, and, like them, it is liable to abuse. There is little doubt that, if a subject is repeatedly hypnotized and made to entertain all kinds of absurd
delusions and to carry out very frequently post-hypnotic suggestions, he may be liable to some ill-defined harm; also, that an unprincipled hypnotizer might secure an undue influence over a naturally weak subject.

But there is no ground for the belief that hypnotic treatment, applied with good intentions and reasonable care and judgment, does or can produce deleterious effects, such as weakening of the will or liability to fall spontaneously into hypnosis. All physicians of large experience in hypnotic practice are in agreement in respect to this point. But some difference of opinion exists as to the possibility of deliberately inducing a subject to commit improper or criminal actions during hypnosis or by post-hypnotic suggestion. There is, however, no doubt that subjects retain even in deep hypnosis a very considerable power of resistance to any suggestion that is repugnant to their moral nature; and it has been shown that, on some cases in which a subject in hypnosis is made to perform some ostensibly criminal action, such as firing an unloaded pistol at a bystander or putting poison into a cup for him to drink, he is aware, however obscurely, of the unreal nature of the situation. Nevertheless it must be admitted that a person lacking in moral sentiments might be induced to commit actions from which in the normal state he would abstain, if only from fear of punishment; and it is probable that a skilful and evil-intentioned operator could in some cases so deceive a well-disposed subject as to lead him into wrong-doing. The proper precaution against such dangers is legislative regulation of the practice of hypnotism such as is already enforced in some countries.

Bibliography.-The literature of hypnotism has increased in volume at a rapid rate during recent years. Of recent writings the following may be mentioned as among the most important: -Treatment by Hypnotism and Suggestion by C. Lloyd Tuckey, M.D. (5th ed., London, 1907); Hypnotism, its History, Practice and Theory, by J. Milne Bramwell, M.B. (2nd ed., London, 1906); Hypnotism, by Albert Moll (5th ed., London, 1901). All these three books give good general accounts of hypnotism, the first being the most strictly medical, the last the most general in its treatment. See also Hypnotism: or Suggestion in Psycho-Therapy, by August Forel (translated from the 5th German ed. by G. H. W. Armit, London, 1906); a number of papers by Ed. Gurney, and by Ed. Gurney and F. W. H. Myers in Proc. of the Soc. for Psychical Research, especially "The Stages of Hypnotism," in vol. ii.; also some more recent papers in the same journal by other hands; chapter on Hypnotism in Human Personality and its Survival of bodily Death, by F. W. H. Myers (London, 1903); The Psychology of Suggestion, by Boris Sidis, Ph.D. (New York, 1898); "Zur Psychologie der Suggestion," by Prof. Th. Lipp, and other papers in the Zeitschrift für Hypnotismus. Of special historical interest are the following:-Étude sur le zoomagnétisme, par A. A. Liébeault (Paris, 1883); Hypnotisme, suggestion, psycho-thérapie, par Prof. Bernheim (Paris, 1891); Braid on Hypnotism (a new issue of James Braid's Neurypnology), edited by A. E. Waite (London, 1899); Traité du somnambulisme, by A. Bertrand (Paris, 1826). A full bibliography is appended to Dr Milne Bramwell's Hypnotism.
(W. McD.)

HYPOCAUST (Gr. ப́пóк chamber formed under the floors of the Roman baths, through which the hot air from the furnace passed, sometimes to a single flue, as in the case of the tepidarium, but in the calidarium and sweating-room to a series of flues placed side by side forming the lining of the walls. The floor of the hot-air chamber consisted of tiles, 2 ft . square, laid on a bed of concrete; on this a series of dwarf piers 2 ft . high were built of 8 -in. square tiles placed about 16 in . apart, which carried the floor of the hall or room; this floor was formed of a bed of concrete covered with layers of pounded bricks and marble cement, on which the marble pavement in slabs or tesserae was laid. In colder countries, as for instance in Germany and England, the living rooms were all heated in a similar way, and round Trèves (Trier) both systems have been found in two or three Roman villas, with the one flue for the ordinary rooms and several wall flues for the hot baths. In England these hypocausts are found in every Roman settlement, and the chief interest in these is centred in the magnificent mosaic pavements with which the principal rooms were laid. Many of the pavements found in London and elsewhere have been preserved in the British or the Guildhall museums; and in some of the provincial towns, such as Leicester and Lincoln, they remain in situ many feet below the present level of the town.

HYPOCHONDRIASIS (synonyms-"the spleen," "the vapours"), a medical term (from tò
 cartilage of the breast-bone) given by the ancients, and indeed by physicians down to the time of William Cullen, to diseases or derangements of one or more of the abdominal viscera. Cullen (Clinical Lectures, 1777) classified it amongst nervous diseases, and Jean Pierre Falret (17941870) more fully described it as a morbid condition of the nervous system characterized by depression of feeling and false beliefs as to an impaired state of the health. The subjects of hypochondriasis are for the most part members of families in which hereditary predisposition to degradation of the nervous system is strong, or those who have suffered from morbid influences affecting this system during the earlier years of life. It may be dependent on depressing disease affecting the general system, but under such circumstances it is generally so complicated with the symptoms of hysteria as to render differentiation difficult (see Hysteria). Hypochondriasis is often handed down from one generation to another in its individual form, but it is also not unfrequently to be met with in an individual as the sole manifestation in him of a family tendency to insanity. In its most common form it is manifested by simple false belief as to the state of the health, the intellect being otherwise unaffected. We may instance the "vapourish" woman or the "splenetic" as terms society has applied to its milder manifestations. Such persons are constantly asserting a weak state of health although no palpable cause can be discovered. In its more definite phases pain or uneasy sensations are referred by the patient to some particular region, generally the abdomen, the heart or the head. That these are subjective is apparent from the fact that the general health is good: all the functions of the various systems are duly performed; the patient eats and sleeps well; and, when any circumstance temporarily overrides the false belief, he is happy and comfortable. No appeal to the reason is of any avail, and the hypochondriac idea so dominates his existence as to render him unable to perform the ordinary duties of life. In its most aggravated form hypochondriasis amounts to actual insanity, delusions arising as to the existence of living creatures in the intestines or brain, or to the effect that the body is materially changed; e.g. into glass, wood, \&c. The symptoms of this condition may be remittent; they may even disappear for years, and only return on the advent of some exciting cause. Suicide is occasionally committed in order to escape from the constant misery. Recovery can only be looked for by placing the patient under such morally hygienic conditions as may help to turn his mind to other matters. (See also Neuropathology.)

HYPOCRISY, pretence, or false assumption of a high character, especially in regard to religious belief or practice. The Greek úпóкрıбıs, from which the word is derived through the Old French, meant primarily the acting of a part on the stage, from úmoкрíveठӨal, to give an answer, to speak dialogue, play a part on the stage, hence to practice dissimulation.

HYPOSTASIS, in theology, a term frequently occurring in the Trinitarian controversies of the 4 th and 5th centuries. According to Irenaeus (i. 5, 4) it was introduced into theology by Gnostic writers, and in earliest ecclesiastical usage appears, as among the Stoics, to have been synonymous with oủoí . Thus Dionysius of Rome (cf. Routh, Rel. Sacr. iii. 373) condemns the attempt to sever the Godhead into three separate hypostases and three deities, and the Nicene
 persistent interchange there was a desire to distinguish between the terms, and to confine $\dot{\text { únó } \sigma \tau \alpha \sigma \iota \text { to the Divine persons. This tendency arose in Alexandria, and its progress may be }}$ seen in comparing the early and later writings of Athanasius. That writer, in view of the Arian trouble, felt that it was better to speak of oủoí $\alpha$ as "the common undifferentiated substance of Deity," and úróбtaбıৎ as "Deity existing in a personal mode, the substance of Deity with certain special properties" (oủoí $\alpha \mu \tau \alpha \dot{\alpha} \tau \iota \nu \omega \nu$ í $\delta \iota \omega \mu \alpha ́ \tau \omega \nu$ ). At the council of Alexandria in 362 the phrase т $\rho \varepsilon$ ĩऽ ن́побта́бعıৎ was permitted, and the work of this council was supplemented by Basil, Gregory of Nazianzus and Gregory of Nyssa in the formula $\mu i ́ \alpha$ оủбí $\alpha$, т $\rho \varepsilon i ̃ \varsigma ~ u ́ m o \sigma \tau \alpha ́ \sigma \varepsilon ı \varsigma ~ o r ~$


The results arrived at by these Cappadocian fathers were stated in a later age by John of Damascus (De orth. fid. iii. 6), quoted in R. L. Ottley, The Doctrine of the Incarnation, ii. 257.

HYPOSTYLE, in architecture, the term applied to a hall, the flat ceiling of which is supported by columns, as in the Hall of Columns at Karnak. In this case the columns flanking the central avenue are of greater height than those of the side aisles, and this admits of openings in the wall above the smaller columns, through which light is admitted over the aisle roof, through clerestory windows.

HYPOSULPHITE OF SODA, the name originally given to the substance known in chemistry as sodium thiosulphate, $\mathrm{Na}_{2} \mathrm{~S}_{2} \mathrm{O}_{3}$; the earlier name is still commonly used, especially by photographers, who employ this chemical as a fixer. In systematic chemistry, sodium hyposulphite is a salt of hyposulphurous acid, to which Schutzenberger gave the formula $\mathrm{H}_{2} \mathrm{SO}_{2}$, but which Bernthsen showed to be $\mathrm{H}_{2} \mathrm{~S}_{2} \mathrm{O}_{4}$. (See Sulphur.)

HYPOTHEC (Lat. hypotheca, Gr. ט́поӨ́́кп), in Roman law, the most advanced form of the contract of pledge. A specific thing may be given absolutely to a creditor on the understanding that it is to be given back when the creditor's debt is paid; or the property in the thing may be assigned to the creditor while the debtor is allowed to remain in possession, the creditor as owner being able to take possession if his debt is not discharged. Here we have the kind of security known as pledge and mortgage respectively. In the hypotheca, the property does not pass to the creditor, nor does he get possession, but he acquires a preferential right to have his debt paid out of the hypothecated property; that is, he can sell it and pay himself out of the proceeds, or in default of a purchaser he can become the owner himself. The name and the principle have passed into the law of Scotland, which distinguishes between conventional hypothecs, as bottomry and respondentia, and tacit hypothecs established by law. Of the latter the most important is the landlord's hypothec for rent (corresponding to distress in the law of England), which extends over the produce of the land and the cattle and sheep fed on it, and over stock and horses used in husbandry. The law of agricultural hypothec long caused much discontent in Scotland; its operation was restricted by the Hypothec Amendment (Scotland) Act 1867, and finally by the Hypothec Abolition (Scotland) Act 1880 it was enacted that the "landlord's right of hypothec for the rent of land, including the rent of any buildings thereon, exceeding two acres in extent, let for agriculture or pasture, shall cease and determine." By the same act and by the Agricultural Holdings (Scotland) Act 1883 other rights and remedies for rent, where the right of hypothec had ceased, were given to the landlord.

HYPOTHESIS (from Gr. únotı日éval, to put under; cf. Lat. suppositio, from sub-ponere), in ordinary language, an explanation, supposition or assumption, which is put forward in the absence of ascertained facts or causes. Both in ordinary life and in the acquisition of scientific knowledge hypothesis is all-important. A detective's work consists largely in forming and testing hypothesis. If an astronomer is confronted by some phenomenon which has no obvious explanation he may postulate some set of conditions which from his general knowledge of the subject would or might give rise to the phenomenon in question; he then tests his hypothesis until he discovers whether it does or does not conflict with the facts. An example of this process is that of the discovery of the planet Neptune: certain perturbations of the orbit of Uranus had been observed, and it was seen that these could be explained on the hypothesis of the existence of a then unknown planet, and this hypothesis was verified by actual observation. The progress of inductive knowledge is by the formation of successive hypotheses, and it frequently happens that the demolition of one or even many hypotheses is the direct road to a new and accurate
hypothesis, i.e. to fresh knowledge. A hypothesis may, therefore, turn out to be entirely wrong, yet it may be of the greatest practical use.

The recognition of the importance of hypotheses has led to various attempts at drawing up exact rules for their formation, but logicians are generally agreed that only very elementary principles can be laid down. Thus a hypothesis must contain nothing which is at variance with known facts or principles: it should not postulate conditions which cannot be verified empirically. J. S. Mill (Logic III. xiv. 4) laid down the principle that a hypothesis is not "genuinely scientific" if it is "destined always to remain a hypothesis": it must "be of such a nature as to be either proved or disproved by comparison with observed facts": in the same spirit Bacon said that in searching for causes in nature "Deum semper excipimus." Mill's principle, though sound in the abstract, has, except in a few cases, little practical value in determining the admissibility of hypotheses, and in practice any rule which tends to discourage hypothesis is in general undesirable. The most satisfactory check on hypothesis is expert knowledge in the particular field of research by which rigorous tests may be applied. This test is roughly of two kinds, first by the ultimate principles or presuppositions on which a particular branch of knowledge rests, and second by the comparison of correlative facts. Useful light is shed on this distinction by Lotze, who contrasts (Logic, § 273) postulates ("absolutely necessary assumptions without which the content of the observation with which we are dealing would contradict the laws of our thought") with hypotheses, which he defines as conjectures, which seek "to fill up the postulate thus abstractly stated by specifying the concrete causes, forces or processes, out of which the given phenomenon really arose in this particular case, while in other cases maybe the same postulate is to be satisfied by utterly different though equivalent combinations of forces or active elements." Thus a hypothesis may be ruled out by principles or postulates without any reference to the concrete facts which belong to that division of the subject to explain which the hypothesis is formulated. A true hypothesis, therefore, seeks not merely to connect or colligate two separate facts, but to do this in the light of and subject to certain fundamental principles. Various attempts have been made to classify hypotheses and to distinguish "hypothesis" from a "theory" or a mere "conjecture": none of these have any great practical importance, the differences being only in degree, not in kind.

The adjective "hypothetical" is used, in the same sense, both loosely in contradistinction to "real" or "actual," and technically in the phrases "hypothetical judgment" and "hypothetical syllogism." (See Logic and Syllogism.)

See Naville, La Logique de l'hypothèse (1880), and textbooks of logic, e.g. those of Jevons, Bosanquet, Joseph; Liebmann, Der Klimax d. Theorien.
 architecture, the space between the annulet of the echinus and the upper bed of the shafts, including, according to C. R. Cockerell, the three grooves or sinkings found in some of the older examples, as in the temple of Neptune at Paestum and the temple of Aphaea at Aegina; there being only one groove in the Parthenon, the Theseum and later examples. In the temple of Ceres and the so-called Basilica at Paestum the hypotrachelium consists of a concave sinking carved with vertical lines suggestive of leaves, the tops of which project forward. A similar decoration is found in the capital of the columns flanking the tomb of Agamemnon at Mycenae, but here the hypotrachelium projects forward with a cavetto moulding, and is carved with triple leaves like the buds of a rose. In the Roman Doric Order the term was sometimes applied to that which is generally known as the "necking," the space between the fillet and the annulet.

HYPSOMETER (Gr. ט̈ $\neq \varsigma$, height, $\mu \varepsilon ́ \tau \rho \circ \nu$, a measure), an instrument for measuring heights which employs the principles that the boiling-point of a liquid is lowered by diminishing the pressure, and that the barometric pressure varies with the height of the point of observation. The instrument consists of a cylindrical vessel in which the liquid, usually water, is boiled, surmounted by a jacketed column, in the outer partitions of which the vapour circulates, while in the central one a thermometer is placed. To deduce the height of the station from the observed boiling-point, it is necessary to know the relation existing between the boiling-point and pressure, and also between the pressure and height of the atmosphere.

HYRACOIDEA, a suborder of ungulate mammals represented at the present day only by the Syrian hyrax (Procavia syriaca), the "coney" of the Bible, and its numerous African relatives, all of which may be included in the single genus Procavia (or Hyrax), and consequently in the family Procaviidae. These creatures have no proper English name, and are generally known as hyraxes, from the scientific term (Hyrax) by which they were for many years designated-a term which has unfortunately had to give place to the earlier Procavia. In size these animals may be compared roughly to rabbits and hares; and they have rodent-like habits, hunching up their backs after the fashion of some foreign members of the hare-family, more especially the Liu-Kiu rabbit. In the matter of nomenclature these animals have been singularly unfortunate. In the title "hyrax" they have, for instance, usurped the Greek name for the shrew-mouse; while in the Bible they have been given the old English name for the rabbit. Perhaps rock-rabbit would be the best name. At the Cape they are known to the Dutch as dass (badger), which has been anglicized into "dassie."


Fig. 1.-The Cape Hyrax (Procavia capensis).
As regards the recent forms, the dentition in the fully adult animal consists only of incisors and cheek-teeth, the formula being i. $1 / 2, c .4, p .4 / 4 \mathrm{~m} .3 / 3$. There is, however, a minute upper canine developed at first, which is early shed; and in extinct forms this tooth was functional and molar-like. The upper incisors have persistent pulps, and are curved longitudinally, forming a semicircle as in rodents; they are, however, not flattened from before backwards as in that order, but prismatic, with an antero-external, an antero-internal and a posterior surface, the first two only being covered with enamel; their tips are consequently not chisel-shaped, but sharp-pointed. They are preceded by functional, rooted milk-teeth. The lower incisors have long tapering roots, but not of persistent growth; and are straight, directed somewhat forwards, with awl-shaped, tri-lobed crowns. Behind the incisors is a considerable gap, followed by the cheekteeth, which are all contiguous, and formed almost exactly on the pattern of some of the perissodactyle ungulates. The milk-dentition includes three pairs of incisors and one of canines in each jaw. The hyoid arch is unlike that of any known mammal. The dorsal and lumbar vertebrae are very numerous, 28 to 30 , of which 21 or 22 bear ribs. The tail is extremely short. There are no clavicles. In the fore foot, the three middle toes are subequally developed, the fifth is present, but smaller, and the first is rudimentary, although, in one species at least, all its normal bones are present. The terminal phalanges of the four outer digits are small, somewhat conical and flattened in form. The carpus has a distinct os centrale. There is a slight ridge on the femur in the place of a third trochanter. The fibula is complete, thickest at its upper end, where it generally unites with the tibia. The articulation between the tibia and astragalus is more complex than in other mammals, the end of the malleolus entering into it. The hind-foot is very like that of a rhinoceros, having three well-developed toes. There is no trace of a first toe, and the fifth meta-tarsal is represented by a small nodule. The terminal phalange of the inner (or second) digit is deeply cleft, and has a peculiar long curved claw, the others having short broad nails. The stomach is formed upon much the same principle as that of the horse or rhinoceros, but is more elongated transversely and divided by a constriction into two cavities-a large left cul de sac, lined by a very dense white epithelium, and a right pyloric cavity, with a thick, soft, vascular lining. The intestinal canal is long, and has, in addition to the ordinary short, but capacious and sacculated caecum at the commencement of the colon, lower down, a pair of large, conical, pointed caeca. The liver is much subdivided, and there is no gall-bladder. The brain resembles that of typical ungulates far more than that of rodents. The testes are
permanently abdominal. The ureters open into the fundus of the bladder as in some Rodents. The female has six teats, of which four are inguinal and two axillary, and the placenta is zonary and deciduous. There is a gland on the back.


Fig. 2.-Skull and Dentition of Tree-Hyrax (Procavia dorsalis).
The more typical members of the genus are terrestrial in their habits, and their cheek-teeth have nearly the same pattern as in rhinoceroses; while the interval between the upper incisors is less than the width of the teeth; and the lower incisors are only slightly notched at the cutting edge. Vertebrae: C. 7, D. 22, L. 8, S. 6, C. 6. Of this form the earliest known species, $P$. capensis, is the type; but there are many other species, as P. syriaca, and P. brucei from Syria and eastern Africa. They inhabit mountainous and rocky regions, and live on the ground. In a second section the molar teeth have the same pattern as in Palaeotherium (except that the third lower molar has but two lobes); the interval between the upper incisors exceeds the width of the teeth; and the lower incisors have distinctly tri-lobed crowns. Vertebrae: C. 7, D. 21, L. 7, S. $5, \mathrm{C} .10$. The members of this section frequent the trunks and large branches of trees, sleeping in holes. There are several species from Western and South Africa, as P. arboreus and P. dorsalis. The members of both groups appear to have a power like that possessed by geckos of clinging to vertical surfaces of rocks and trees by the soles of their feet.

Extinct Hyracoids.-For many years extinct representatives of the Hyracoidea were unknown, partly owing to the fact that certain fossils were not recognized as really belonging to that group. The longest known of these was originally named Leptodon graecus, but, on account of the preoccupation of the generic title, the designation has been changed to Pliohyrax graecus. This animal, whose remains occur in the Lower Pliocene of both Attica and Samos, was about the size of a donkey, and possessed three pairs of upper incisor teeth, of which the innermost were large and trihedral, recalling those of the existing genus. On the other hand, the two outer pairs of incisors were in contact with one another and with the canines, so as to form on each side a series continuous with the cheek-teeth.

The next representatives of the group occur in the Upper Eocene beds of the Fayum district of Egypt, where the genera Saghatherium and Megalohyrax occur. These are regarded as representing a distinct family, the Saghatheriidae, characterized by the possession of the full series of twenty-two teeth in the upper jaw, among which the first pair of incisors was modified to form trihedral rootless tusks, while the two remaining pairs were separated from one another and from the teeth in front by gaps. The canine was like a premolar, and in contact with the first tooth of that series; and the cheek-teeth were short-crowned, with the premolar simpler than the molars, and a third lobe to the last lower tooth of the latter series. The members of this genus were small or medium-sized ungulates with single-rooted incisors. On the other hand, the representatives of the contemporary genus Megalohyrax were approximately as large as Pliohyrax, and in some instances had double roots to the second and third incisors.

It is now possible to define the suborder Hyracoidea as including ungulates with a centrale in the carpus, plantigrade feet, in which the first and fifth toes are reduced in greater or less degree, and clavicles and a foramen in the lower end of the humerus are absent. The femur has a small third trochanter, the radius and ulna and tibia and fibula are respectively separate, at least in the young, and the fibula articulates with the astragalus. The earlier forms had the full series of 44 teeth, with the premolars simpler than the molars; but in the later types the canines and some of the incisors disappear, and at least the hinder premolars become molarlike. In all cases the first upper incisors are large and rootless.

That the group originated in Africa there can be no reasonable doubt; and it is remarkable that so early as the Upper Eocene the types in existence differed comparatively little in structure from the modern forms. In fact the hyraxes were then almost as distinct from other mammals as they are at the present day.

HYRCANIA. (1) An ancient district of Asia, south of the Caspian Sea, and bounded on the E. by the river Oxus, called Virkana, or "Wolf's Land," in Old Persian. It was a wide and indefinite tract. Its chief city is called Tape by Strabo, Zadracarta by Arrian (probably the modern Astarabad). The latter is evidently the same as Carta, mentioned by Strabo as an important city. Little is known of the history of the country. Xenophon says it was subdued by the Assyrians; Curtius that 6000 Hyrcanians were in the army of Darius III. (2) Two towns named Hyrcania are mentioned, one in Hyrcania, the other in Lydia. The latter is said to have derived its name from a colony of Hyrcanians, transported thither by the Persians.

HYRCANUS ('Yркаvóৎ), a Greek surname, of unknown origin, borne by several Jews of the Maccabaean period.

John Hyrcanus I., high priest of the Jews from 135 to 105 b.c., was the youngest son of Simon Maccabaeus. In 137 в.с. he, along with his brother Judas, commanded the force which repelled the invasion of Judaea led by Cendebeus, the general of Antiochus VII. Sidetes. On the assassination of his father and two elder brothers by Ptolemy, governor of Jericho, his brother-in-law, in February 135, he succeeded to the high priesthood and the supreme authority in Judaea. While still engaged in the struggle with Ptolemy, he was attacked by Antiochus with a large army (134), and compelled to shut himself up in Jerusalem; after a severe siege peace was at last secured only on condition of a Jewish disarmament, and the payment of an indemnity and an annual tribute, for which hostages were taken. In 129 he accompanied Antiochus as a vassal prince on his ill-fated Parthian expedition; returning, however, to Judaea before winter, he escaped the final disaster. By the judicious mission of an embassy to Rome he now obtained confirmation of the alliance which his father had previously made with the growing western power; at the same time he availed himself of the weakened state of the Syrian monarchy under Demetrius II. to overrun Samaria, and also to invade Idumaea, which he completely subdued, compelling its inhabitants to receive circumcision and accept the Jewish faith. After a long period of rest he directed his arms against the town of Samaria, which, in spite of the intervention of Antiochus, his sons Antigonus and Aristobulus ultimately took, and by his orders razed to the ground (c. 100 в.с.). He died in 105, and was succeeded by Aristobulus, the eldest of his five sons. The external policy of Hyrcanus was marked by considerable energy and tact, and, aided as it was by favouring circumstances, was so successful as to leave the Jewish nation in a position of independence and of influence such as it had not known since the days of Solomon. During its later years his reign was much disturbed, however, by the contentions for ascendancy which arose between the Pharisees and Sadducees, the two rival sects or parties which then for the first time (under those names at least) came into prominence. Josephus has related the curious circumstances under which he ultimately transferred his personal support from the former to the latter.

John Hyrcanus II., high priest from 78 to 40 b.c., was the eldest son of Alexander Jannaeus by his wife Alexandra, and was thus a grandson of the preceding. When his father died in 78, he was by his mother forthwith appointed high priest, and on her death in 69 he claimed the succession to the supreme civil authority also; but, after a brief and troubled reign of three months, he was compelled to abdicate both kingly and priestly dignities in favour of his more energetic and ambitious younger brother Aristobulus II. In 63 it suited the policy of Pompey that he should be restored to the high priesthood, with some semblance of supreme command, but of much of this semblance even he was soon again deprived by the arrangement of the proconsul Gabinius, according to which Palestine was in 57 в.c. divided into five separate circles ( $\sigma$ v́voठoı, ouvદ́ठ $\rho \iota \alpha$ ). For services rendered to Caesar after the battle of Pharsalia, he was again rewarded with the sovereignty ( $\pi \rho \circ \sigma \tau \alpha \sigma \sigma^{\alpha} \alpha$ тои̃ $\varepsilon \theta$ vous, Jos. Ant. xx. 10) in 47 b.c., Antipater of Idumaea, however, being at the same time made procurator of Judaea. In 41 b.c. he was practically superseded by Antony's appointment of Herod and Phasael to be tetrarchs of Judaea; and in the following year he was taken prisoner by the Parthians, deprived of his ears that he might be permanently disqualified for priestly office, and carried to Babylon. He was permitted in 33 в.с. to return to Jerusalem, where on a charge of treasonable correspondence
with Malchus, king of Arabia, he was put to death in 30 в.с.
See Josephus (Ant. xiii. 8-10; xiv. 5-13; Bell. Jud. i. 2; i. 8-13). Also Maccabees, History.

HYSSOP (Hyssopus officinalis), a garden herb belonging to the natural order Labiatae, formerly cultivated for use in domestic medicine. It is a small perennial plant about 2 ft . high, with slender, quadrangular, woody stems; narrowly elliptical, pointed, entire, dotted leaves, about 1 in . long and $1 / 3 \mathrm{in}$. wide, growing in pairs on the stem; and long terminal, erect, halfwhorled, leafy spikes of small violet-blue flowers, which are in blossom from June to September. Varieties of the plant occur in gardens with red and white flowers, also one having variegated leaves. The leaves have a warm, aromatic, bitter taste, and are believed to owe their properties to a volatile oil which is present in the proportion of $1 / 4$ to $1 / 2 \%$. Hyssop is a native of the south of Europe, its range extending eastward to central Asia. A strong tea made of the leaves, and sweetened with honey, was formerly used in pulmonary and catarrhal affections, and externally as an application to bruises and indolent swellings.

The hedge hyssop (Gratiola officinalis) belongs to the natural order Scrophulariaceae, and is a native of marshy lands in the south of Europe, whence it was introduced into Britain more than 300 years ago. Like Hyssopus officinalis, it has smooth opposite entire leaves, but the stems are cylindrical, the leaves twice the size, and the flowers solitary in the axils of the leaves and having a yellowish-red veined tube and bluish-white limb, while the capsules are oval and many-seeded. The herb has a bitter, nauseous taste, but is almost odourless. In small quantities it acts as a purgative, diuretic and emetic when taken internally. It was formerly official in the Edinburgh Pharmacopoeia, being esteemed as a remedy for dropsy. It is said to have formed the basis of a celebrated nostrum for gout, called Eau médicinale, and in former times was called Gratia Dei. When growing in abundance, as it does in some damp pastures in Switzerland, it becomes dangerous to cattle. G. peruviana is known to possess similar properties.

The hyssop ('ezob) of Scripture (Ex. xii. 22; Lev. xiv. 4, 6; Numb. xix. 6, 18; 1 Kings v. 13 (iv. 33); Ps. li. 9 (7); John xix. 29), a wall-growing plant adapted for sprinkling purposes, has long been the subject of learned disputation, the only point on which all have agreed being that it is not to be identified with the Hyssopus officinalis, which is not a native of Palestine. No fewer than eighteen plants have been supposed by various authors to answer the conditions, and Celsius has devoted more than forty pages to the discussion of their several claims. By Tristram (Oxford Bible for Teachers, 1880) and others the caper plant (Capparis spinosa) is supposed to be meant; but, apart from other difficulties, this identification is open to the objection that the caper seems to be, at least in one passage (Eccl. xii. 5), otherwise designated ('abiy-yônah). Thenius (on 1 Kings v. 13) suggests Orthotrichum saxatile. The most probable opinion would seem to be that found in Maimonides and many later writers, according to which the Hebrew 'ezob is to be identified with the Arabic sa'atar, now understood to be Satureja Thymus, a plant of very frequent occurrence in Syria and Palestine, with which Thymus Serpyllum, or wild thyme, and Satureja Thymbra are closely allied. Its smell, taste and medicinal properties are similar to those of H. officinalis. In Morocco the sa'atar of the Arabs is Origanum compactum; and it appears probable that several plants of the genera Thymus, Origanum and others nearly allied in form and habit, and found in similar localities, were used under the name of hyssop.

HYSTASPES (the Greek form of the Persian Vishtāspa). (1) A semi-legendary king (kava), praised by Zoroaster as his protector and a true believer, son of Aurvataspa (Lohrasp). The later tradition and the Shahname of Firdousi makes him (in the modern form Kai Gushtāsp) king of Iran. As Zoroaster probably preached his religion in eastern Iran, Vishtāspa must have been a dynast in Bactria or Sogdiana. The Zoroastrian religion was already dominant in Media in the time of the Assyrian king Sargon (c. 715 в.с.), and had been propagated here probably in much earlier times (cf. Persia); the time of Zoroaster and Vishtāspa may therefore be put at $c$. 1000 b.c. (2) A Persian, father of Darius I., under whose reign he was governor of Parthia, as Darius himself mentions in the Behistun inscription (2. 65). By Ammianus Marcellinus, xxiii. 6. 32 , and by many modern authors he has been identified with the protector of Zoroaster, which is equally impossible for chronological and historical reasons, and from the evidence of the

HYSTERESIS (Gr. ט́бтદ́pŋбıऽ, from ú $\sigma \tau \varepsilon ́ \rho \varepsilon ı v$, to lag behind), a term added to the vocabulary of physical science by J. A. Ewing, who defines it as follows: When there are two qualities M and N such that cyclic variations of N cause cyclic variations of M , then if the changes of M lag behind those of N , we may say that there is hysteresis in the relation of M to N (Phil. Trans., 1885,176, p. 524). The phenomenon is best known in connexion with magnetism. If an iron bar is subjected to a magnetic force which is first gradually increased to a maximum and then gradually diminished, the resulting magnetization of the bar for any given value of the magnetic force will be greater when the force is decreasing than when it is increasing; the iron always tends to retain the magnetic condition which it has previously acquired, and changes of its magnetization consequently lag behind changes of the magnetic force. Thus there is hysteresis in the relation of magnetization to magnetic force. In consequence of hysteresis the process of magnetizing a piece of iron to a certain intensity and then restoring it to its original condition, or of effecting a double reversal of its magnetization, involves the expenditure of energy, which is dissipated as heat in the iron. Electrical generators and transformers often contain pieces of iron the magnetization of which is reversed many times in a second, and in order to economize power and to avoid undue heating it is essential that hysteresis should in such cases be as small as possible. Iron and mild steels showing remarkably little hysteresis are now specially manufactured for use in the construction of electrical machinery. (See Magnetism.)

HYSTERIA, a term applied to an affection which may manifest itself by a variety of symptoms, and which depends upon a disordered condition of the highest nervous centres. It is characterized by psychical peculiarities, while in addition there is often derangement of the functions subserved by the lower cerebral and spinal centres. Histological examination of the nervous system has failed to disclose associated structural alterations.

By the ancients and by modern physicians down to the time of Sydenham the symptoms of hysteria were supposed to be directly due to disturbances of the uterus (Gr. ט́otépa, whence the name). This view is now universally recognized to be erroneous. The term "functional" is often used by English neurologists as synonymous with hysterical, a nomenclature which is tentatively advantageous since it is at least non-committal. P. J. Möbius has defined hysteria as "a state in which ideas control the body and produce morbid changes in its functions." P. Janet, who has done much to popularize the psychical origin of the affection, holds that there is "a limitation of the field of consciousness" comparable to the contraction of the visual fields met with in the disease. The hysterical subject, according to this view, is incapable of taking into the field of consciousness all the impressions of which the normal individual is conscious. Strong momentary impressions are no longer controlled so efficiently because of the defective simultaneous impressions of previous memories. Hence the readiness with which the impulse of the moment is obeyed, the loss of emotional control and the increased susceptibility to external suggestion, which are so characteristic. A secondary subconscious mental state is engendered by the relegation of less prominent impressions to a lower sphere. The dual personality which is typically exemplified in somnambulism and in the hypnotic state is thus induced. The explanation of hysterical symptoms which are independent of the will, and of the existence of which the individual may be unaware, is to be found in a relative preponderance of this secondary subconscious state as compared with the primary conscious personality. An elaboration of this theory affords an explanation of hysterical symptoms dependent upon a "fixed idea." The following definition of hysteria has recently been advanced by J. F. F. Babinski: "Hysteria is a psychical condition manifesting itself principally by signs that may be termed primary, and in an accessory sense others that we may call secondary. The characteristic of the primary signs is that they may be exactly reproduced in certain subjects by suggestion and dispelled by persuasion. The characteristic of the secondary signs is that they are closely related to the primary phenomena."

The causes of hysteria may be divided into (a) the predisposing, such as hereditary
predisposition to nervous disease, sex, age and national idiosyncrasy; and (b) the immediate, such as mental and physical exhaustion, fright and other emotional influences, pregnancy, the puerperal condition, diseases of the uterus and its appendages, and the depressing influence of injury or general disease. Perhaps, taken over all, hereditary predisposition to nerve-instability may be asserted as the most prolific cause. There is frequently direct inheritance, and cases of epilepsy and insanity or other form of nervous disease are rarely wanting when the family history is carefully enquired into. As regards age, the condition is apt to appear at the evolution periods of life-puberty, pregnancy and the climacteric-without any further assignable cause except that first spoken of. It is rare in young children, but very frequent in girls between the ages of fifteen and twenty-five, while it sometimes manifests itself in women at the menopause. It is much more common in the female than in the male-in the proportion of 20 to 1 . Certain races are more liable to the disease than others; thus the Latin races are much more prone to hysteria than are those who come of a Teutonic stock, and in more aggravated and complex forms. In England it has been asserted that an undue proportion of cases occur among Jews. Occupation, or be it rather said want of occupation, is a prolific cause. This is noticeable more especially in the higher classes of society.

An hysterical attack may occur as an immediate sequel to an epileptic fit. If the patient suffers only from petit mal (see Epilepsy), unaccompanied by true epileptic fits, the significance of the hysterical seizure, which is really a post-epileptic phenomenon, may remain unrecognized.

It is convenient to group the very varied symptoms of hysteria into paroxysmal and chronic. The popular term "hysterics" is applied to an explosion of emotionalism, generally the result of mental excitement, on which convulsive fits may supervene. The characters of these vary, and may closely resemble epilepsy. The hysterical fit is generally preceded by an aura or warning. This sometimes takes the form of a sensation as of a lump in the throat (globus hystericus). The patient may fall, but very rarely is injured in so doing. The eyes are often tightly closed, the body and limbs become rigid, and the back may become so arched that the patient rests on her heels and head (opisthotonos). This stage is usually followed by violent struggling movements. There is no loss of consciousness. The attack may last for half-an-hour or even longer. Hysterical fits in their fully-developed form are rarely seen in England, though common in France. In the chronic condition we find an extraordinary complexity of symptoms, both physical and mental. The physical symptoms are extremely diverse. There may be a paralysis of one or more limbs associated with rigidity, which may persist for weeks, months or years. In some cases, the patient is unable to walk; in others there are peculiarities of the gait quite unlike anything met with in organic disease. Perversions of sensation are usually present; a common instance is the sensation of a nail being driven through the vertex of the head (clavus hystericus). The region of the spine is a very frequent seat of hysterical pain. Loss of sensation (anaesthesia), of which the patient may be unaware, is of common occurrence. Very often this sensory loss is limited exactly to one-half of the body, including the leg, arm and face on that side (hemianaesthesia). Sensation to touch, pain, heat and cold, and electrical stimuli may have completely disappeared in the anaesthetic region. In other cases, the anaesthesia is relative or it may be partial, certain forms of sensation remaining intact. Anaesthesia is almost always accompanied by an inability to recognize the exact position of the affected limb when the eyes are closed. When hemianaesthesia is present, sight, hearing, taste and smell are usually impaired on that side of the body. Often there is loss of voice (hysterical aphonia). It is to such cases of hysterical paralysis and sensory disturbance that the wonderful cures effected by quacks and charlatans may be referred. The mental symptoms have not the same tendency to pass away suddenly. They may be spoken of as inter-paroxysmal and paroxysmal. The chief characteristics of the former are extreme emotionalism combined with obstructiveness, a desire to be an object of interest and a constant craving for sympathy which is often procured at an immense sacrifice of personal comfort. Obstructiveness is the invariable symptom. Hysteria may pass into absolute insanity.

The treatment of hysteria demands great tact and firmness on the part of the physician. The affection is a definite entity and has to be clearly distinguished from malingering, with which it is so often erroneously regarded as synonymous. Drugs are of little value. The moral treatment is all-important. In severe cases, removal from home surroundings and isolation, either in a hospital ward or nursing home, are essential, in order that full benefit may be derived from psychotherapeutic measures.

Bibliography.-Charcot, Leçons sur les maladies du système nerveuse (1877); S. Weir Mitchell, Lectures on Diseases of the Nervous System especially in Women (1885); Buzzard, Simulation of Hysteria by Organic Nervous Disease (1891); Pitres, Leçons cliniques sur l'hystérie et l'hypnotisme (1891); Richer, Études cliniques sur la grande hystérie (1891); Gilles de la Tourette, Traité clinique et thérapeutique de l'hystérie (1891); Bastian, Hysterical or Functional Paralysis (1893); Ormerod, Art. "Hysteria," in Clifford Allbutt's System of Medicine (1899); Camus and Pagnez, Isolement et Psychotherapie (1904).
 which the order of words or phrases is inverted, and that which should logically or naturally come last is put first, to secure emphasis for the principal idea; the classical example is Virgil's " moriamur et in media arma ruamus," "let us die and charge into the thick of the fight" (Aen. ii. 358). The term is also applied to any inversion in order of events, arguments, \&c.

HYTHE, a market town and watering-place, one of the Cinque Ports, and a municipal and parliamentary borough of Kent, England, 67 m. S.E. by E. of London on a branch of the South Eastern \& Chatham railway. Pop. (1901) 5557. It is beautifully situated at the foot of a steep hill near the eastern extremity of Romney Marsh, about half a mile from the sea, and consists principally of one long street running parallel with the shore, with which it is connected by a straight avenue of wych elms. On account of its fine situation and picturesque and interesting neighbourhood, it is a favourite watering-place. A sea-wall and parade extend eastward to Sandgate, a distance of 3 m . There is communication with Sandgate by means of a tramway along the front. On the slope of the hill above the town stands the fine church of St Leonard, partly Late Norman, with a very beautiful Early English chancel. The tower was rebuilt about 1750. In a vault under the chancel there is a collection of human skulls and bones supposed to be the remains of men killed in a battle near Hythe in 456. Lionel Lukin (1742-1834), inventor of the life-boat, is buried in the churchyard. Hythe possesses a guildhall founded in 1794 and two hospitals, that of St Bartholomew founded by Haimo, bishop of Rochester, in 1336, and that of St John (rebuilt in 1802), of still greater antiquity but unknown date, founded originally for the reception of lepers. A government school of musketry, in which instructors for the army are trained, was established in 1854, and has been extended since, and the Shorncliffe military camp is within $2 \frac{1}{2} \mathrm{~m}$. of the town.

Lympne, which is now 3 m . inland, is thought to have been the original harbour which gave Hythe a place among the Cinque Ports. The course of the ancient estuary may be distinctly traced from here along the road to Hythe, the sea-sand lying on the surface and colouring the soil. Here are remains of a Roman fortress, and excavations have brought to light many remains of the Roman Portus Lemanis. Large portions of the fortress walls are standing. At the south-west corner is one of the circular towers which occurred along the line of wall. The site is now occupied by the fine old castellated mansion of Studfall castle, formerly a residence of the archdeacons of Canterbury. The name denotes a fallen place, and is not infrequently thus applied to ancient remains. The church at Lympne is Early English, with a Norman tower built by Archbishop Lanfranc, and Roman material may be traced in the walls. A short distance east is Shipway or Shepway Cross, where some of the great assemblies relating to the Cinque Ports were held. A mile north from Hythe is Saltwood Castle, of very ancient origin, but rebuilt in the time of Richard II. The castle was granted to the see of Canterbury in 1026, but escheated to the crown in the time of Henry II., when the murder of Thomas à Beckett is said to have been concerted here, and having been restored to the archbishops by King John remained a residence of theirs until the time of Henry VIII. It was restored as a residence in 1882. About 2 m . N.W. of Saltwood are remains of the fortified 14th-century manor-house of Westenhanger. It is quadrangular and surrounded by a moat, and of the nine towers (alternately square and round) by which the walls were defended, three remain.

The parliamentary borough of Hythe, which includes Folkestone, Sandgate and a number of neighbouring villages, returns one member. The town is governed by a mayor, 4 aldermen and 12 councillors. Area 2617 acres.

Hythe (Heda, Heya, Hethe, Hithe, i.e. landing-place) was known as a port in Saxon times, and was granted by Halfden, a Saxon thegn, to Christ Church, Canterbury. In the Domesday Survey the borough is entered among the archbishop's lands as appurtenant to his manor of Saltwood, and the bailiff of the town was appointed by the archbishop. Hythe was evidently a Cinque Port before the Conquest, as King John in 1205 confirmed the liberties, viz. freedom from toll, the right to be impleaded only at the Shepway court, \&c., which the townsmen had under Edward the Confessor. The liberties of the Cinque Ports were confirmed in Magna Carta and later by

Edward I. in a general charter, which was confirmed, often with additions, by subsequent kings down to James II. John's charter to Hythe was confirmed by Henry IV., Henry V. and Henry VI. These charters were granted to the Cinque Ports in return for the fifty-seven ships which they supplied for the royal service, of which five were contributed by Hythe. The ports were first represented in the parliament of 1365 , to which they each sent four members.

Hythe was governed by twelve jurats until 1574, when it was incorporated by Elizabeth under the title of the mayor, jurats and commonalty of Hythe; a fair for the sale of fish, \&c., was also granted, to be held on the feast of St Peter and St Paul. As the sea gradually retreated from Hythe and the harbour became choked up with sand, the town suffered the fate of other places near it, and lost its old importance.

I
the ninth letter of the English and Latin alphabet, the tenth in the Greek and Phoenician, because in these the symbol Teth (the Greek $\theta$ ) preceded it. Teth was not included in the Latin alphabet because that language had no sound corresponding to the Greek $\theta$, but the symbol was metamorphosed and utilized as the numeral $\mathbf{C}=100$, which took this form through the influence of the initial letter of the Latin centum. The name of I in the Phoenician alphabet was Yōd. Though in form it seems the simplest of letters it was originally much more complex. In Phoenician it takes the form $\mathcal{Z}$, which is found also in the earliest Syriac and Palestinian inscriptions with little modification. Ultimately in Hebrew it became reduced to a very small symbol, whence comes its use as a term of contempt for things of no importance as in "not one jot or tittle" (Matthew v. 18). The name passed from Phoenician to Greek, and thence to the Latin of the vulgate as iōta, and from the Latin the English word is derived. Amongst the Greeks of Asia it appears only as the simple upright $\mathbf{I}$, but in some of the oldest alphabets elsewhere, as Crete, Thera, Attica, Achaia and its colonies in lower Italy, it takes the form $\boldsymbol{S}$ or $\mathbf{S}$, while at Corinth and Corcyra it appears first in a form closely resembling the later Greek sigma $\Sigma$. It had originally no cross-stroke at top and bottom. I being not $i$ but $z$. The Phoenician alphabet having no vowel symbols, the value of yōd was that of the English y. In Greek, where the consonant sound had disappeared or been converted into $h, \mathbf{I}$ is regularly used as a vowel. Occasionally, as in Pamphylian, it is used dialectically as a glide between $i$ and another vowel, as in the proper name $\Delta \alpha \mu \alpha ́ \tau \rho \iota \iota \zeta$. In Latin I was used alike for both vowel and consonant, as in iugum (yoke). The sound represented by it was approximately that still assigned to $i$ on the continent. Neither Greek nor Latin made any distinction in writing between short and long $i$, though in the Latin of the Empire the long sound was occasionally represented by a longer form of the symbol $\mathbf{I}$. The dot over the $i$ begins in the 5 th or 6 th century a.d. In pronunciation the English short $i$ is a more open sound than that of most languages, and does not correspond to the Greek and Latin sound. Nor are the English short and long $i$ of the same quality. The short $i$ in Sweet's terminology is a high-front-wide vowel, the long $i$, in English often spelt ee in words like seed, is diphthonged, beginning like the short vowel but becoming higher as it proceeds. The Latin short $i$, however, in final syllables was open and ultimately became $e, e . g$. in the neuter of $i$-stems as utile from utili-s. Medially both the short and the long sounds are very common in syllables which were originally unaccented, because in such positions many other sounds passed into i: officio but facio, redimo but emo, quidlibet but lubet (libet is later); collīdo but laedo, fido from an older feido, istis (dative plural) from an earlier istois.

IAMBIC, the term employed in prosody to denote a succession of verses, each consisting of a foot or metre called an iambus (í $\alpha \mu \beta$ oऽ), formed of two syllables, of which the first is short and the second long ( $\cup-)$. After the dactylic hexameter, the iambic trimeter was the most popular metre of ancient Greece. Archilochus is said to have been the inventor of this iambic verse, the $\tau \rho i ́ \mu \varepsilon \tau \rho о \varsigma$ consisting of three iambic fed. In the Greek tragedians an iambic line is formed of six feet arranged in obedience to the following scheme:-


Much of the beauty of the verse depends on the caesura, which is usually In the middle of the third foot, and far less frequently in the middle of the fourth. The English language runs more naturally in the iambic metre than in any other. The normal blank verse in English is founded upon an iambic basis, and Milton's line-

> And swims | or sinks | or wades | or creeps | or flies | -
exhibits it in its primitive form. The ordinary alexandrine of French literature is a hexapod iambic, but in all questions of quantity in modern prosody great care has to be exercised to recollect that all ascriptions of classic names to modern forms of rhymed or blank verse are merely approximate. The octosyllabic, or four-foot iambic metre, has found great favour in English verse founded on old romances. Decasyllabic iambic lines rhyming together form an "heroic" metre.

IAMBLICHUS (d. c. a.d. 330), the chief representative of Syrian Neoplatonism, is only imperfectly known to us in the events of his life and the details of his creed. We learn, however, from Suidas, and from his biographer Eunapius, that he was born at Chalcis in Coele-Syria, the scion of a rich and illustrious family, that he studied under Anatolius and afterwards under Porphyry, the pupil of Plotinus, that he himself gathered together a large number of disciples of different nations with whom he lived on terms of genial friendship, that he wrote "various philosophical books," and that he died during the reign of Constantine,-according to Fabricius, before A.D. 333. His residence (probably) at his native town of Chalcis was varied by a yearly visit with his pupils to the baths of Gadara. Of the books referred to by Suidas only a fraction has been preserved. His commentaries on Plato and Aristotle, and works on the Chaldaean theology and on the soul, are lost. For our knowledge of his system we are indebted partly to the fragments of these writings preserved by Stobaeus and others, and to the notices of his successors, especially Proclus, partly to his five extant books, the sections of a great work on the Pythagorean philosophy. Besides these, Proclus (412-485) seems to have ascribed to him ${ }^{1}$ the authorship of the celebrated book On the Egyptian Mysteries (so-called), and although its differences in style and in some points of doctrine from the writings just mentioned make it improbable that the work was by Iamblichus himself, it certainly emanated from his school, and in its systematic attempt to give a speculative justification of the polytheistic cultus of the day, marks the turning-point in the history of thought at which Iamblichus stood.

As a speculative theory Neoplatonism (q.v.) had received its highest development from Plotinus. The modifications introduced by Iamblichus were the elaboration in greater detail of its formal divisions, the more systematic application of the Pythagorean number-symbolism, and chiefly, under the influence of Oriental systems, the thorough-going mythic interpretation of what the previous philosophy had still regarded as notional. It is on the last account, probably, that Iamblichus was looked upon with such extravagant veneration. As a philosopher he had learning indeed, but little originality. His aim was to give a philosophical rendering of the popular religion. By his contemporaries he was accredited with miraculous powers (which he, however, disclaimed), and by his followers in the decline of Greek philosophy, and his admirers on its revival in the 15th and 16th centuries, his name was scarcely mentioned without the epithet "divine" or "most divine," while, not content with the more modest eulogy of Eunapius that he was inferior to Porphyry only in style, the emperor Julian regarded him as not even second to Plato, and said that he would give all the gold of Lydia for one epistle of Iamblichus.

Theoretically, the philosophy of Plotinus was an attempt to harmonize the principles of the various Greek schools. At the head of his system he placed the transcendent incommunicable
 which in turn gives birth to $\varphi$ úoıs, the realm of nature. Immediately after the absolute one, Iamblichus introduced a second superexistent unity to stand between it and the many as the producer of intellect, and made the three succeeding moments of the development (intellect, soul and nature) undergo various modifications. He speaks of them as intellectual ( $\theta$ عoì vorpoí),
 represented under the three stages of (objective) being (óv), (subjective) life ( $\zeta \omega \eta$ ), and (realized) intellect (voṽऽ) -is distinguished by him into spheres of intelligible gods ( $\theta$ عol̀ vorpoí) and of intellectual gods ( $\theta$ عoi vorpoí), each subdivided into triads, the latter sphere being the place of ideas, the former of the archetypes of these ideas. Between these two worlds, at once separating and uniting them, some scholars think there was inserted by Iamblichus, as afterwards by Proclus, a third sphere partaking of the nature of both ( $\theta$ عol̀ vontoì kà vorpoí). But this supposition depends on a merely conjectural emendation of the text. We read, however, that "in the intellectual hebdomad he assigned the third rank among the fathers to the Demiurge." The Demiurge, Zeus, or world-creating potency, is thus identified with the perfected voũ̧, the intellectual triad being increased to a hebdomad, probably (as Zeller supposes) through the subdivision of its first two members. As in Plotinus voũऽ produced nature by mediation of $\psi \cup \chi \eta$, so here the intelligible gods are followed by a triad of psychic gods. The first of these is incommunicable and supramundane, while the other two seem to be mundane though rational. In the third class, or mundane gods ( $\theta \varepsilon o \grave{\iota} \varepsilon$ ह́үкó $\sigma \mu \circ$ ), there is a still greater wealth of divinities, of various local position, function, and rank. We read of gods, angels, demons and heroes, of twelve heavenly gods whose number is increased to thirty-six or three hundred and sixty, and of seventy-two other gods proceeding from them, of twenty-one chiefs
 particular individuals and nations. The world is thus peopled by a crowd of superhuman beings influencing natural events, possessing and communicating knowledge of the future, and not inaccessible to prayers and offerings.
The whole of this complex theory is ruled by a mathematical formulism of triad, hebdomad, \&c., while the first principle is identified with the monad, voũऽ with the dyad, and $\psi \cup \chi \emptyset ́$ with the triad, symbolic meanings being also assigned to the other numbers. "The theorems of mathematics," he says, "apply absolutely to all things," from things divine to original matter ( $\because \lambda \eta$ ). But though he thus subjects all things to number, he holds elsewhere that numbers are independent existences, and occupy a middle place between the limited and unlimited.
Another difficulty of the system is the account given of nature. It is said to be "bound by the indissoluble chains of necessity which men call fate," as distinguished from divine things which are not subject to fate. Yet, being itself the result of higher powers becoming corporeal, a continual stream of elevating influence flows from them to it, interfering with its necessary laws and turning to good ends the imperfect and evil. Of evil no satisfactory account is given; it is said to have been generated accidentally.

In his doctrine of man Iamblichus retains for the soul the middle place between intellect and nature which it occupies in the universal order. He rejects the passionless and purely intellectual character ascribed to the human soul by Plotinus, distinguishing it sharply both from those above and those below it. He maintains that it moves between the higher and lower spheres, that it descends by a necessary law (not solely for trial or punishment) into the body, and, passing perhaps from one human body to another, returns again to the supersensible. This return is effected by the virtuous activities which the soul performs through its own power of free will, and by the assistance of the gods. These virtues were classified by Porphyry as political, purifying ( $\kappa \alpha \theta \alpha \rho \tau(\kappa \alpha$ ) $)$, theoretical, and paradigmatic; and to these Iamblichus adds a fifth class of priestly virtues (í $\rho \alpha \tau \tau \kappa \alpha \dot{\alpha} \alpha \rho \varepsilon \tau \alpha$ í), in which the divinest part of the soul raises itself above intellect to absolute being.

Iamblichus does not seem ever to have attained to that ecstatic communion with and absorption in deity which was the aim of earlier Neoplatonism, and which Plotinus enjoyed four times in his life, Porphyry once. Indeed his tendency was not so much to raise man to God as to bring the gods down to men-a tendency shown still more plainly in the "Answer of Abamon the master to Porphyry's letter to Anebo and solutions of the doubts therein expressed," afterwards entitled the Liber de mysteriis, and ascribed to Iamblichus.
In answer to questions raised and doubts expressed by Porphyry, the writer of this treatise appeals to the innate idea all men have of the gods as testifying to the existence of divinities countless in number and various in rank (to the correct arrangement of which he, like Iamblichus, attaches the greatest importance). He holds with the latter that above all principles of being and intelligence stands the absolute one, from whom the first god and king spontaneously proceeds; while after these follow the ethereal, empyrean, and heavenly gods, and the various orders of archangels, angels, demons, and heroes distinguished in nature, power, and activity, and in greater profusion than even the imagination of Iamblichus had conceived. He says that all the gods are good (though he in another place admits the existence of evil demons who must be propitiated), and traces the source of evil to matter; rebuts the objection that their answering prayer implies passivity on the part of gods or demons; defends divination, soothsaying, and theurgic practices as manifestations of the divine activity; describes the appearances of the different sorts of divinities; discusses the various kinds of
sacrifice, which he says must be suitable to the different natures of the gods, material and immaterial, and to the double condition of the sacrificer as bound to the body or free from it (differing thus in his psychology from Iamblichus); and, in conclusion, states that the only way to happiness is through knowledge of and union with the gods, and that theurgic practices alone prepare the mind for this union-again going beyond his master, who held assiduous contemplation of divine things to be sufficient. It is the passionless nature of the soul which permits it to be thus united to divine beings,-knowledge of this mystic union and of the worship associated with it having been derived from the Egyptian priests, who learnt it from Hermes.

On one point only does the author of the De mysteriis seem not to go so far as Iamblichus in thus making philosophy subservient to priestcraft. He condemns as folly and impiety the worship of images of the gods, though his master held that these simulacra were filled with divine power, whether made by the hand of man or (as he believed) fallen from heaven. But images could easily be dispensed with from the point of view of the writer, who not only held that all things were full of gods ( $\pi \alpha \dot{\nu} \tau \alpha \pi \lambda n ́ \rho \eta ~ \theta \varepsilon \omega \tilde{\omega}$, as Thales said), but thought that each man


The following are the extant works of Iamblichus: (1) On the Pythagorean (Life Пعрì toũ ПuӨ $\neq$ үорккои̃ $\beta$ íou), ed. T. Kiessling (1815), A. Nauck (St Petersburg, 1884); for a discussion of the authorities used see E. Rohde in Rheinisches Museum, xxvi., xxvii. (1871, 1872); Eng. trans.
 $\varphi(\lambda \circ \sigma \circ \varphi i ́ \alpha v)$, ed. T. Kiessling (1813); H. Piselli (1888). (3) The treatise On the General Science


 عủ $\chi n ̃ \varsigma)$, ed. S. Tennulius (1688), the Arithmetic by H. Pistelli (1894). (5) The Theological
 Ast (Leipzig, 1817). Two lost books, treating of the physical and ethical signification of numbers, stood fifth and sixth, while books on music, geometry and astronomy followed. The emperor Julian had a great admiration for Iamblichus, whom he considered "intellectually not inferior to Plato"; but the Letters to Iamblicus the Philosopher which bear his name are now generally considered spurious.

The so-called Liber de mysteriis was first edited, with Latin translation and notes, by T. Gale (Oxford, 1678), and more recently by C. Parthey (Berlin, 1857); Eng. trans. by Thomas Taylor (1821).

There is a monograph on Iamblichus by G. E. Hebenstreit (De Iamblichi, philosophi Syri, doctrina, Leipzig, 1764), and one of the De myst. by Harless (Das Buch v. d. ägypt. Myst., Munich, 1858). The best accounts of Iamblichus are those of Zeller, Phil. d. Griechen, iii. 2, pp. 613 sq., 2nd ed.; E. Vacherot, Hist. de l'école d'Alexandrie (1846), ii. 57 sq.; J. Simon, Hist. de l'école d'Alexandrie (1845); A. E. Chaignet, Histoire de la psychologie des Grecs (Paris, 1893) v. 67-108; T. Whittaker, The Neo-Platonists (Cambridge, 1901).
(W. R. So.)

1 Besides the anonymous testimony prefixed to an ancient MS. of Proclus, De Myst. viii. 3 seems to be quoted by the latter as Iamblichus's. Cf. Meiners. "Judicium de libro qui de Myst. Aeg. inscribitur," in Comment. Soc. Reg. Sci. Gott., vol. iv., 1781, p. 77.

IAMBLICHUS, of Syria, the earliest of the Greek romance writers, flourished in the 2nd century a.d. He was the author of B $\beta \beta \cup \lambda \omega \nu \iota \alpha \kappa \alpha$, the loves of Rhodanes and Sinonis, of which an epitome is preserved in Photius (cod. 94). Garmus, a legendary king of Babylon, forces Sinonis to marry him and throws Rhodanes into prison. The lovers manage to escape, and after many singular adventures, in which magic plays a considerable part, Garmus is overthrown by Rhodanes, who becomes king of Babylon. According to Suidas, Iamblichus was a freedman, and a scholiast's note on Photius further informs us that he was a native Syrian (not descended from Greek settlers); that he borrowed the material for his romance from a love story told him by his Babylonian tutor, and that he subsequently applied himself with great success to the study of Greek. A MS. of the original in the library of the Escorial is said to have been destroyed by fire in 1670. Only a few fragments have been preserved, in addition to Photius's epitome.

See Scriptores erotici, ed. A. Hirschig (1856) and R. Hercher (1858); A. Mai, Scriptorum veterum nova collectio, ii.; E. Rohde, Der griechische Roman (1900).

IANNINA (i.e. "the city of St John"; Gr. Ioannina; Turk Yaniá; also written Janina, Jannina, and, according to its Albanian pronunciation, Yanina), the capital of the vilayet of Iannina, Albania, European Turkey. Pop. (1905) about 22,000. The largest ethnical groups in the population are the Albanian and Greek; the purest form of colloquial Greek is spoken here among the wealthy and highly educated merchant families. The position of Iannina is strikingly picturesque. At the foot of the grey limestone mass of Mount Mitzekeli ( 1500 ft. ), which forms part of the fine range of hills running north from the Gulf of Arta, there lies a valley (the Hellopia of antiquity) partly occupied by a lake; and the city is built on the slopes of a slight eminence, stretching down to the western shore. It has greatly declined from the state of barbaric prosperity which it enjoyed from 1788 to 1822 , when it was the seat of Ali Pasha (q.v.), and was estimated to have from 30,000 to 50,000 inhabitants. The fortress-Demir Kule or Iron Castle, which, like the principal seraglio, was built on a promontory jutting into the lake-is now in ruins. But the city is the seat of a Greek archbishop, and still possesses many mosques and churches, besides synagogues, a Greek college (gymnasium), a library and a hospital. Sayades (opposite Corfu) and Arta are the places through which it receives its imports. The rich gold and silver embroidery for which the city has long been famous is still one of the notable articles in its bazaar; but the commercial importance of Iannina has notably declined since the cession of Arta and Thessaly to Greece in 1881. Iannina had previously been one of the chief centres of the Thessalian grain trade; it now exports little except cheese, hides, bitumen and sheepskins to the annual value of about $£ 120,000$; the imports, which supply only the local demand for provisions, textile goods, hardware, \&c., are worth about double that sum.

The lake of Iannina (perhaps to be identified with the Pambotus or Pambotis of antiquity) is 6 m . long, and has an area of 24 sq.m., with an extreme depth of less than 35 ft . In time of flood it is united with the smaller lake of Labchistas to the north. There are no affluents of any considerable size, and the only outlets are underground passages or katavothra extending for many miles through the calcareous rocks.

The theory supported by W. M. Leake (Northern Greece, London, 1835) that the citadel of Iannina is to be identified with Dodona, is now generally abandoned in favour of the claims of a more southern site. As Anna Comnena, in describing the capture of the town ( (خ̀ 'loávvivo) by Bohemond in 1082, speaks of the walls as being dilapidated, it may be supposed that the place existed before the 11 th century. It is mentioned from time to time in the Byzantine annals, and on the establishment of the lordship of Epirus by Michael Angelus Comnenus Ducas, it became his capital. In the middle ages it was successively attacked by Serbs, Macedonians and Albanians; but it was in possession of the successors of Michael when the forces of the Sultan Murad appeared before it in 1430 (cf. Hahn, Alban. Studien, Jena [1854], pp. 319-322). Since 1431 it has continued under Turkish rule.

Descriptions of Iannina will be found in Holland's Travels (1815); Hughes, Travels in Greece, \&c. (1830); H. F. Tozer, Researches in the Highlands of Turkey (London, 1869). See also Albania and the authorities there cited.

IAPETUS, in Greek mythology, son of Uranus and Gaea, one of the Titans, father of Atlas, Prometheus, Epimetheus and Menoetius, the personifications of certain human qualities (Hesiod, Theog. 507). As a punishment for having revolted against Zeus, he was imprisoned in Tartarus (Homer, Iliad, viii. 479) or underneath the island of Inarime off the coast of Campania (Silius Italicus xii. 148). Hyginus makes him the son of Tartarus and Gaea, and one of the giants. Iapetus was considered the original ancestor of the human race, as the father of Prometheus and grandfather of Deucalion. The name is probably identical with Japhet (Japheth), and the son of Noah in the Greek legend of the flood becomes the ancestor of (Noah) Deucalion. Iapetus as the representative of an obsolete order of things is described as warring against the new order under Zeus, and is naturally relegated to Tartarus.

See F. G. Welcker, Griechische Götterlehre, i. (1857); C. H. Völcker, Die Mythologie des Iapetischen Geschlechtes (1824); M. Mayer, Giganten und Titanen (1887).

IAPYDES, or Iapodes, one of the three chief peoples of Roman Illyria. They occupied the interior of the country on the north between the Arsia (Arsa) and Tedanius (perhaps the Zermanja), which separated them from the Liburnians. Their territory formed part of the modern Croatia. They are described by Strabo as a mixed race of Celts and Illyrians, who used Celtic weapons, tattooed themselves, and lived chiefly on spelt and millet. They were a warlike race, addicted to plundering expeditions. In 129 в.с. C. Sempronius Tuditanus celebrated a triumph over them, and in 34 в.c. they were finally crushed by Augustus. They appear to have had a foedus with Rome, but subsequently rebelled.

See Strabo iv. 207, vii. 313-315; Dio Cassius xlix. 35; Appian, Illyrica, 10, 14, 16; Livy, Epit. lix. 131; Tibullus iv. 1. 108; Cicero, Pro Balbo, 14.

IATROCHEMISTRY (coined from Gr. ${ }^{\alpha} \alpha \tau \rho o ́ \varsigma$, a physician, and "chemistry"), a stage in the history of chemistry, during which the object of this science was held to be "not to make gold but to prepare medicines." This doctrine dominated chemical thought during the 16 th century, its foremost supporters being Paracelsus, van Helmont and de la Boë Sylvius. But it gave way to the new definition formulated by Boyle, viz. that the proper domain of chemistry was "to determine the composition of substances." (See Chemistry: I. History; Medicine.)

IAZYGES, a tribe of Sarmatians first heard of on the Maeotis, where they were among the allies of Mithradates the Great. Moving westward across Scythia, and hence called Metanastae, they were on the lower Danube by the time of Ovid, and about a.d. 50 occupied the plains east of the Theiss. Here, under the general name of Sarmatae, they were a perpetual trouble to the Roman province of Dacia. They were divided into freemen and serfs (Sarmatae Limigantes), the latter of whom had a different manner of life and were probably an older settled population enslaved by nomad masters. They rose against them in A.D. 334, but were repressed by foreign aid. Nothing is heard of Iazyges or Sarmatae after the Hunnish invasions. Graves at Keszthely and elsewhere in the Theiss valley, shown by their contents to belong to nomads of the first centuries A.D., are referred to the Iazyges.

IBADAN, a town of British West Africa, in Yorubaland, Southern Nigeria, 123 m . by rail N.E. of Lagos, and about 50 m . N.E. of Abeokuta. Pop. 1910 estimated at 150,000. The town occupies the slope of a hill, and stretches into the valley through which the river Ona flows. It is enclosed by mud walls, which have a circuit of 18 m ., and is encompassed by cultivated land 5 or 6 m . in breadth. The native houses are all low, thatched structures, enclosing a square court, and the only break in the mud wall is the door. There are numerous mosques, orishas (idolhouses) and open spaces shaded with trees. There are a few buildings in the European style. Most of the inhabitants are engaged in agriculture; but a great variety of handicrafts is also carried on. Ibadan is the capital of one of the Yoruba states and enjoys a large measure of autonomy. Nominally the state is subject to the alafin (ruler) of Oyo; but it is virtually independent. The administration is in the hands of two chiefs, a civil and a military, the bale and the balogun; these together form the highest court of appeal. There is also an iyaloda or mother of the town, to whom are submitted all the disputes of the women. Ibadan long had a feud with Abeokuta, but on the establishment of the British protectorate the intertribal wars were stopped. In 1862 the people of Ibadan destroyed Ijaya, a neighbouring town of 60,000 inhabitants. A British resident and a detachment of Hausa troops are stationed at Ibadan.

IBAGUÉ, or San Bonifacio de Ibagué, a city of Colombia, and capital of the department of Tolima, about $60 \mathrm{~m} . \mathrm{W}$. of Bogotá and 18 m . N.W. of the Nevado de Tolima. Pop. (1900, estimate) 13,000 . Ibagué is built on a beautiful plain between the Chipalo and Combeima, small affluents of the Cuello, a western tributary of the Magdalena. Its elevation, 4300 ft . above the sea, gives it a mild, subtropical climate. The plain and the neighbouring valleys produce cacao, tobacco, rice and sugar-cane. There are two thermal springs in the vicinity, and undeveloped mines of sulphur and silver. The city has an endowed college. It is an important commercial centre, being on the road which crosses the Quindio pass, or paramo, into the Cauca valley. Ibagué was founded in 1550 and was the capital of the republic for a short time in 1854.

IBARRA, a city of Ecuador and capital of the province of Imbabura, about 50 m . N.N.E. of Quito, on a small fertile plain at the northern foot of Imbabura volcano, 7300 ft . above sea-level. Pop. (1900, estimate) 5000. It stands on the left bank of the Tahuando, a small stream whose waters flow north and west to the Pacific through the Mira, and is separated from the higher plateau of Quito by an elevated transverse ridge of which the Imbabura and Mojanda volcanoes form a part. The surrounding country is mountainous, the valleys being very fertile. Ibarra itself has a mild, humid climate, and is set in the midst of orchards and gardens. It is the see of a bishop and has a large number of churches and convents, and many substantial residences. Ibarra has manufactures of cotton and woollen fabrics, hats, sandals (alpargates), sacks and rope from cabulla fibre, laces, sugar and various kinds of distilled spirits and cordials made from the sugar-cane grown in the vicinity. Mules are bred for the Colombian markets of Pasto and Popayan. Ibarra was founded in 1597 by Alvaro de Ibarra, the president of Quito. It has suffered from the eruptions of Imbabura, and more severely from earthquakes, that of 1859 causing great damage to its public buildings, and the greater one of the 16th of August 1868 almost completely destroyed the town and killed a large number of its inhabitants. The village of Carranqui, $11 / 4 \mathrm{~m}$. from Ibarra, is the birthplace of Atahualpa, the Inca sovereign executed by Pizarro, and close by is the small lake called Yaguarcocha where the army of Huaynacapac, the father of Atahualpa, inflicted a bloody defeat on the Carranquis. Another aboriginal battle-field is that of Hatuntaqui, near Ibarra, where Huaynacapac won a decisive victory and added the greater part of Ecuador to his realm. The whole region is full of tolas, or Indian burial mounds.

IBERIANS (Iberi, " $ß \eta \rho \varepsilon \varsigma)$ ) an ancient people inhabiting parts of the Spanish peninsula. Their ethnic affinities are not known, and our knowledge of their history is comparatively slight. It is almost impossible to make any statement in regard to them which will meet with general agreement. At the same time, the general lines of Iberian controversy are clear enough The principal sources of information about the Iberians are (1) historical, (2) numismatic, (3) linguistic, (4) anthropological.

1. Historical.-The name seems to have been applied by the earlier Greek navigators to the peoples who inhabited the eastern coast of Spain; probably it originally meant those who dwelt by the river Iberus (mod. Ebro). It is possible (Boudard, Études sur l'alphabet ibérien (Paris, 1852) that the river-name itself represents the Basque phrase ibay-erri "the country of the river." On the other hand, even in older Greek usage (as in Thuc. vi. 1) the term Iberia is said to have embraced the country as far east as the Rhone (see Herodorus of Heraclea, Fragm. Hist. Gr. ii. 34), and by the time of Strabo it was the common Greek name for the Spanish peninsula. Iberians thus meant sometimes the population of the peninsula in general and sometimes, it would appear, the peoples of some definite race (үर́vos) which formed one element in that population. Of the tribal distribution of this race, of its linguistic, social and political characteristics, and of the history of its relation to the other peoples of Spain, we have only the most general, fragmentary and contradictory accounts. On the whole, the historical evidence indicates that in Spain, when it first became known to the Greeks and Romans there existed many separate and variously civilized tribes connected by at least apparent identity of race, and by similarity (but not identity) of language, and sufficiently distinguished by their general characteristics from Phoenicians, Romans and Celts. The statement of Diodorus Siculus that the mingling of these Iberians with the immigrant Celts gave rise to the Celtiberians is in itself probable. Varro and Dionysius Afer proposed to identify the Iberians of Spain with the Iberians of the Caucasus, the one regarding the eastern, and other the western, settlements as the
earlier.
2. Numismatic.-Knowledge of ancient Iberian language and history is mainly derived from a variety of coins, found widely distributed in the peninsula, ${ }^{1}$ and also in the neighbourhood of Narbonne. They are inscribed in an alphabet which has many points of similarity with the western Greek alphabets, and some with the Punic alphabet; but which seems to retain a few characters from an older script akin to those of Minoan Crete and Roman Libya. ${ }^{2}$ The same Iberian alphabet is found also rarely in inscriptions. The coinage began before the Roman conquest was completed; the monetary system resembles that of the Roman republic, with values analogous to denarii and quinarii. The coin inscriptions usually give only the name of the town, e.g. plplis (Bilbilis), klaQriqs (Calagurris), seqbrics (Segobriga), tmaniav (Dumania). The types show late Greek and perhaps also late Punic influence, but approximate later to Roman models. The commonest reverse type, a charging horseman, reappears on the Roman coins of Bilbilis, Osca, Segobriga and other places. Another common type is one man leading two horses or brandishing a sword or a bow. The obverse has usually a male head, sometimes inscribed with what appears to be a native name.
3. Linguistic.-The survival of the non-Aryan language among the Basques around the west Pyrenees has suggested the attempt to interpret by its means a large class of similar-sounding place-names of ancient Spain, some of which are authenticated by their occurrence on the inscribed coins, and to link it with other traces of non-Aryan speech round the shores of the Western Mediterranean and on the Atlantic seaboard of Europe. This phase of Iberian theory opens with K. W. Humboldt (Prüfung der Untersuchungen über die Urbewohner Hispaniens vermittelst der waskischen Sprache, Berlin, 1821), who contended that there existed once a single great Iberian people, speaking a distinct language of their own; that an essentially "Iberian" population was to be found in Sicily, Sardinia and Corsica, in southern France, and even in the British Isles; and that the Basques of the present day were remnants of this race, which had elsewhere been expelled or absorbed. This last was the central and the seminal idea of the work, and it has been the point round which the battle of scholarship has mainly raged. The principal evidence which Humboldt adduced in its support was the possibility of explaining a vast number of the ancient topographical names of Spain, and of other asserted Iberian districts, by the forms and significations of Basque. In reply, Graslin (De l'Ibérie, Paris, 1839), maintained that the name Iberia was nothing but a Greek misnomer of Spain, and that there was no proof that the Basque people had ever occupied a wider area than at present; and Bladé (Origine des Basques, Paris, 1869) took the same line of argument, holding that Iberia is a purely geographical term, that there was no proper Iberian race, that the Basques were always shut in by alien races, that their affinity is still to seek, and that the whole Basque-Iberian theory is a figment. His main contention has met with some acceptance, ${ }^{3}$ but the great current of ethnographical speculation still flows in the direction indicated by Humboldt.
4. Anthropological.-Humboldt's "Iberian theory" depended partly on linguistic comparisons, but partly on his observation of widespread similarity of physical type among the population of south-western Europe. Since his time the anthropological researches of Broca, Thurnam and Davis, Huxley, Busk, Beddoe, Virchow, Tubino and others have proved the existence in Europe, from Neolithic times, of a race, small of stature, with long or oval skulls, and accustomed to bury their dead in tombs. Their remains have been found in Belgium and France, in Britain, Germany and Denmark, as well as in Spain; and they bear a close resemblance to a type which is common among the Basques as well as all over the Iberian peninsula. This Neolithic race has consequently been nicknamed "Iberians," and it is now common to speak of the "Iberian" ancestry of the people of Britain, recognizing the racial characteristics of "Iberians" in the "small swarthy Welshman," the "small dark Highlander," and the "Black Celts to the west of the Shannon," as well as in the typical inhabitants of Aquitania and Brittany. ${ }^{4}$ Later investigators went further. M. d'Arbois de Jubainville, for example (Les Premiers habitants de l'Europe, Paris, 1877), maintained that besides possessing Spain, Gaul, Italy and the British Isles, "Iberian" peoples penetrated into the Balkan peninsula, and occupied a part of northern Africa, Corsica and Sardinia; and it is now generally accepted that a race with fairly uniform characteristics was at one time in possession of the south of France (or at least of Aquitania), the whole of Spain from the Pyrenees to the straits, the Canary Islands (the Guanches) a part of northern Africa and Corsica. Whether this type is more conveniently designated by the word Iberian, or by some other name ("Eur-african," "Mediterranean," \&c.) is a matter of comparative indifference, provided that there is no misunderstanding as to the steps by which the term Iberian attained its meaning in modern anthropology.

Authorities.-K. W. von Humboldt, "Über die cantabrische oder baskische Sprache" in Adelung, Mithridates iv. (1817), and Prüfung d. Untersuchungen ü. die Urbewohner Hispaniens vermittelst der waskischen Sprache (Berlin, 1821); L. F. Graslin, De l'Ibérie (Paris, 1838); T. B. G. M. Bory de St Vincent, Essai géologique sur le genre humain (1838); G. Lagneau, "Sur l'ethnologie des peuples ibériens," in Bull. soc. anthrop. (1867), pp. 146-161; J. F. Bladé, Études
sur l'origine des Basques (Paris, 1869), Défense des études, \&c. (Paris, 1870); Phillips, Die Einwanderung der Iberer in die pyren. Halbinsel (Vienna, 1870), Über das iberische Alphabet (Vienna, 1870); W. Boyd Dawkins, "The Northern Range of the Basques," in Fortnightly Rev. N.S. xvi. 323-337 (1874); W. T. van Eys, "La Langue ibérienne et la langue basque," in Revue de linguistique, pp. 3-15 (1874); W. Webster, "The Basque and the Kelt," in Journ. Anthrop. Inst. v. 5-29 (1875); F. M. Tubino, Los Aborigines ibericos o los Berberos en la peninsula (Madrid, 1876); A. Luchaire, Les Origines linguistiques de l'Aquitaine (Paris, 1877); W. Boyd Dawkins, Early Man in Britain (London, 1880); A. Castaing, "Les Origines des Aquitains," Mém. Soc. Eth. N.S. 1, pp. 183-328 (1884); G. C. C. Gerland, "Die Basken und die Iberer" in Gröber, Grundriss d. roman. Philologie, 1, pp. 313-334 (1888); M. H. d’Arbois de Jubainville, Les Premiers habitants de l'Europe (1889-1894); J. F. Bladé, Les Vascons avant leur établissement en Novempopulanie, Agen. (1891); W. Webster, "The Celt-iberians," Academy xl. 268-269 (and consequent correspondence) (1891); J. Rhys, "The Inscriptions and Language of the Northern Picts," Proc. Soc. Ant. Scot. xxvi. 263-351 (1892); F. Fita, "El Vascuence en las inscripciones ógmicas," Bol. Real. Acad. Hist. Madrid (June 1893), xxii. 579-587; G. v. d. Gabelentz, "Baskisch u. Berberisch," Sitz. k. preuss. Akad. Wiss. 593-613 (Berlin, 1893), Die Verwandtschaft der Baskischen mit der Berber-Sprache Nordafrikas nachgewiesen (Braunschweig, 1894); M. H. d’Arbois de Jubainville, "Les Celtes en Espagne," Rev. celtique, xiv. 357-395 (1894); G. Buschan, "Über die iberische Rasse," Ausland, lxvi. 342-344 (1894); F. Olóriz y Aguilera, Distribucion geografica del indice cefalico en España (Madrid, 1894), "La Talla humana en España" in Discursos R. Acad. Medicina xxxvi. 389 (Madrid, 1896); R. Collignon, "La Race basque," L'Anthropologie, v. 276-287 (1894); T. de Aranzadi, "Le Peuple basque, résumé" Bull. soc. d'anth. 510-520 (1894), "Consideraciones acerca de la raza basca" Euskel-Erria xxxv. 33, 65, 97, 129 (1896); H. Schuchhardt, Baskische Studien, i. "Über die Entstehung der Bezugsformen des baskischen Zeitworts"; Denkschriften der K. Akad. der Wiss., Phil.-Hist., Classe, Bd. 42, Abh. 3. (Wien, 1893); Ph. Salmon, Rev. mens. Éc. d'anthr. v. 155-181, 214-220 (1895); R. Collignon, "Anthr. du S.-O. de la France," Mém. Soc. Anthr. § 3. 1. 4. p. 1-129 (1895), Ann. de géogr. v. 156-166 (1896), and with J. Deniker, "Les Maures de Sénégal," L'Anthr. vii. 57-69 (1897); G. Hervé, Rev. mens. Éc. d'anthr. vi. 97-109 (1896); G. Sergi, Africa: Anthropologia della stirpe Camitica (Turin, 1897), Arii ed Italici (1898); L. de Hoyos Sainz, "L'Anthropologie et la préhistorique en Espagne et en Portugal en 1897," L'Anthropologie, ix. 37-51 (1898); J. Deniker (see Collignon) "Les Races de l'Europe," L'Anthropologie, ix. 113-133 (1898); M. Gèze, "De quelques rapports entre les langues berbère et basque," Mém. soc. arch. du Midi de la France, xiii. See also the works quoted in the footnotes; and the bibliography under Basques.

1 For the prehistoric civilization of the peninsula as a whole see Spain.
2 P. A. Boudard's Études sur l'alphabet ibérien (Paris, 1852). and Numismatique ibérienne (Béziers, 1859); Aloiss Heiss, Notes sur les monnaies celtibériennes (Paris, 1865), and Description générale des monnaies antiques de l'Espagne (Paris, 1870); Phillips, Über das iberische Alphabet (Vienna, 1870), Die Einwanderung der Iberer in die pyren. Halbinsel (Vienna, 1870); W. M. Flinders Petrie, Journ. Anthr. Inst. xxix. (1899) 204, and above all E. Hübner, Monumenta linguae Ibericae.
3 W. van Eys, for example, "La Langue ibérienne et la langue basque," in Revue de linguistique, goes against Humboldt; but Prince Napoleon and to a considerable extent A. Luchaire maintain the justice of his method and the value of many of his results. See Luchaire, Les Origines linguistiques de l'Aquitaine (Paris, 1877).

Compare the interesting résumé of the whole question in Boyd Dawkins's Early Man in Britain (London, 1880).

IBEX, one of the names of the Alpine wild goat, otherwise known as the steinbok and bouquetin, and scientifically as Capra ibex. Formerly the ibex was common on the mountainranges of Germany, Switzerland and Tirol, but is now confined to the Alps which separate Valais from Piedmont, and to the lofty peaks of Savoy, where its existence is mainly due to game-laws. The ibex is a handsome animal, measuring about $41 / 2 \mathrm{ft}$. in length and standing about 40 in . at the shoulder. The skin is covered in summer with a short fur of an ashy-grey colour, and in winter with much longer yellowish-brown hair concealing a dense fur beneath. The horns of the male rise from the crest of the skull, and after bending gradually backwards terminate in smooth tips; the front surface of the remainder carrying bold transverse ridges or knots. About 1 yd . is the maximum recorded length of ibex-horns. The fact that the fore-legs are somewhat shorter than those behind enables the ibex to ascend mountain slopes with more facility than it can descend, while its hoofs are as hard as steel, rough underneath and when walking over a flat surface capable of being spread out. These, together with its powerful
sinews, enable it to take prodigious leaps, to balance itself on the smallest foothold and to scale almost perpendicular rocks. Ibex live habitually at a greater height than chamois or any other Alpine mammals, their vertical limit being the line of perpetual snow. There they rest in sunny nooks during the day, descending at night to the highest woods to graze. Ibex are gregarious, feeding in herds of ten to fifteen individuals; but the old males generally live apart from, and usually at greater elevations than, the females and young. They utter a sharp whistling sound not unlike that of the chamois, but when greatly irritated or frightened make a peculiar snorting noise. The period of gestation in the female is ninety days, after which she producesusually at the end of June-a single young one which is able at once to follow its mother. Kids when caught young and fed on goat's milk can be readily tamed; and in the 16 th century young tamed ibex were frequently driven to the mountains along with the goats, in whose company they would afterwards return. Even wild ibex have been known to stray among the herds of goats, although they shun the society of chamois. Its flesh is said to resemble mutton, but has a flavour of game.


The Ibex (Capra ibex).
By naturalists the name "ibex" has been extended to embrace all the kindred species of wild goats, while by sportsmen it is used in a still more elastic sense, to include not only the true wild goat (known in India as the Sind ibex) but even the short-horned Hemitragus hylocrius of the Nilgiris. Dealing only with species zoologically known as ibex, the one nearest akin to the European kind is the Asiatic or Siberian ibex (Capra sibirica), which, with several local phases, extends from the northern side of Kashmir over an enormous area in Central Asia. These ibex, especially the race from the Thian Shan, are incomparably finer than the European species, their bold knotted horns sometimes attaining a length of close on 60 in . The Arabian, or Nubian, ibex (C. nubiana) is characterized by the more slender type of horn, in which the front edge is much narrower; while the Simien ibex (C. vali) of Central Abyssinia is a very large and dark-coloured animal, with the horns black instead of brownish, and bearing only slightly marked front ridges. The Caucasian ibex (C. caucasica), or tur, is a wholly fox-coloured animal, in which the horns are still flatter in front, and thus depart yet further from the ibex type. In the Spanish ibex ( $C$. pyrenaica) the horns are flattened, with ill-defined knobs, and a spiral twist. (See Goat.)
(W. H. F.; R. L.*)

IBIS, one of the sacred birds of the ancient Egyptians. James Bruce identified this bird with the Abu-Hannes or "Father John" of the Abyssinians, and in 1790 it received from Latham (Index ornithologicus, p. 706) the name of Tantalus aethiopicus. This determination was placed beyond question by Cuvier (Ann. du Muséum, iv. 116-135) and Savigny (Hist. nat. et mythol. de l'ibis) in 1805. They, however, removed it from the Linnaean genus Tantalus and, Lacépède
having some years before founded a genus Ibis, it was transferred thither, and is now generally known as I. aethiopica, though some speak of it as I. religiosa. No attempt can here be made to treat the ibis from a mythological or antiquarian point of view. Savigny's memoir contains a great deal of matter on the subject. Wilkinson (Ancient Egyptians, ser. 2, vol. ii. pp. 217-224) added some of the results of later research, and Renouf in his Hibbert Lectures explains the origin of the myth.

The ibis is chiefly an inhabitant of the Nile basin from Dongola southward, as well as of Kordofan and Sennar; whence about midsummer it moves northwards to Egypt. ${ }^{1}$ In Lower Egypt it bears the name of Abu-mengel, or "father of the sickle," from the form of its bill, but it does not stay long in that country, disappearing when the Nile has subsided. Hence most travellers have failed to meet with it there ${ }^{2}$ (since their acquaintance with the birds of Egypt is limited to those which frequent the country in winter), and writers have denied generally to this species a place in its modern fauna (cf. Shelley, Birds of Egypt, p. 261). However, in 1864, von Heuglin (Journ. für Ornithologie, 1865, p. 100) saw a young bird which had been shot in the Delta, and E. C. Taylor (Ibis, 1878, p. 372) saw an adult which had been killed near Lake Menzal in 1877. The story told to Herodotus of its destroying snakes is, according to Savigny, devoid of truth, but Cuvier states that he discovered partly digested remains of a snake in the stomach of a mummied ibis.

The ibis is somewhat larger than a curlew, Numenius arquata, which bird it resembles, with a much stouter bill and stouter legs. The head and greater part of the neck are bare and black. The plumage is white, except the primaries, which are black, and a black plume, formed by the secondaries, tertials and lower scapulars, and richly glossed with bronze, blue and green, which curves gracefully over the hind-quarters. The bill and feet are also black. The young lack the ornamental plume, and in them the head and neck are clothed with short black feathers, while the bill is yellow. The nest is placed in bushes or high trees, the bird generally building in companies, and in the middle of August von Heuglin (Orn. Nord-Ost-Afrikas, p. 1138) found that it had from two to four young or much incubated eggs. ${ }^{3}$ These are of a dingy white, splashed, spotted and speckled with reddish-brown.

Congeneric with the typical ibis are two or three other species, the I. melanocephala of India, the I. molucca or I. strictipennis, of Australia, and the I. bernieri of Madagascar, all of which closely resemble I. aethiopica; while many other forms not very far removed from it, though placed by authors in distinct genera, ${ }^{4}$ are known. Among these are several beautiful species such as the Japanese Geronticus nippon, the Lophotibis cristata of Madagascar, and the scarlet ibis, ${ }^{5}$ Eudocimus ruber, of America. The glossy ibis, Plegadis falcinellus, found throughout the West Indies, Central and the south-eastern part of North America, as well as in many parts of Europe (whence it not unfrequently strays to the British Islands), Africa, Asia and Australia. This bird, believed to be the second kind of ibis spoken of by Herodotus, is rather smaller than the sacred ibis, and mostly of a dark chestnut colour with brilliant green and purple reflections on the upper parts, exhibiting, however, when young none of the rufous hue. This species lays eggs of a deep sea-green colour, having wholly the character of heron's eggs, and it often breeds in company with herons, while the eggs of all other ibises whose eggs are known resemble those of the sacred ibis. Though ibises resemble the curlews externally, there is no affinity between them. The Ibididae are more nearly related to the storks, Ciconiidae, and still more to the spoonbills, Plataleidae, with which latter many systematists consider them to form one group, the Hemiglottides of Nitzsch. Together these groups form the sub-order Ciconiae of the order Ciconiiformes. The true ibises are also to be clearly separated from the wood-ibises, Tantalidae, of which there are four or five species, by several not unimportant structural characters. Fossil remains of a true ibis, I. pagana, have been found in considerable numbers in the middle Tertiary beds of France. ${ }^{6}$

1 It has been said to occur occasionally in Europe (Greece and southern Russia).
2 E. C. Taylor remarked (Ibis, 1859, p. 51), that the buff-backed heron, Ardea bubulcus, was made by the tourists' dragomans to do duty for the "sacred ibis," and this seems to be no novel practice, since by it, or something like it, Hasselqvist was misled, and through him Linnaeus.

4 For some account of these may be consulted Dr Reichenow's paper in Journ. für Ornithologie (1877), pp. 143-156; Elliot's in Proc. Zool. Society (1877), pp. 477-510; and that of Oustalet in Nouv. Arch. du Muséum, ser. 2, vols. i. pp. 167-184.

5 It is a popular error-especially among painters-that this bird was the sacred ibis of the Egyptians.

IBLIS, or Eblis, in Moslem mythology the counterpart of the Christian and Jewish devil. He figures oftener in the Koran under the name Shaitan, Iblis being mentioned 11 times, whereas Shaitan appears in 87 passages. He is chief of the spirits of evil, and his personality is adapted to that of his Jewish prototype. Iblis rebelled against Allah and was expelled from Paradise. The Koranic legend is that his fall was a punishment for his refusal to worship Adam. Condemned to death he was afterwards respited till the judgment day (Koran vii. 13).

See Gustav Weil, The Bible, the Koran and the Talmud (London, 1846).

IBN 'ABD RABBIHI [Abū 'Umar Aḥmad ibn Maḥommed ibn 'Abd Rabbihi] (860-940), Arabian poet, was born in Cordova and descended from a freed slave of Hishām, the second Spanish Omayyad caliph. He enjoyed a great reputation for learning and eloquence. No diwan of his is extant, but many selections from his poems are given in the Yatīmat ud-Dahr, i. 412436 (Damascus, 1887). More widely known than his poetry is his great anthology, the 'Iqd ulFarīd ("The Precious Necklace"), a work divided into twenty-five sections, the thirteenth being named the middle jewel of the necklace, the chapters on either side of this being named after other jewels. It is an adab book (see Arabia: Literature, section "Belles Lettres") resembling Ibn Qutaiba's 'Uyūn ul-Akhbār, from which it borrows largely. It has been printed, several times in Cairo (1876, 1886, \&c.).
(G. W. T.)

IBN ‘ARABĪ [Muḥyiuddīn Abū ‘Abdallāh ibn ul-'Arabī] (1165-1240), Moslem theologian and mystic, was born in Murcia and educated in Seville. When thirty-eight he travelled in Egypt, Arabia, Bagdad, Mosul and Asia Minor, after which he lived in Damascus for the rest of his life. In law he was a Zahirite, in theology a mystic of the extreme order, though professing orthodox Ash'arite theology and combating in many points the Indo-Persian mysticism (pantheism). He claims to have had conversations with all the prophets past and future, and reports conversations with God himself. Of his numerous works about 150 still exist. The most extensive is the twelve-volume Futūhāt ul-Makkīyāt ("Meccan Revelations"), a general encyclopaedia of Sufic beliefs and doctrines. Numerous extracts from this work are contained in Sha'rānī's (d. 1565) manual of Sufic dogma (Yawāqīt) published several times in Cairo. A short account of these works is given in A. von Kremer's Geschichte der herrschenden Ideen des Islams, pp. 102-109 (Leipzig, 1868). Another characteristic and more accessible work of Ibn 'Arabi is the Fuṣūs ul-Hikam, on the nature and importance of the twenty-seven chief prophets, written in 1230 (ed. Bulāq, 1837) and with the Commentary (Cairo, 1891) of Qāshāni (d. 1350); cf. analysis by M. Schreiner in Journal of German Oriental Society, lii. 516-525.

Of some 289 works said to have been written by Ibn 'Arabī 150 are mentioned in C. Brockelmann's Gesch. der arabischen Litteratur, vol. i. (Weimar, 1898), pp. 441-448. See also R. A. Nicholson, A Literary History of the Arabs, pp. 399-404 (London, 1907).
(G. W. T.)

IBN ATHĪR, the family name of three brothers, all famous in Arabian literature, born at Jazīrat ibn 'Umar in Kurdistan. The eldest brother, known as Majd ud-Dīn (1149-1210), was long in the service of the amir of Mosul, and was an earnest student of tradition and language. His dictionary of traditions (Kitāb un-Nihāya) was published at Cairo (1893), and his dictionary of
family names (Kitāb ul-Murassa) has been edited by Seybold (Weimar, 1896). The youngest brother, known as DiyĀ ud-Dīn (1163-1239), served Saladin from 1191 on, then his son, al-Malik ul-Afdal, and was afterwards in Egypt, Samosata, Aleppo, Mosul and Bagdad. He was one of the most famous aesthetic and stylistic critics in Arabian literature. His Kitāb ul-Mathal, published in Bulāq in 1865 (cf. Journal of the German Oriental Society, xxxv. 148, and Goldziher's Abhandlungen, i. 161 sqq.), contains some very independent criticism of ancient and modern Arabic verse. Some of his letters have been published by D. S. Margoliouth "On the Royal Correspondence of Diya ed-Din el-Jazari" in the Actes du dixième congrès international des orientalistes, sect. 3, pp. 7-21.

The brother best known by the simple name of Ibn Athīr was Abu-l-Hasan 'Izzuddīn Mahommed Ibn Ul-Athír (1160-1234), who devoted himself to the study of history and tradition. At the age of twenty-one he settled with his father in Mosul and continued his studies there. In the service of the amir for many years, he visited Bagdad and Jerusalem and later Aleppo and Damascus. He died in Mosul. His great history, the Kāmil, extends to the year 1231; it has been edited by C. J. Tornberg, Ibn al-Athiri Chronicon quod perfectissimum inscribitur (14 vols., Leiden, 18511876), and has been published in 12 vols. in Cairo (1873 and 1886). The first part of this work up to A.н. 310 (A.D. 923) is an abbreviation of the work of Tabarī (q.v.) with additions. Ibn Athīr also wrote a history of the Atabegs of Mosul, published in the Recueil des historiens des croisades (vol. ii., Paris); a work (Usd ul-Ghāba), giving an account of 7500 companions of Mahomet (5 vols., Cairo, 1863), and a compendium (the Lubāb) of Sam'āni’s Kitāb ul-Anṣāb (cf. F. Wüstenfeld's Specimen el-Lobabi, Göttingen, 1835).
(G. W. T.)

IBN BATUTA, i.e. Abu Abdullah Mahommed, surnamed Ibn Batuta (1304-1378), the greatest of Moslem travellers, was born at Tangier in 1304. He entered on his travels at twenty-one (1325) and closed them in 1355. He began by traversing the coast of the Mediterranean from Tangier to Alexandria, finding time to marry two wives on the road. After some stay at Cairo, then probably the greatest city in the world (excluding China), and an unsuccessful attempt to reach Mecca from Aidhab on the west coast of the Red Sea, he visited Palestine, Aleppo and Damascus. He then made the pilgrimage to Mecca and Medina, and visited the shrine of Ali at Mashhad-Ali, travelling thence to Basra, and across the mountains of Khuzistan to Isfahan, thence to Shiraz and back to Kufa and Bagdad. After an excursion to Mosul and Diarbekr, he made the haj a second time, staying at Mecca three years. He next sailed down the Red Sea to Aden (then a place of great trade), the singular position of which he describes, noticing its dependence for water-supply upon the great cisterns restored in modern times. He continued his voyage down the African coast, visiting, among other places, Mombasa and Quiloa (Kilwa). Returning north he passed by the chief cities of Oman to New Ormuz (Hurmuz), which had about 15 years before, c. 1315, been transferred to its famous island-site from the mainland (Old Ormuz). After visiting other parts of the gulf he crossed the breadth of Arabia to Mecca, making the haj for the third time. Crossing the Red Sea, he made a journey of great hardship to Syene, and thence along the Nile to Cairo. After this, travelling through Syria, he made a circuit among the petty Turkish states into which Asia Minor was divided after the fall of the kingdom of Rum (Iconium). He now crossed the Black Sea to Kaffa, then mainly occupied by the Genoese, and apparently the first Christian city he had seen, for he was much perturbed by the bell-ringing. He next travelled into Kipchak (the Mongol khanate of Russia), and joined the camp of the reigning khan Mahommed Uzbeg, from whom the great and heterogeneous Uzbeg race is perhaps named. Among other places in this empire he travelled to Bolghar ( $54^{\circ} 54^{\prime} \mathrm{N}$.) in order to witness the shortness of the summer night, and desired to continue his travels north into the "Land of Darkness" (in the extreme north of Russia), of which wonderful things were told, but was obliged to forego this. Returning to the khan's camp he joined the cortège of one of the Khatuns, who was a Greek princess by birth (probably illegitimate) and in her train travelled to Constantinople, where he had an interview with the emperor Andronikos III. the Younger (1328-1341). He tells how, as he passed the city gates, he heard the guards muttering Sarakinu. Returning to the court of Uzbeg, at Sarai on the Volga, he crossed the steppes to Khwarizm and Bokhara; thence through Khorasan and Kabul, and over the Hindu Kush (to which he gives that name, its first occurrence). He reached the Indus, on his own statement, in September, 1333. This closes the first part of his narrative.

From Sind, which he traversed to the sea and back again, he proceeded to Multan, and eventually, on the invitation of Mahommed Tughlak, the reigning sovereign, to Delhi. Mahommed was a singular character, full of pretence at least to many accomplishments and virtues, the founder of public charities, and a profuse patron of scholars, but a parricide, a
fratricide, and as madly capricious, bloodthirsty and unjust as Caligula. "No day did his palace gate fail to witness the elevation of some abject to affluence and the torture and murder of some living soul." He appointed the traveller to be kazi of Delhi, with a present of 12,000 silver dinars (rupees), and an annual salary of the same amount, besides an assignment of village lands. In the sultan's service Ibn Batuta remained eight years; but his good fortune stimulated his natural extravagance, and his debts soon amounted to four or five times his salary. At last he fell into disfavour and retired from court, only to be summoned again on a congenial duty. The emperor of China, last of the Mongol dynasty, had sent a mission to Delhi, and the Moor was to accompany the return embassy (1342). The party travelled through central India to Cambay and thence sailed to Calicut, classed by the traveller with the neighbouring Kaulam (Quilon), Alexandria, Sudak in the Crimea, and Zayton (Amoy harbour) in China, as one of the greatest trading havens in the world-an interesting enumeration from one who had seen them all. The mission party was to embark in Chinese junks (the word used) and smaller vessels, but that carrying the other envoys and the presents, which started before Ibn Batuta was ready, was wrecked totally; the vessel that he had engaged went off with his property, and he was left on the beach of Calicut. Not daring to return to Delhi, he remained about Honore and other cities of the western coast, taking part in various adventures, among others the capture of Sindabur (Goa), and visiting the Maldive Islands, where he became kazi, and married four wives, and of which he has left the best medieval account, hardly surpassed by any modern. In August 1344 he left the Maldives for Ceylon; here he made the pilgrimage to the "Footmark of our Father Adam." Thence he betook himself to Maabar (the Coromandel coast), where he joined a Mussulman adventurer, residing at Madura, who had made himself master of much of that region. After once more visiting Malabar, Canara and the Maldives, he departed for Bengal, a voyage of forty-three days, landing at Sadkawan (Chittagong). In Bengal he visited the famous Moslem saint Shaykh Jalaluddin, whose shrine (Shah Jalal at Silhet) is still maintained. Returning to the delta, he took ship at Sunarganw (near Dacca) on a junk bound for Java (i.e. Java Minor of Marco Polo, or Sumatra). Touching the coast of Arakan or Burma, he reached Sumatra in forty days, and was provided with a junk for China by Malik al Dhahir, a zealous disciple of Islam, which had recently spread among the states on the northern coast of that island. Calling (apparently) at Cambodia on his way, Ibn Batuta reached China at Zayton (Amoy harbour), famous from Marco Polo; he also visited Sin Kalan or Canton, and professes to have been in Khansa (Kinsay of Marco Polo, i.e. Hangchau), and Khanbalik (Cambaluc or Peking). The truth of his visit to these two cities, and especially to the last, has been questioned. The traveller's history, not least in China, singularly illustrates the free masonry of Islam, and its power of carrying a Moslem doctor over the known world of Asia and Africa. On his way home he saw the great bird Rukh (evidently, from his description, an island lifted by refraction); revisited Sumatra, Malabar, Oman, Persia, Bagdad, and crossed the great desert to Palmyra and Damascus, where he got his first news of home, and heard of his father's death fifteen years before. Diverging to Hamath and Aleppo, on his return to Damascus, he found the Black Death raging, so that two thousand four hundred died in one day. Revisiting Jerusalem and Cairo he made the haj a fourth time, and finally reappeared at Fez (visiting Sardinia en route) on the 8th of November 1349, after twenty-four years' absence. Morocco, he felt, was, after all, the best of countries. "The dirhems of the West are but little; but then you get more for them." After going home to Tangier, Ibn Batuta crossed into Spain and made the round of Andalusia, including Gibraltar, which had just then stood a siege from the "Roman tyrant Adfunus" (Alphonso XI. of Castile, 1312-1350). In 1352 the restless man started for Central Africa, passing by the oases of the Sahara (where the houses were built of rock-salt, as Herodotus tells, and roofed with camel skins) to Timbuktu and Gogo on the Niger, a river which he calls the Nile, believing it to flow down into Egypt, an opinion maintained by some up to the date of Lander's discovery. Being then recalled by his own king, he returned to Fez (early in 1354) via Takadda, Haggar and Tuat. Thus ended his twenty-eight years' wanderings which in their main lines alone exceeded $75,000 \mathrm{~m}$. By royal order he dictated his narrative to Mahommed Ibn Juzai, who concludes the work, 13th of December 1355 (A.D.) with the declaration: "This Shaykh is the traveller of our age; and he who should call him the traveller of the whole body of Islam would not exceed the truth." Ibn Batuta died in 1378, aged seventythree.

Ibn Batuta's travels have only been known in Europe during the 19th century; at first merely by Arabic abridgments in the Gotha and Cambridge libraries. Notices or extracts had been published by Seetzen (c. 1808), Kosegarten (1818), Apetz (1819), and Burckhardt (1819), when in 1829 Dr S. Lee published for the Oriental Translation Fund a version from the abridged MSS. at Cambridge, which attracted much interest. The French capture of Constantina afforded MSS. of the complete work, one of them the autograph of Ibn Juzai. And from these, after versions of fragments by various French scholars, was derived at last (1858-1859) the standard edition and translation of the whole by M. Défrémery and Dr Sanguinetti, in 4 vols. See also Sir Henry Yule, Cathay, ii. 397-526; C. Raymond Beazley, Dawn of Modern Geography, iii. 535-538. Though there are some singular chronological difficulties in the narrative, and a good many cursory
inaccuracies and exaggerations, there is no part of it except, perhaps, certain portions of the journeys in north China, which is open to doubt. The accounts of the Maldive Islands, and of the Negro countries on the Niger, are replete with interesting and accurate particulars. The former agrees surprisingly with that given by the only other foreign resident we know of, Pyrard de la Val, two hundred and fifty years later. Ibn Batuta's statements and anecdotes regarding the showy virtues and solid vices of Sultan Muhammad Tughlak are in entire agreement with Indian historians, and add many fresh details.
(H. Y.; C. R. B.)

IBN DURAID [Abū Bakr Mahommed ibn ul-Ḥasan ibn Duraid ul-Azdī] (837-934), Arabian poet and philologist, was born at Baṣra of south Arabian stock. At his native place he was trained under various teachers, but fled in 871 to Oman at the time Baṣra was attacked by the negroes, known as the Zanj, under Muhallabī. After living twelve years in Oman he went to Persia, and, under the protection of the governor, 'Abdallāh ibn Mahommed ibn Mīkāl, and his son, Isma'īl, wrote his chief works. In 920 he went to Bagdad, where he received a pension from the caliph Moqtadir.

The Maqsūra, a poem in praise of Ibn Mīkāl and his son, has been edited by A. Haitsma (1773) E. Scheidius (1786) and N. Boyesen (1828). Various commentaries on the poem exist in MS. (cf. C. Brockelmann, Gesch. der ar. Lit., i. 211 ff., Weimar, 1898), The Jamhara fi-l-Lugha is a large dictionary written in Persian but not printed. Another work is the Kitāb ul-Ishtiqāq ("Book of Etymology"), edited by F. Wüstenfeld (Göttingen, 1854); it was written in opposition to the anti-Arabian party to show the etymological connexion of the Arabian tribal names.
(G. W. T.)

IBN FARADĪ [Abū-l-Walīd 'Abdallāh ibn ul-Faradi] (962-1012), Arabian historian, was born at Cordova and studied law and tradition. In 992 he made the pilgrimage and proceeded to Egypt and Kairawān, studying in these places. After his return in 1009 he became cadi in Valencia, and was killed at Cordova when the Berbers took the city.

His chief work is the History of the Learned Men of Andalusia, edited by F. Codera (Madrid, 1891-1892). He wrote also a history of the poets of Andalusia.
(G. W. T.)

IBN FĀRID [Abū-l-Qāsim 'Umar ibn ul-Fāriḍ] (1181-1235), Arabian poet, was born in Cairo, lived for some time in Mecca and died in Cairo. His poetry is entirely Sufic, and he was esteemed the greatest mystic poet of the Arabs. Some of his poems are said to have been written in ecstasies. His diwan has been published with commentary at Beirūt, 1887, \&c.; with the commentaries of Burīnī (d. 1615) and 'Abdul-Ghānī (d. 1730) at Marseilles, 1853, and at Cairo; and with the commentary of Rushayyid Ghālib (19th century) at Cairo, 1893. One of the separate poems was edited by J. von Hammer Purgstall as Das arabische hohe Lied der Liebe (Vienna, 1854).

See R. A. Nicholson, A Literary History of the Arabs (London, 1907), pp. 394-398.
(G. W. T.)

IBN GABIROL [Solomon ben Judah], Jewish poet and philosopher, was born at Malaga, probably about 1021. The early part of his troublous life was spent at Saragossa, but few personal details of it are recorded. His parents died while he was a child and he was under the protection first of a certain Jekuthiel, who died in 1039, and afterwards of Samuel ha-Nagid,
the well-known patron of learning. His passionate disposition, however, embittered no doubt by his misfortunes, involved him in frequent difficulties and led to his quarrelling with Samuel. It is generally agreed that he died young, although the date is uncertain. Al Harizi ${ }^{1}$ says at the age of twenty-nine, and Moses b. Ezra ${ }^{2}$ about thirty, but Abraham Zaccuto ${ }^{3}$ states that he died (at Valencia) in 1070. M. Steinschneider ${ }^{4}$ accepts the date 1058.

His literary activity began early. He is said to have composed poems at the age of sixteen, and elegies by him are extant on Hai Gaon (died in 1038) and Jekuthiel (died in 1039), each of which was written probably soon after the death of the person commemorated. About the same time he also wrote his 'Anaq, a poem on grammar, of which only 97 lines out of 400 are preserved. Moses ben Ezra says of him that he imitated Moslem models, and was the first to open to Jewish poets the door of versification, ${ }^{5}$ meaning that he first popularized the use of Arabic metres in Hebrew. It is as a poet that he has been known to the Jews to the present day, and admired for the youthful freshness and beauty of his work, in which he may be compared to the romantic school in France and England in the early 19th century. Besides his lyrical and satirical poems, he contributed many of the finest compositions to the liturgy (some of them with the acrostic "Shelomoh ha-qațō"), which are widely different from the artificial manner of the earlier payyeṭanim. The best known of his longer liturgical compositions are the philosophical Kether Malkūth (for the Day of Atonement) and the Azharōth, on the 613 precepts (for Shebhu'öth). Owing to his pure biblical style he had an abiding influence on subsequent liturgical writers.

Outside the Jewish community he was known as the philosopher Avicebron (Avencebrol, Avicebrol, \&c.) The credit of identifying this name as a medieval corruption of Ibn Gabirol is due to S. Munk, who showed that selections made by Shem Tōbh Palqera (or Falqera) from the Meqōr Hayyīm (the Hebrew translation of an Arabic original) by Ibn Gabirol, corresponded to the Latin Fons Vitae of Avicebron. The Latin version, made by Johannes Hispalensis and Gundisalvi about one hundred years after the author's death, had at once become known among the Schoolmen of the 12th century and exerted a powerful influence upon them, although so little was known of the author that it was doubted whether he was a Christian or a Moslem. The teaching of the Fons Vitae was entirely new to the country of its origin, and being drawn largely from Neoplatonic sources could not be expected to find favour with Jewish thinkers. Its distinctive doctrines are: (1) that all created beings, spiritual or corporeal, are composed of matter and form, the various species of matter being but varieties of the universal matter, and similarly all forms being contained in one universal form; (2) that between the primal One and the intellect (the voũऽ of Plotinus) there is interposed the divine Will, which is itself divine and above the distinction of form and matter, but is the cause of their union in the being next to itself, the intellect, in which Avicebron holds that the distinction does exist. The doctrine that there is a material, as well as a formal, element in all created beings was explicitly adopted from Avicebron by Duns Scotus (as against the view of Albertus Magnus and Thomas Aquinas), and perhaps his exaltation of the will above the intellect is due to the same influence. Avicebron develops his philosophical system throughout quite independently of his religious views-a practice wholly foreign to Jewish teachers, and one which could not be acceptable to them. Indeed, this charge is expressly brought against him by Abraham ben David of Toledo (died in 1180). It is doubtless this non-religious attitude which accounts for the small attention paid to the Fons Vitae by the Jews, as compared with the wide influence of the philosophy of Maimonides.

The other important work of Ibn Gabirol is Iṣlāḥ al-akhlāq (the improvement of character), a popular work in Arabic, translated into Hebrew (Tiqqūn middōth ha-nephesh) by Judah ibn Tibbon. It is widely different in treatment from the Fons, being intended as a practical not a speculative work.

The collection of moral maxims, compiled in Arabic but best known (in the Hebrew translation of Judah ibn Tibbon) as Mibḥar ha-peninim, is generally ascribed to Ibn Gabirol, though on less certain grounds.

Bibliography.-Texts of the liturgical poems are to be found in the prayer-books: others in Dukes and Edelmann, Treasures of Oxford (Oxford, 1850); Dukes, Shīrē Shelomoh (Hanover, 1858); S. Sachs, Shīr ha-shīrīm asher li-Shelomoh (Paris, 1868, incomplete); Brody, Die weltlichen Gedichte des ... Gabirol (Berlin, 1897, \&c.).
"Avencebrolis Fons Vitae" (Latin text) in Clemens Bäumker's Beiträge zur Gesch. d. Philosophie, Bd. i. Hefte 2-4 (Münster, 1892); The Improvement of the Moral Qualities [Arabic and English] ed. by S. S. Wise (New York, 1901); A Choice of Pearls [Hebrew and English] ed. by Ascher (London, 1859).

On the philosophy in general: S. Munk, Mélanges (quoted above); Guttmann, Die Philosophie des Sal.-ibn Gabirol (Göttingen, 1889); D. Kaufmann, Studien über Sal.-ibn Gabirol (Budapest, 1899); S. Horovitz, "Die Psychologie Ibn Gabirols," in the Jahresbericht des jüd. theol. Seminars

Fränckel'scher Stiftung (Breslau, 1900); Wittmann, "Zur Stellung Avencebrols ..." (in Bäumker's Beiträge, Bd. v. Heft 1, Münster, 1905).

1 Jud. Har. Macamæ, ed. Lagarde (Göttingen, 1883), p. 89, 1. 61.
See the passage quoted by Munk, Mélanges de philosophie arabe et juive (Paris, 1859), pp. 264 and 517.

Liber Juchassin, ed. Filipowski (London, 1857), p. 217.
Hebr. Übersetzungen (Berlin, 1893), § 219, note 70; cf. Kaufmann, Studien über Sal.-ibn Gabirol (Budapest, 1899), p. 79, note 2.
See Munk, op. cit. pp. 515-516, transl. on pp. 263-264. Metre had been already used by Dunash.

IBN HAUKAL, strictly Ibn Hauqal, a 10th century Arabian geographer. Nothing is known of his life. His work on geography, written in 977, is only a revision and extension of the Masālik ul-Mamālik of al-Isțakhrī, who wrote in 951. This itself was a revised edition of the Kitāb ulAshkāl or Suwar ul-Aqālim of Abū Zaid ul-Balkhī, who wrote about 921. Ibn Haukal's work was published by M. J. de Goeje (Leiden, 1873). An anonymous epitome of the book was written in 1233.

See M. J. de Goeje, "Die Ișṭahrī-Balhī Frage," in the Zeitschrift der deutschen Morgenländischen Gesellschaft, xxv. 42 sqq.

IBN HAZM [Abū Maḥommed 'Alī ibn Aḥmad ibn Ḥazm] (994-1064), Moslem theologian, was born in a suburb of Cordova. He studied history, law and theology, and became a vizier as his father had been before him, but was deposed for heresy, and spent the rest of his life quietly in the country. In legal matters he belonged first to the Shāfi'ite school, but came to adopt the views of the Zāhirites, who admitted only the external sense of the Koran and tradition, disallowing the use of analogy (Qiyās) and Taqlīd (appeal to the authority of an imām), and objecting altogether to the use of individual opinion ( $R a^{\prime} y$ ). Every sentence of the Koran was to be interpreted in a general and universal sense; the special application to the circumstances of the time it was written was denied. Every word of the Koran was to be taken in a literal sense, but that sense was to be learned from other uses in the Koran itself, not from the meaning in other literature of the time. The special feature of Ibn Hazm's teaching was that he extended the application of these principles from the study of law to that of dogmatic theology. He thus found himself in opposition at one time to the Mo'tazilites, at another to the Ash'arites. He did not, however, succeed in forming a school. His chief work is the Kitāb ul-Milal wan-Niḥal, or "Book of Sects" (published in Cairo, 1899).

For his teaching cf. I. Goldziher, Die Zahiriten, pp. 116-172 (Leipzig, (1884), and M. Schreiner in the Journal of the German Oriental Society, lii. 464-486. For a list of his other works see C. Brockelmann's Geschichte der arabischen Literatur, vol. i. (Weimar, 1898), p. 400.
(G. W. T.)

IBN HISHĀM [Abū Maḥommed 'Abdulmalik ibn Hishām ibn Ayyūb ul-Himyarī] (d. 834), Arabian biographer, studied in Kufa but lived afterwards in Fostāt (old Cairo), where he gained a name as a grammarian and student of language and history. His chief work is his edition of Ibn Isḥāq's (q.v.) Life of the Apostle of God, which has been edited by F. Wüstenfeld (Göttingen, 1858-1860). An abridged German translation has been made by G. Weil (Stuttgart, 1864; cf. P. Brönnle, Die Commentatoren des Ibn Isḥaq und ihre Scholien, Halle, 1895). Ibn Hishām is said to have written a work explaining the difficult words which occur in poems on the life of the Apostle, and another on the genealogies of the Himyarites and their princes.

IBN ISHĀQ [Mahommed ibn Isḥāq Abū 'Abdallāh] (d. 768), Arabic historian, lived in Medina, where he interested himself to such an extent in the details of the Prophet's life that he was attacked by those to whom his work seemed to have a rationalistic tendency. He consequently left Medina in 733, and went to Alexandria, then to Kufa and Hira, and finally to Bagdad, where the caliph Manṣūr provided him with the means of writing his great work. This was the Life of the Apostle of God, which is now lost and is known to us only in the recension of Ibn Hishām (q.v.). The work has been attacked by Arabian writers (as in the Fihrist) as untrustworthy, and it seems clear that he introduced forged verses (cf. Journal of the German Oriental Society, xiv. 288 sqq.). It remains, however, one of the most important works of the age.
(G. W. T.)

IBN JUBAIR [Abū-l Husain Maḥommed ibn Aḥmad ibn Jubair] (1145-1217), Arabian geographer, was born in Valencia. At Granada he studied the Koran, tradition, law and literature, and later became secretary to the Mohad governor of that city. During this time he composed many poems. In 1183 he left the court and travelled to Alexandria, Jerusalem, Medina, Mecca, Damascus, Mosul and Bagdad, returning in 1185 by way of Sicily.

The Travels of Ibn Jubair were edited by W. Wright (Leiden, 1852); and a new edition of this text, revised by M. J. de Goeje, was published by the Gibb Trustees (London, 1907). The part relating to Sicily was published, with French translation and notes, by M. Amari in the Journal asiatique (1845-1846) and a French translation alone of the same part by G. Crolla in Museon, vi. 123-132.
(G. W. T.)

IBN KHALDŪN [Abū Zaid ibn Maḥommed ibn Maḥommed ibn Khaldūn] (1332-1406), Arabic historian, was born at Tunis. He studied the various branches of Arabic learning with great success. In 1352 he obtained employment under the Marīnid sultan Abū Inān (Faris I.) at Fez. In the beginning of 1356 , his integrity having been suspected, he was thrown into prison until the death of Abū Inān in 1358, when the vizier al-Hasan ibn Omar set him at liberty and reinstated him in his rank and offices. He here continued to render great service to Abu Salem (Ibrahim III.), Abū Inān's successor, but, having offended the prime minister, he obtained permission to emigrate to Spain, where, at Granada, he was received with great cordiality by Ibn al Ahmar, who had been greatly indebted to his good offices when an exile at the court of Abu Salem. The favours he received from the sovereign excited the jealousy of the vizier, and he was driven back to Africa (1364), where he was received with great cordiality by the sultan of Bougie, Abu Abdallah, who had been formerly his companion in prison. On the fall of Abu Abdallah Ibn Khaldūn raised a large force amongst the desert Arabs, and entered the service of the sultan of Tlemçen. A few years later he was taken prisoner by Abdalaziz ('Abd ul 'Azīz), who had defeated the sultan of Tlemçen and seized the throne. He then entered a monastic establishment, and occupied himself with scholastic duties, until in 1370 he was sent for to Tlemçen by the new sultan. After the death of 'Abd ul 'Azīz he resided at Fez, enjoying the patronage and confidence of the regent. After some further vicissitudes in 1378 he entered the service of the sultan of his native town of Tunis, where he devoted himself almost exclusively to his studies and wrote his history of the Berbers. Having received permission to make the pilgrimage to Mecca, he reached Cairo, where he was presented to the sultan, al-Malik udhDhahir Barkuk, who insisted on his remaining there, and in the year 1384 made him grand cadi of the Malikite rite for Cairo. This office he filled with great prudence and probity, removing many abuses in the administration of justice in Egypt. At this time the ship in which his wife and family, with all his property, were coming to join him, was wrecked, and every one on board lost. He endeavoured to find consolation in the completion of his history of the Arabs of Spain. At the same time he was removed from his office of cadi, which gave him more leisure for his work. Three years later he made the pilgrimage to Mecca, and on his return lived in retirement in the Fayum until 1399, when he was again called upon to resume his functions as
cadi. He was removed and reinstated in the office no fewer than five times.
In 1400 he was sent to Damascus, in connexion with the expedition intended to oppose Timur or Tamerlane. When Timur had become master of the situation, Ibn Khaldūn let himself down from the walls of the city by a rope, and presented himself before the conqueror, who permitted him to return to Egypt. Ibn Khaldūn died on the 16th of March 1406, at the age of sixty-four.

The great work by which he is known is a "Universal History," but it deals more particularly with the history of the Arabs of Spain and Africa. Its Arabic title is Kitāb ul'Ibar, wa dīwān el Mubtada wa'l Khabar, fī ayyām ul 'Arab wa'l'Ajām wa'l Berber, that is, "The Book of Examples and the Collection of Origins and Information respecting the History of the Arabs, Foreigners and Berbers." It consists of three books, an introduction and an autobiography. Book i. treats of the influence of civilization upon man; book ii. of the history of the Arabs and other peoples from the remotest antiquity until the author's own times; book iii. of the history of the Berber tribes and of the kingdoms founded by that race in North Africa. The introduction is an elaborate treatise on the science of history and the development of society, and the autobiography contains the history, not only of the author himself, but of his family and of the dynasties which ruled in Fez, Tunis and Tlemçen during his lifetime. An edition of the Arabic text has been printed at Būlāq, (7 vols., 1867) and a part of the work has been translated by the late Baron McG. de Slane under the title of Histoire des Berbères (Algiers, 1852-1856); it contains an admirable account of the author and analysis of his work. Vol. i., the Muqaddama (preface), was published by M. Quatremère (3 vols., Paris, 1858), often republished in the East, and a French translation was made by McG. de Slane (3 vols., Paris, 1862-1868). The parts of the history referring to the expeditions of the Franks into Moslem lands were edited by C. J. Tornberg (Upsala, 1840), and the parts treating of the Banu-l Ahmar kings of Granada were translated into French by M. Gaudefroy-Demombynes in the Journal asiatique, ser. 9, vol. xiii. The Autobiography of Ibn Khaldūn was translated into French by de Slane in the Journal asiatique, ser. 4, vol. iii. For an English appreciation of the philosophical spirit of Ibn Khaldūn see R. Flint's History of the Philosophy of History (Edinburgh, 1893), pp. 157-170.
(E. H. P.; G. W. T.)

IBN KHALLIKĀN [Abū-l 'Abbās Aḥmad ibn Khallikān] (1211-1282), Arabian biographer, was born at Arbela, the son of a professor reputed to be ascended from the Barmecides of the court of Harun al-Rashid. When eighteen he went to Aleppo, where he studied for six years, then to Damascus, and in 1238 to Alexandria and Cairo. In 1252 he married and became chief cadi of Syria in Damascus in 1261. Having held this office for ten years, he was professor in Cairo until 1278, when he again took office in Damascus for three years. In 1281 he accepted a professorship in the same city, but died in the following year.

His great work is the Kitab Wafayāt ul-A'yān, "The Obituaries of Eminent Men." It contains in alphabetical order the lives of the most celebrated persons of Moslem history and literature, except those of Mahomet, the four caliphs and the companions of Mahomet and their followers (the Tābiūn). The work is anecdotal and contains many brief extracts from the poetry of the writers. It was published by F. Wüstenfeld (Göttingen, 1835-1843), in part by McG. de Slane (Paris, 1838-1842), and also in Cairo (1859 and 1882). An English translation by McG. de Slane was published for the Oriental Translation Fund in 4 vols. (London, 1842-1871). Thirteen extra biographies from a manuscript in Amsterdam were published by Pijnappel (Amsterdam, 1845). A Persian translation exists in manuscript, and various extracts from the work are known. Several supplements to the book have been written, the best known being that of Mahommed ibn Shākir (d. 1362), published at Cairo 1882. A collection of poems by Ibn Khallikān is also extant.
(G. W. T.)

IBN QUTAIBA, or Kotaiba [Abū Mahommed ibn Muslim ibn Qutaiba] (828-889), Arabian writer, was born at Bagdad or Kufa, and was of Iranian descent, his father belonging to Merv. Having studied tradition and philology he became cadi in Dinawār and afterwards teacher in Bagdad, where he died. He was the first representative of the eclectic school of Bagdad philologists that succeeded the schools of Kufa and Baṣra (see Arabia: Literature, section "Grammar"). Although engaged also in theological polemic (cf. I. Goldziher, Muhammedanische Studien, ii. 136, Halle, 1890), his chief works were directed to the training of the ideal
secretary. Of these five may be said to form a series. The Adab ul-Kātib ("Training of the Secretary") contains instruction in writing and is a compendium of Arabic style. It has been edited by Max Grünert (Leiden, 1900). The Kitāb ush-Sharāb is still in manuscript. The Kitāb ulMa'ārif has been edited by F. Wüstenfeld as the Handbuch der Geschichte ${ }^{1}$ (Göttingen, 1850); the Kitāb ush-Shi'r wash-Shu'arāi ("Book of Poetry and Poets") edited by M. J. de Goeje (Leiden, 1904). ${ }^{2}$ The fifth and most important is the 'Uyūn ul-Akhbār, which deals in ten books with lordship, war, nobility, character, science and eloquence, asceticism, friendship, requests, foods and women, with many illustrations from history, poetry and proverb (ed. C. Brockelmann, Leiden, 1900 sqq.).

For other works (which were much quoted by later Arabian writers) see C. Brockelmann, Gesch. der arabischen Literatur, vol. i. (Weimar, 1898), pp. 120-122.
(G. W. T.)

[^1]IBN ṢA'D [Abū 'Abdallāh Maḥommed ibn Ṣa'd ibn Mani' uz-Zuhrī], often called Kātib ulWaqidī ("secretary of Waqidī") of Baṣra] (d. 845), Arabian biographer, received his training in tradition from Waqidī and other celebrated teachers. He lived for the most part in Bagdad, and had the reputation of being both trustworthy and accurate in his writings, which, in consequence, were much used by later writers. His work, the Kitāb ul-Tabaqāt ul-Kabīr (15 vols.) contains the lives of Mahomet, his Companions and Helpers (including those who fought at Badr as a special class) and of the following generation (the Followers) who received their traditions from the personal friends of the Prophet.

This work has been edited under the superintendence of E. Sachau (Leiden, 1904 sqq.); cf. O. Loth, Das Classenbuch des Ibn Sa'd (Leipzig, 1869).
(G. W. T.)

IBN TIBBON, a family of Jewish translators, who flourished in Provence in the 12th and 13th centuries. They all made original contributions to philosophical and scientific literature, but their permanent fame is based on their translations. Between them they rendered into Hebrew all the chief Jewish writings of the middle ages. These Hebrew translations were, in their turn, rendered into Latin (by Buxtorf and others) and in this form the works of Jewish authors found their way into the learned circles of Europe. The chief members of the Ibn Tibbon family were (1) Judah Ben Saul (1120-1190), who was born in Spain but settled in Lunel. He translated the works of Baḥya, Halevi, Saadiah and the grammatical treatises of Janah. (2) His son, Samuel (1150-1230), translated the Guide of the Perplexed by Maimonides. He justly termed his father "the father of the Translators," but Samuel's own method surpassed his father's in lucidity and fidelity to the original. (3) Son of Samuel, Moses (died 1283). He translated into Hebrew a large number of Arabic books (including the Arabic form of Euclid). The Ibn Tibbon family thus rendered conspicuous services to European culture, and did much to further among Jews who did not understand Arabic the study of science and philosophy.
(I. A.)

IBN TִUFAIL, or Țofail [Abū Bakr Maḥommed ibn 'Abd-ul-Malik ibn Ṭufail ul-Qaisī] (d. 1185), Moslem philosopher, was born at Guadix near Granada. There he received a good training in philosophy and medicine, and is said to have been a pupil of Avempace (q.v.). He became secretary to the governor of Granada, and later physician and vizier to the Mohad caliph, Abu Ya'qūb Yūsuf. He died at Morocco.

His chief work is a philosophical romance, in which he describes the awakening and growth of intellect in a child removed from the influences of ordinary life. Its Arabic title is Risālat Hayy ibn Yaqzān; it was edited by E. Pococke as Philosophus autodidactus (Oxford, 1671; 2nd ed., 1700), and with a French translation by L. Gauthier (Algiers, 1900). An English translation by S. Ockley was published in 1708 and has been reprinted since. A Spanish translation by F. Pons Boigues was published at Saragossa (1900). Another work of Ibn Țufail, the Kitāb Asrār ulHikma ul-mashraqīyya ("Secrets of Eastern Science"), was published at Bulāq (1882); cf. S. Munk, Mélanges (1859), pp. 410 sqq., and T. J. de Boer, Geschichte der Philosophie im Islam (Stuttgart, 1901), pp. 160 sqq. (also an English translation).
(G. W. T.)

IBN USAIBI'A [Muwaffaquddīn Abū-l-'Abbās Aḥmad ibn ul-Qāsim ibn Abī Usaibi'a] (12031270), Arabian physician, was born at Damascus, the son of an oculist, and studied medicine at Damascus and Cairo. In 1236 he was appointed by Saladin physician to a new hospital in Cairo, but surrendered the appointment the following year to take up a post given him by the amir of Damascus in Salkhad near that city. There he lived and died. He wrote 'Uyūn ul-Anba'fi Țabaqāt ul-Ațibba' or "Lives of the Physicians," which in its first edition (1245-1246) was dedicated to the vizier of Damascus. This he enlarged, though it is uncertain whether the new edition was made public in the lifetime of the author.

Edition by A. Müller (Königsberg, 1884).
(G. W. T.)

IBO, a district of British West Africa, on the lower Niger immediately above the delta, and mainly on the eastern bank of the river. The chief town, frequently called by the same name (more correctly Abo or Áboh), lies on a creek which falls into the main stream about 150 m . from its mouth and contains from 6000 to 8000 inhabitants. The Ibo are a strong well-built Negro race. Their women are distinguished by their embonpoint. The language of the Ibo is one of the most widely spoken on the lower Niger. The Rev. J. F. Schön began its reduction in 1841, and in 1861 he published a grammar (Oku Ibo Grammatical Elements, London, Church Miss. Soc.). (See Nigeria.)

IBRAHĪM AL-MAUŞILĪ (742-804), Arabian singer, was born of Persian parents settled in Kufa. In his early years his parents died and he was trained by an uncle. Singing, not study, attracted him, and at the age of twenty-three he fled to Mosul, where he joined a band of wild youths. After a year he went to Rai (Rei, Rhagae), where he met an ambassador of the caliph Manṣūr, who enabled him to come to Baṣra and take singing lessons. His fame as a singer spread, and the caliph Mahdì brought him to the court. There he remained a favourite under Hādī, while Harūn al-Rashīd kept him always with him until his death, when he ordered his son (Ma'mūn) to say the prayer over his corpse. Ibrahīm, as might be expected, was no strict Moslem. Two or three times he was knouted and imprisoned for excess in wine-drinking, but was always taken into favour again. His powers of song were far beyond anything else known at the time. Two of his pupils, his son Isḥāq and Muḥāriq, attained celebrity after him.

See the Preface to W. Ahlwardt's Abu Nowas (Greifswald, 1861), pp. 13-18, and the many stories of his life in the Kitāb ul-Aghāni, v. 2-49.
(G. W. T.)
son of Mehemet Ali, pasha of Egypt. He is also and more commonly called his son. He was born in his father's native town, Kavala in Thrace. During his father's struggle to establish himself in Egypt, Ibrahim, then sixteen years of age, was sent as a hostage to the Ottoman capitan pasha (admiral), but when Mehemet Ali was recognized as pasha, and had defeated the English expedition under General A. M. Fraser, he was allowed to return to Egypt. When Mehemet Ali went to Arabia to prosecute the war against the Wahhabis in 1813, Ibrahim was left in command in Upper Egypt. He continued the war with the broken power of the Mamelukes, whom he suppressed. In 1816 he succeeded his brother Tusun in command of the Egyptian forces in Arabia. Mehemet Ali had already begun to introduce European discipline into his army, and Ibrahim had probably received some training, but his first campaign was conducted more in the old Asiatic style than his later operations. The campaign lasted two years, and terminated in the destruction of the Wahhabis as a political power. Ibrahim landed at Yembo, the port of Medina, on the 30th of September 1816. The holy cities had been recovered from the Wahhabis, and Ibrahim's task was to follow them into the desert of Nejd and destroy their fortresses. Such training as the Egyptian troops had received, and their artillery, gave them a marked superiority in the open field. But the difficulty of crossing the desert to the Wahhabi stronghold of Deraiya, some 400 m . east of Medina, and the courage of their opponents, made the conquest a very arduous one. Ibrahim displayed great energy and tenacity, sharing all the hardships of his army, and never allowing himself to be discouraged by failure. By the end of September 1818 he had forced the Wahhabi leader to surrender, and had taken Deraiya, which he ruined. On the 11th of December 1819 he made a triumphal entry into Cairo. After his return he gave effective support to the Frenchman, Colonel Sève (Suleiman Pasha), who was employed to drill the army on the European model. Ibrahim set an example by submitting to be drilled as a recruit. When in 1824 Mehemet Ali was appointed governor of the Morea by the sultan, who desired his help against the insurgent Greeks, he sent Ibrahim with a squadron and an army of 17,000 men. The expedition sailed on the 10 th of July 1824 , but was for some months unable to do more than come and go between Rhodes and Crete. The fear of the Greek fire ships stopped his way to the Morea. When the Greek sailors mutinied from want of pay, he was able to land at Modon on the 26th of February 1825. He remained in the Morea till the capitulation of the 1 st of October 1828 was forced on him by the intervention of the Western powers. Ibrahim's operations in the Morea were energetic and ferocious. He easily defeated the Greeks in the open field, and though the siege of Missolonghi proved costly to his own troops and to the Turks who operated with him, he brought it to a successful termination on the 24 th of April 1826. The Greek guerrilla bands harassed his army, and in revenge he desolated the country and sent thousands of the inhabitants into slavery in Egypt. These measures of repression aroused great indignation in Europe, and led first to the intervention of the English, French and Russian squadrons (see Navarino, Battle of), and then to the landing of a French expeditionary force. By the terms of the capitulation of the 1st of October 1828, Ibrahim evacuated the country. It is fairly certain that the Turkish government, jealous of his power, had laid a plot to prevent him and his troops from returning to Egypt. English officers who saw him at Navarino describe him as short, grossly fat and deeply marked with smallpox. His obesity did not cause any abatement of activity when next he took the field. In 1831, his father's quarrel with the Porte having become flagrant, Ibrahim was sent to conquer Syria. He carried out his task with truly remarkable energy. He took Acre after a severe siege on the 27th of May 1832, occupied Damascus, defeated a Turkish army at Homs on the 8th of July, defeated another Turkish army at Beilan on the 29th of July, invaded Asia Minor, and finally routed the grand vizier at Konia on the 21st of December. The convention of Kutaiah on the 6th of May left Syria for a time in the hands of Mehemet Ali. Ibrahim was undoubtedly helped by Colonel Sève and the European officers in his army, but his intelligent docility to their advice, as well as his personal hardihood and energy, compare most favourably with the sloth, ignorance and arrogant conceit of the Turkish generals opposed to him. He is entitled to full credit for the diplomatic judgment and tact he showed in securing the support of the inhabitants, whom he protected and whose rivalries he utilized. After the campaign of 1832 and 1833 Ibrahim remained as governor in Syria. He might perhaps have administered successfully, but the exactions he was compelled to enforce by his father soon ruined the popularity of his government and provoked revolts. In 1838 the Porte felt strong enough to renew the struggle, and war broke out once more. Ibrahim won his last victory for his father at Nezib on the 24th of June 1839. But Great Britain and Austria intervened to preserve the integrity of Turkey. Their squadrons cut his communications by sea with Egypt, a general revolt isolated him in Syria, and he was finally compelled to evacuate the country in February 1841. Ibrahim spent the rest of his life in peace, but his health was ruined. In 1846 he paid a visit to western Europe, where he was received with some respect and a great deal of curiosity. When his father became imbecile in 1848 he held the regency till his own death on the 10th of November 1848.

See Edouard Gouin, L'Égypte au XIX ${ }^{e}$ siècle (Paris, 1847); Aimé Vingtrinier, Soliman-Pasha (Colonel Sève) (Paris, 1886). A great deal of unpublished material of the highest interest with regard to Ibrahim's personality and his system in Syria is preserved in the British Foreign

IBSEN, HENRIK (1828-1906), Norwegian dramatic and lyric poet, eldest son of Knud Henriksen Ibsen, a merchant, and of his wife Marichen Cornelia Altenburg, was born at Skien on the 20th of March 1828. For five generations the family had consisted on the father's side of a blending of the Danish, German and Scottish races, with no intermixture of pure Norwegian. In 1836 Knud Ibsen became insolvent, and the family withdrew, in great poverty, to a cottage in the outskirts of the town. After brief schooling at Skien, Ibsen was, towards the close of 1843, apprenticed to an apothecary in Grimstad; here he remained through seven dreary years of drudgery, which set their mark upon his spirit. In 1847, in his nineteenth year, he began to write poetry. He made a gloomy and almost sinister impression upon persons who met him at this time, and one of his associates of those days has recorded that Ibsen "walked about Grimstad like a mystery sealed with seven seals." He had continued, by assiduous reading, his self-education, and in 1850 he contrived to come up as a student to Christiania. In the same year he published his first work, the blank-verse tragedy of Catilina, under the pseudonym Brynjolf Bjarme. A second drama, The Viking's Barrow, was acted (but not printed) a few months later; Ibsen was at this time entirely under the influence of the Danish poet Oehlenschläger. During the next year or two he made a very precarious livelihood in Christiania as a journalist, but in November 1851 he had the good fortune to be appointed "stage-poet" at the little theatre of Bergen, with a small but regular salary. He was practically manager at this house, and he also received a travelling stipend. In 1852, therefore, he went for five months to study the stage, to Copenhagen and to Dresden. Among many dramatic experiments which Ibsen made in Bergen, the most considerable and most satisfactory is the saga-drama of Mistress Inger at Östraat, which was produced in 1855; and printed at Christiania in 1857; here are already perceptible some qualities of his mature character. Much less significant, although at the time more successful, is The Feast at Solhaug, a tragedy produced in Bergen in 1856; here for a moment Ibsen abandoned his own nascent manner for an imitation of the popular romantic dramatist of Denmark, Henrik Hertz. It is noticeable that Ibsen, by far the most original of modern writers for the stage, was remarkably slow in discovering the true bent of his genius. His next dramatic work was the romantic tragedy of Olaf Liljekrans, performed in 1857, but unprinted until 1898. This was the last play Ibsen wrote in Bergen. In the summer of the former year his five years' appointment came to an end, and he returned to Christiania. Almost immediately he began the composition of a work which showed an extraordinary advance on all that he had written before, the beautiful saga-drama of The Warriors in Helgeland, in which he threw off completely the influence of the Danish romantic tragedians, and took his material directly from the ancient Icelandic sources. This play marks an epoch in the development of Norwegian literature. It was received by the managers, both in Christiania and Copenhagen, with contemptuous disapproval, and in the autumn of 1857 Ibsen could not contrive to produce it even at the new theatre of which he was now the manager. The Warriors was printed at Christiania in 1858, but was not acted anywhere until 1861. During these years Ibsen suffered many reverses and humiliations, but he persisted in his own line in art. Some of his finest short poems, among others the admirable seafaring romance, Terje Vigen, belong to the year 1860. The annoyances which Ibsen suffered, and the retrograde and ignorant conditions which he felt around him in Norway, developed the ironic qualities in his genius, and he became an acid satirist. The brilliant rhymed drama, Love's Comedy, a masterpiece of lyric wit and incisive vivacity, was published in 1862. This was a protest against the conventionality which deadens the beauty of all the formal relations between men and women, and against the pettiness, the publicity, and the prosiness of betrothed and married life among the middle classes in Norway; it showed how society murders the poetry of love. For some time past Ibsen had been meditating another saga-drama in prose, and in 1864 this appeared, Kongsemnerne (The Pretenders). These works, however, now so universally admired, contained an element of strangeness which was not welcome when they were new. Ibsen's position in Christiania grew more and more disagreeable, and he had positive misfortunes which added to his embarrassment. In 1862 his theatre became bankrupt, and he was glad to accept the poorly-paid post of "aesthetic adviser" at the other house. An attempt to obtain a poet's pension (digtergage) was unsuccessful; the Storthing, which had just voted one to Björnson, refused to do the same for Ibsen. His cup was full of disillusion and bitterness, and in April 1864 he started, by Berlin and Trieste, ultimately to settle in Rome. His anger and scorn gave point to the satirical arrows which he shot back to his thankless fatherland from Italy in the splendid poem of Brand, published in Copenhagen in 1866, a fierce attack on the Laodicean
state of religious and moral sentiment in the Norway of that day; the central figure, the stern priest Brand, who attempts to live like Christ and is snubbed and hounded away by his latitudinarian companions, is one of the finest conceptions of a modern poet. Ibsen had scarcely closed Brand before he started a third lyrico-dramatic satire. Peer Gynt (1867), which remains, in a technical sense, the most highly finished of all his metrical works. In Brand the hero had denounced certain weaknesses which Ibsen saw in the Norwegian character, but these and other faults are personified in the hero of Peer Gynt; or rather, in this figure the poet pictured, in a type, the Norwegian nation in all the egotism, vacillation, and lukewarmness which he believed to be characteristic of it. Ibsen, however, acted better than he preached, and he soon forgot his abstraction in the portrait of Peer Gynt as a human individual. In this magnificent work modern Norwegian literature first rises to a level with the finest European poetry of the century. In 1869 Ibsen wrote the earliest of his prose dramas, the political comedy, The Young Men's League, in which for the first time he exercised his extraordinary gift for perfectly natural and yet pregnant dialogue. Ibsen was in Egypt, in October 1869, when his comedy was put on the stage in Christiania, amid violent expressions of hostility; on hearing the news, he wrote his brilliant little poem of defiance, called At Port Saïd. By this time, however, he had become a successful author; Brand sold largely, and has continued to be the most popular of Ibsen's writings. In 1866, moreover, the Storthing had been persuaded to vote him a "poet's pension," and there was now an end of Ibsen's long struggle with poverty. In 1868 he left Rome, and settled in Dresden until 1874, when he returned to Norway. But after a short visit he went back to Germany, and lived first at Dresden, afterwards at Munich, and did not finally settle in Christiania until 1891. His shorter lyrical poems were collected in 1871, and in that year his name and certain of his writings were for the first time mentioned to the English public. At this time he was revising his old works, which were out of print, and which he would not resign again to the reading world until he had subjected them to what in some instances (for example, Mistress Inger at Östraat) amounted to practical recomposition. In 1873 he published a double drama, each part of which was of unusual bulk, the whole forming the tragedy of Emperor and Galilean; this, Ibsen's latest historical play, has for subject the unsuccessful struggle of Julian the Apostate to hold the world against the rising tide of Christianity. The work is of an experimental kind, and takes its place between the early poetry and the later prose of the author. Compared with the series of plays which Ibsen had already inaugurated with The Young Men's League, Emperor and Galilean preserves a colour of idealism and even of mysticism which was for many years to be absent from Ibsen's writings, but to reappear in his old age with The Master-builder. There is some foundation for the charge that Ibsen has made his romantic Greek emperor needlessly squalid, and that he has robbed him, at last, too roughly of all that made him a sympathetic exponent of Hellenism. Ibsen was now greatly occupied by the political spectacle of Germany at war first in Denmark, then in France, and he believed that all things were conspiring to start a new epoch of individualism. He was therefore deeply disgusted by the Paris commune, and disappointed by the conservative reaction which succeeded it. This disillusion in political matters had a very direct influence upon Ibsen's literary work. It persuaded him that nothing could be expected in the way of reform from democracies, from large blind masses of men moved capriciously in any direction, but that the sole hope for the future must lie in the study of personality, in the development of individual character. He set himself to diagnose the conditions of society, which he had convinced himself lay sick unto death. Hitherto Ibsen had usually employed rhymed verse for his dramatic compositions, or, in the case of his saga-plays, a studied and artificial prose. Now, in spite of the surprising achievements of his poetry, he determined to abandon versification, and to write only in the language of everyday conversation. In the first drama of this his new period, The Pillars of Society (1877), he dealt with the problem of hypocrisy in a small commercial centre of industry, and he drew in the Bernick family a marvellous picture of social egotism in a prosperous seaport town. There was a certain similarity between this piece and A Doll's House (1879), although the latter was much the more successful in awakening curiosity. Indeed, no production of Ibsen's has been so widely discussed as this, which is nevertheless not the most coherently conceived of his plays. Here also, social hypocrisy, was the object of the playwright's satire, but this time mainly in relation to marriage. In $A$ Doll's House Ibsen first developed his views with regard to the individualism of woman. In his previous writings he had depicted woman as a devoted and willing sacrifice to man; here he begins to explain that she has no less a duty to herself, and must keep alive her own conception of honour and of responsibility. The conclusion of $A$ Doll's House was violently and continuously discussed through the length and breadth of Europe, and to the situation of Nora Helmer is probably due more than to anything else the long tradition that Ibsen is "immoral." He braved convention still more audaciously in Ghosts (1881), perhaps the most powerful of the series of plays in which Ibsen diagnoses the diseases of modern society. It was received in Norway with a tumult of ill-will, and the author was attacked no less venomously than he had been twenty years before. Ibsen was astonished and indignant at the reception given to Ghosts, and at the insolent indifferentism of the majority to all ideas of social reform. He wrote, more as a
pamphlet than as a play, what is yet one of the most effective of his comedies, An Enemy of the People (1882). Dr Stockmann, the hero of that piece, discovers that the drainage system of the bathing-station on which the little town depends is faulty, and the water impure and dangerous. He supposes that the corporation will be grateful to have these deficiencies pointed out; on the contrary, they hound him out of their midst as an "enemy of the people." In this play occurs Ibsen's famous and typical saying, "a minority may be right-a majority is always wrong." This polemical comedy seemed at first to be somewhat weakened by the personal indignation which runs through it, but it has held the stage. Ibsen's next drama, The Wild Duck (1884), was written in singular contrast with the zest and fire which had inspired An Enemy of the People. Here he is squalid and pessimistic to a degree elsewhere unparalleled in his writings; it is not quite certain that he is not here guilty of a touch of parody of himself. The main figure of the play is an unhealthy, unlucky enthusiast, who goes about making hopeless mischief by exposing weak places in the sordid subterfuges of others. This drama contains a figure, Hjálmar Ekdal, who claims the bad pre-eminence of being the meanest scoundrel in all drama. The Wild Duck is the darkest, the least relieved, of Ibsen's studies of social life, and his object in composing it is not obvious. With Rosmersholm (1886) he rose to the height of his genius again; this is a mournful, but neither a pessimistic nor a cynical play. The fates which hang round the contrasted lives of Rosmer and Rebecca, the weak-willed scrupulous man and the strong-willed unshrinking woman, the old culture and the new, the sickly conscience and the robust one, create a splendid dramatic antithesis. Ibsen then began to compose a series of dramas, of a more and more symbolical and poetic character; the earliest of these was the mystical The Lady from the Sea (1888). At Christmas 1890 he brought out Hedda Gabler, two years later The Master-builder (Bygmester Solnaes), in which many critics see the highest attainment of his genius; at the close of 1894 Little Eyolf; in 1896 John Gabriel Borkman; and in 1900 When We Dead Awaken. On the occasion of his seventieth birthday (1898) Ibsen was the recipient of the highest honours from his own country and of congratulations and gifts from all parts of the world. A colossal bronze statue of him was erected outside the new National Theatre, Christiania, in September 1899. In 1901 his health began to decline, and he was ordered by the physician to abandon every species of mental effort. The evil advanced, and he became unconscious of the passage of events. After lingering in this sad condition he died, without suffering, on the 23 rd of May 1906, and was accorded a public funeral, with the highest national honours.

No recent writer belonging to the smaller countries of Europe has had so widely spread a fame as that of Ibsen, and although the value of his dramatic work is still contested, it has received the compliment of vivacious discussion in every part of the world. There would, perhaps, have been less violence in this discussion if it had been perceived that the author does not pose as a moral teacher, but as an imaginative investigator. He often and with much heat insisted that he was not called upon as a poet to suggest a remedy for the diseases of society, but to diagnose them. In this he was diametrically opposed to Tolstoi, who admitted that he wrote his books for the healing of the nations. If the subjects which Ibsen treats, or some of them, are open to controversy, we are at least on firm ground in doing homage to the splendour of his art as a playwright. He reintroduced into modern dramatic literature something of the velocity and inevitability of Greek tragic intrigue. It is very rarely that any technical fault can be found with the architecture of his plots, and his dialogue is the most lifelike that the modern stage has seen. His long apprenticeship to the theatre was of immense service to him in this respect. In every country, though least perhaps in England, the influence of Ibsen has been marked in the theatrical productions of the younger school. Even in England, on the rare occasions when his dramas are acted, they awaken great interest among intelligent playgoers.

The editions of Ibsen's works are numerous, but the final text is included in the Samlede Vaerker, with a bibliography by J. B. Halvorsen, published in Copenhagen, in 10 vols. (18981902). They have been translated into the principal European languages, and into Japanese. The study of Ibsen in English was begun by Mr Gosse in 1872, and continued by Mr William Archer, whose version of Ibsen's prose dramas appeared in 5 vols. (1890, 1891; new and revised edition, 1906). Other translators have been Mr C. Herford, Mr R. A. Streatfield, Miss Frances Lord and Mr Adie. His Correspondence was edited, in 2 vols., under the supervision of his son, Sigurd Ibsen, in 1904 (Eng. trans., 1905). Critical studies on the writings and position of Ibsen are innumerable, and only those which were influential in guiding opinion, during the early part of his career, in the various countries, can be mentioned here: Georg Brandes Ästhetiske Studier (Copenhagen, 1868); Les Quesnel, Poésie scandinave (Paris 1874); Valfrid Valsenius, Henrik Ibsen (Helsingfors, 1879); Edmund Gosse, Studies in Northern Literature (London, 1879); L. Passarge, Henrik Ibsen (Leipzig, 1883); G. Brandes, Björnson och Ibsen (Stockholm, 1882); Henrik Jaeger, Henrik Ibsen 1828-1888 (Copenhagen, 1888; Eng. trans., 1890); T. Terwey, Henrik Ibsen (Amsterdam, 1882); G. Bernard Shaw, The Quintessence of Ibsen (London, 1892). In France Count Moritz Prozor carried on an ardent propaganda in favour of Ibsen from 1885, and Jules Lemaître's articles in his Les Contemporains and Impressions de théâtre did much to encourage discussion. W. Archer forwarded the cause in

England from 1878 onwards. In Germany Ibsen began to be known in 1866, when John Grieg, P. F. Siebold and Adolf Strodtmann successively drew attention to his early dramas; but his real popularity among the Germans dates from 1880.

IBYCUS, of Rhegium in Italy, Greek lyric poet, contemporary of Anacreon, flourished in the 6 th century в.c. Notwithstanding his good position at home, he lived a wandering life, and spent a considerable time at the court of Polycrates, tyrant of Samos. The story of his death is thus related: While in the neighbourhood of Corinth, the poet was mortally wounded by robbers. As he lay dying he saw a flock of cranes flying overhead, and called upon them to avenge his death. The murderers betook themselves to Corinth, and soon after, while sitting in the theatre, saw the cranes hovering above. One of them, either in alarm or jest, ejaculated, "Behold the avengers of Ibycus," and thus gave the clue to the detection of the crime (Plutarch, De Garrulitate, xiv.). The phrase, "the cranes of Ibycus," passed into a proverb among the Greeks for the discovery of crime through divine intervention. According to Suidas, Ibycus wrote seven books of lyrics, to some extent mythical and heroic, but mainly erotic (Cicero, Tusc. Disp. iv. 33), celebrating the charms of beautiful youths and girls. F. G. Welcker suggests that they were sung by choruses of boys at the "beauty competitions" held at Lesbos. Although the metre and dialect are Dorian, the poems breathe the spirit of Aeolian melic poetry.

The best editions of the fragments are by F. W. Schneidewin (1833) and Bergk, Poëtae lyrici Graeci.

ICA (Yca, or Ecca), a city of southern Peru and the capital of a department of the same name, 170 m. S.S.E. of Lima, and 46 m . by rail S.E. of Pisco; its port on the Pacific coast. Pop. (1906, official estimate) 6000. It lies in a valley of the foothills of the Cordillera Occidental, which is watered by the Rio de Ica, is made highly fertile by irrigation, and is filled with vineyards and cotton fields; between this valley and the coast is a desert. The original town was founded in 1563, 4 m . E. of its present site, but it was destroyed by the earthquake of 1571 , and again by that of 1664, after which the present town was laid out near the ruins. In 1882 a Chilean marauding expedition inflicted great damage to private property in the town and vicinity. These repeated disasters give the place a partially ruined appearance, but it has considerable commercial and industrial prosperity. It has a large cotton factory and there are some smaller industries. Wine-making is one of the principal industries of the valley, and much brandy, called pisco, is exported from Pisco. A new industry is that of drying the fruits for which this region is celebrated. Ica is the seat of a national college.

The department of Ica lies between the Western Cordillera and the Pacific coast, and extends from the department of Lima S.E. to that of Arequipa. Pop. (1906, official estimate) 68,220; area 8721 sq. m . Ica is in the rainless region of Peru, and the greater part of its surface is barren. It is crossed by the rivers Pisco, Ica and Grande, whose tributaries drain the western slope of the Cordillera, and whose valleys are fertile and highly cultivated. The valley of the Nasca, a tributary of the Grande, is celebrated for an extensive irrigating system constructed by the natives before the discovery of America. The principal products of the department are cotton, grapes, wine, spirits, sugar and fruit. These are two good ports on the northern coast, Tambo de Mora and Pisco, the latter being connected with the capital by a railway across the desert, 46 m . long.

ICE (a word common to Teutonic languages; cf. Ger. Eis), the solid crystalline form which water assumes when exposed to a sufficiently low temperature. It is a colourless crystalline substance, assuming forms belonging to the hexagonal system, and distinguished by a wellmarked habit of twinning, which occasions the beautiful "ice flowers" displayed by hoar-frost. It is frequently precipitated as hoar-frost, snow or hail; and in the glaciers and snows of lofty
mountain systems or of regions of high latitude it exists on a gigantic scale, being especially characteristic of the seas and lands around the poles. In various regions, especially in France and Italy, great quantities of ice form in caves, which, in virtue of their depth below the earth's surface, their height above the sea-level, or their exposure to suitable winds, or to two or more of these conditions in combination, are unaffected by ordinary climatic changes, so that the mean annual temperature is sufficiently low to ensure the permanency of the ice. The temperature at which water freezes, and also at which ice melts, is so readily determined that it is employed as one of the standard temperatures in the graduation of ordinary thermometer scales, this temperature being the zero of the Centigrade and Réaumur scales, and $32^{\circ}$ of the Fahrenheit (see Thermometry). In the act of freezing, water, though its temperature remains unchanged, undergoes a remarkable expansion so that ice at $0^{\circ} \mathrm{C}$. is less dense than water-a fact demonstrated by its power of floating. The sub-aqueous retention of "ground-ice" or "anchor-ice," which forms in certain circumstances at the bottom of streams or pools in which there are many eddies, is due to the cohesion between it and the stones or rocks which compose the bed of the streams or pools. As water expands on freezing, so conversely ice contracts on melting; and the ice-cold water thus formed continues to contract when heated until it has reached its point of maximum density, the temperature at which this occurs being about $39^{\circ}$ Fahr, or $4^{\circ} \mathrm{C}$. Above this point water continuously expands, and at no temperature is it less dense than ice as is shown by the following table:-

| Density of ice at |  | $0^{\circ} \mathrm{C}=.9175$ |
| :---: | :---: | :---: |
| $"$ | water at | $0^{\circ} \mathrm{C}=.99988$ |
| $"$ | $"$ | $4^{\circ} \mathrm{C}=1.00000$ |
| $"$ | $"$ | $10^{\circ} \mathrm{C}=.99976$ |
| $"$ | $"$ | $100^{\circ} \mathrm{C}=.95866$ |

Under the influence of heat, ice itself behaves as most solids do, contracting when cooled, expanding when heated. According to Plücker, the coefficient of cubical dilatation at moderately low temperatures is 0.0001585 . From a series of elaborate experiments, Person deduced 0.505 as the specific heat of ice, or about half that of water.

Though no rise of temperature accompanies the melting of ice, there is yet a definite quantity of heat absorbed, namely, about 80 calories per gram; this is called the latent heat of fusion of water (see Fusion). The same amount of heat is evolved when water becomes ice. That ice can be melted by increase of pressure was first pointed out by James Thomson in 1849. He showed that, since water expands on freezing, the laws of thermodynamics require that its freezingpoint must be lowered by increase of pressure; and he calculated that for every additional atmosphere of pressure the freezing-point of water was lowered by $0.0075^{\circ}$. This result was verified by his brother, Sir William Thomson (Lord Kelvin), in 1850. The Thomsons and H. L. F. Helmholtz successfully applied this behaviour of ice under pressure to the explanation of many properties of the substance. When two blocks of ice at $0^{\circ} \mathrm{C}$. are pressed together or even simply laid in contact, they gradually unite along their touching surfaces till they form one block. This "regelation" is due to the increased pressure at the various points of contact causing the ice there to melt and cool. The water so formed tends to escape, thus relieving the pressure for an instant, refreezing and returning to the original temperature. This succession of melting and freezing, with their accompanying thermal effects, goes on until the two blocks are cemented into one.

Ice forms over fresh water if the temperature of the air has been for a sufficient time at or below the freezing-point; but not until the whole mass of water has been cooled down to its point of maximum density, so that the subsequent cooling of the surface can give rise to no convection currents, is freezing possible. Sea-water, in the most favourable circumstances, does not freeze till its temperature is reduced to about $-2^{\circ} \mathrm{C}$. ; and the ice, when formed, is found to have rejected four-fifths of the salt which was originally present. In the upper provinces of India water is made to freeze during cold clear nights by leaving it overnight in porous vessels, or in bottles which are enwrapped in moistened cloth. The water then freezes in virtue of the cold produced by its own evaporation or by the drying of the moistened wrapper. In Bengal the natives resort to a still more elaborate forcing of the conditions. Pits are dug about 2 ft . deep and filled three-quarters full with dry straw, on which are set flat porous pans containing the water to be frozen. Exposed overnight to a cool dry gentle wind from the northwest, the water evaporates at the expense of its own heat, and the consequent cooling takes place with sufficient rapidity to overbalance the slow influx of heat from above through the cooled dense air or from below through the badly conducting straw.

See Water, and for the manufacture of ice see Refrigerating.

ICEBERG (from ice and Berg, Ger. for hill, mountain), a floating mass of ice broken from the end of a glacier or from an ice-sheet. The word is sometimes, but rarely, applied to the arch of an Arctic glacier viewed from the sea. It is more commonly used to describe huge floating masses of ice that drift from polar regions into navigable waters. They are occasionally encountered far beyond the polar regions, rising into beautiful forms with breakers roaring into their caves and streams of water pouring from their pinnacles in the warmer air. When, however, they rest in comparatively warm water, melting takes place most rapidly at the base and they frequently overturn. Only one-ninth of the mass of ice is seen above water. When a glacier descends to the sea, as in Alaska, and "advances into water, the depth of which approaches its thickness, the ends are broken off and the detached masses float away as icebergs. Many of the bergs are overturned, or at least tilted, as they set sail. If this does not happen at once it is likely to occur later as the result of the wave-cutting and melting which disturb their equilibrium" (T. C. Chamberlin and R. D. Salisbury, Geology: Processes and their Results, 1905). These bergs carry a load of débris from the glacier and gradually strew their load upon the sea floor. They do not travel far before losing all stony and earthy débris, but glacial material found in dredgings shows that icebergs occasionally carry their load far from land. The structure of the iceberg varies with its origin and is always that of the glacier or icesheet from which it was broken. The breaking off of the ice-sheet from a Greenland glacier is called locally the "calving" of the glacier. The constantly renewed material from which the icebergs are formed is brought down by the motion of the glacier. The ice-sheet cracks at the end, and masses break off, owing to the upward pressure of the water upon the lighter ice which is pushed into it. This is accomplished with considerable violence. The disintegration of an Arctic ice-sheet is a simpler matter, as the ice is already floating.

ICELAND (Dan. Island), an island in the North Atlantic Ocean, belonging to Denmark. Its extreme northerly point is touched by the Arctic Circle; it lies between $13^{\circ} 22^{\prime}$ and $24^{\circ} 35^{\prime} \mathrm{W}$., and between $63^{\circ} 12^{\prime}$ and $66^{\circ} 33^{\prime} \mathrm{N}$., and has an area of $40,437 \mathrm{sq}$. m. Its length is 298 m . and its breadth 194 m. , the shape being a rough oval, broken at the north-west, where a peninsula, diversified by a great number of fjords, projects from the main portion of the island. The total length of the coast-line is about 3730 m ., of which approximately one-third belongs to the north-western peninsula. Iceland is a plateau or tableland, built up of volcanic rocks of older and younger formation, and pierced on all sides by fjords and valleys. Compared with the tableland, the lowlands have a relatively small area, namely, one-fourteenth of the whole; but these lowlands are almost the only parts of the island which are inhabited. In consequence of the rigour of its climate, the central tableland is absolutely uninhabitable. At the outside, not more than one-fourth of the area of Iceland is inhabited; the rest consists of elevated deserts, lava streams and glaciers. The north-west peninsula is separated from the main mass of the island by the bays Hunaflói and Breiðifjörðr, so that there are really two tablelands, a larger and a smaller. The isthmus which connects the two is only $41 / 4 \mathrm{~m}$. across, but has an altitude of 748 ft . The mean elevation of the north-west peninsula is 2000 ft . The fjords and glens which cut into it are shut in by precipitous walls of basalt, which plainly shows that they have been formed by erosion through the mass of the plateau. The surface of this tableland is also bare and desolate, being covered with gravel and fragments of rock. Here and there are large straggling snowfields, the largest being Glámu and Drangajökull, ${ }^{1}$ on the culminating points of the plateau. The only inhabited districts are the shores of the fjords, where grass grows capable of supporting sheep; but a large proportion of the population gain their livelihood by fishing. The other and larger tableland, which constitutes the substantial part of Iceland, reaches its culminating point in the south-east, in the gigantic snowfield of Vatnajökull, which covers 3300 sq. m. The axis of highest elevation of Iceland stretches from north-west to south-east, from the head of Hvammsfjörðr to Hornafjörðr, and from this water-parting the rivers descend on both sides. The crest of the water-parting is crowned by a chain of snow-capped mountains, separated by broad patches of lower ground. They are really a chain of minor plateaus which rise 4500 to 6250 ft . above sea-level and 2000 to 3000 ft . above the tableland itself. In the extreme east is Vatnajökull, which is separated from Tungnafellsjökull by Vonarskard ( 3300 ft .). Between Tungnafellsjökull and Hofsjökull lies the broad depression of Sprengisandr ( 2130 ft .). Continuing north-west, between Hofsjökull and the next snow-capped mountain, Langjökull, lies Kjölur (2000 ft.); and between Langjökull and Eiriksjökull, Flosaskard (2630 ft.). To the north of the jöklar last mentioned there are a number of lakes, all well stocked with fish. Numerous valleys or glens penetrate into the tableland, especially on the north and east, and between them long mountain spurs, sections of the tableland which have resisted the action of erosion, thrust themselves towards the sea. Of these the most considerable is the mass
crowned by Mýrdalsjökull, which stretches towards the south. The interior of the tableland consists for the most part of barren, grassless deserts, the surface being covered by gravel, loose fragments of rock, lava, driftsand, volcanic ashes and glacial detritus.

Save the lower parts of the larger glens, there are no lowlands on the north and east. The south coast is flat next the sea; but immediately underneath Vatnajökull there is a strip of gravel and sand, brought down and deposited by the glacial streams. The largest low-lying plain of Iceland, lying between Mýrdalsjökull and Reykjanes, has an area of about $1550 \mathrm{sq} . \mathrm{m}$. In its lowest parts this plain barely keeps above sea-level, but it rises gradually towards the interior, terminating in a ramification of valleys. Its maximum altitude is attained at 381 ft . near Geysir. On the west of Mount Hekla this plain connects by a regular slope directly with the tableland, to the great injury of its inhabited districts, which are thus exposed to the clouds of pumice dust and driftsand that cover large areas of the interior. Nevertheless the greater part of this lowland plain produces good grass, and is relatively well inhabited. The plain is drained by three rivers-Markarfljót, Thjórsá and Oelfusá-all of large volume, and numerous smaller streams. Towards the west there exist a number of warm springs. There is another lowland plain around the head of Faxaflói, nearly 400 sq . m. in extent. As a rule the surface of this second plain is very marshy. Several dales or glens penetrate the central tableland; the eastern part of this lowland is called Borgarfjörðr, the western part Mýrar.

The great bays on the west of the island (Faxaflói and Breiðifjörðr), ${ }^{2}$ as well as the many bays on the north, which are separated from one another by rocky promontories, appear to owe their origin to subsidences of the surface; whereas the fjords of the north-west peninsula, which make excellent harbours, and those of the east coast seem to be the result chiefly of erosion.


Glaciers.-An area of 5170 sq. m. is covered with snowfields and glaciers. This extraordinary development of ice and snow is due to the raw, moist climate, the large rainfall and the low summer temperature. The snow-line varies greatly in different parts of the island, its range being from 1300 to 4250 ft . It is highest on the tableland, on the north side of Vatnajökull, and lowest on the north-west peninsula, to the south of North Cape. Without exception the great névés of Iceland belong to the interior tableland. They consist of slightly rounded domes or billowy snowfields of vast thickness. In external appearance they bear a closer resemblance to the glaciers of the Polar regions than to those of the Alps. The largest snowfields are Vatnajökull (3280 sq. m.), Hofsjökull (520) Langjökull (500) and Mýrdalsjökull (390). The glaciers which stream off from these snowfields are often of vast extent, e.g. the largest glacier of Vatnajökull has an area of 150 to 200 sq. m., but the greater number are small. Altogether, more than 120 glaciers are known in Iceland. It is on the south side of Vatnajökull that they descend lowest; the lower end of Breidamerkurjökull was in the year 1894 only 30 ft . above sea-level. The glaciers of the north-west peninsula also descend nearly to sea-level. The great number of streams of large volume is due to the moist climate and the abundance of glaciers, and the milky white or yellowish-brown colour of their waters (whence the common name

Hvítá, white) is due to the glacial clays. The majority of them change their courses very often, and vary greatly in volume; frequently they are impetuous torrents, forming numerous waterfalls. Iceland also possesses a great number of lakes, the largest being Thingvallavatn ${ }^{3}$ and Thorisvatn, each about 27 sq. m. in area. Mývatn, in the north, is well known from the natural beauty of its surroundings. Above its surface tower a great number of volcanoes and several craters, and its waters are alive with water-fowl, a multitude of ducks of various species breeding on its islands. The lakes of Iceland owe their origin to different causes, some being due to glacial erosion, others to volcanic subsidence. Mývatn fills a depression between lava streams, and has a depth of not more than $83 / 4 \mathrm{ft}$. The group of lakes called Fiskivötn (or Veidivötn), which lie in a desolate region to the west of Vatnajökull, consist for the most part of crater lakes. The groups of lakes which lie north-west from Langjökull occupy basins formed between ridges of glacial gravel; and in the valleys numerous lakes are found at the backs of the old moraines.

Volcanoes.-Iceland is one of the most volcanic regions of the earth; volcanic activity has gone on continuously from the formation of the island in the Tertiary period down to the present time. So far as is known, there have in historic times been eruptions from twenty-five volcanic vents. Altogether 107 volcanoes are known to exist in Iceland, with thousands of craters, great and small. The lava-streams which have flowed from them since the Glacial epoch now cover an area of 4650 sq. m. They are grouped in dense masses round the volcanoes from which they have flowed, the bulk of the lava dating from outbreaks which occurred in prehistoric times. The largest volume of lava which has issued at one outflow within historic times is the stream which came from the craters of Laki at Skaptá. This belongs to the year 1783 , and covers an area of 218 sq. m., and amounts to a volume represented by a cube each of whose sides measures $71 / 2 \mathrm{~m}$. The largest unbroken lava-field in Iceland is Odaðahraun (Lava of Evil Deeds), upon the tableland north from Vatnajökull ( 2000 to 4000 ft . above sea-level). It is the accretion of countless eruptions from over twenty volcanoes, and covers an area of 1300 sq.m. (or, including all its ramifications and minor detached streams, $1700 \mathrm{sq} . \mathrm{m}$. ), and its volume would fill a cube measuring 13.4 m . in every direction. As regards their superficies, the lava-streams differ greatly. Sometimes they are very uneven and jagged (apalhraun), consisting of blocks of lava loosely flung together in the utmost confusion. The great lava-fields, however, are composed of vast sheets of lava, ruptured and riven in divers ways (helluhraun). The smooth surface of the viscous billowy lava is further diversified by long twisted "ropes," curving backwards and forwards up and down the undulations. Moreover, there are gigantic fissures, running for several miles, caused by subsidences of the underlying sections. The best-known fissure of this character is Almannagjá at Thingvellir. On the occasion of outbreaks the fine ashes are scattered over a large portion of the island, and sometimes carried far across the Atlantic. After the eruption of Katla in 1625 the ashes were blown as far as Bergen in Norway, and when Askja was in eruption in 1875 a rain of ashes fell on the west coast of Norway 11 hours 40 minutes, and at Stockholm 15 hours, afterwards. The volcanic ash frequently proves extremely harmful, destroying the pastures so that the sheep and cattle die of hunger and disease. The outbreak of Laki in 1783 occasioned the loss of 11,500 cattle, 28,000 horses and 190,500 sheep-that is to say, $53 \%$ of the cattle in the island, $77 \%$ of the horses and $82 \%$ of the sheep. After that the island was visited by a famine, which destroyed 9500 people, or one-fifth of the total population.

The Icelandic volcanoes may be divided into three classes: (1) cone-shaped, like Vesuvius, built up of alternate layers of ashes, scoriae and lava; (2) cupola-shaped, with an easy slope and a vast crater opening at the top-these shield-shaped cupolas are composed entirely of layers of lava, and their inclination is seldom steeper than $7^{\circ}-8^{\circ}$; (3) chains of craters running close alongside a fissure in the ground. For the most part the individual craters are low, generally not exceeding 300 to 500 ft . These crater chains are both very common and often very long. The chain of Laki, which was formed in 1783, extends 20 m ., and embraces about one hundred separate craters. Sometimes, however, the lava-streams are vomited straight out of gigantic fissures in the earth without any crater being formed. Many of the Icelandic volcanoes during their periods of quiescence are covered with snow and ice. Then when an outbreak occurs the snow and ice melt, and in that way they sometimes give rise to serious catastrophes (jökulhlaup), through large areas being suddenly inundated by great floods of water, which bear masses of ice floating on their surface. Katla caused very serious destruction in this way by converting several cultivated districts into barren wastes. In the same way in the year 1362 Oeræfajökull, the loftiest mountain in Iceland ( 6424 ft .), swept forty farms, together with their inhabitants and live stock, bodily into the ocean. The best-known volcano is Hekla ( 5108 ft .), which was in eruption eighteen times within the historic period down to 1845. Katla during the same period was active thirteen times down to 1860 . The largest volcano is Askja, situated in the middle of the lava-field of Odaðahraun. Its crater measures $34 \mathrm{sq} . \mathrm{m}$. in area. At Mývatn there are several volcanoes, which were particularly active in the years 1724-1730. On several occasions there have been volcanic outbreaks under the sea outside the peninsula of Reykjanes,
islands appearing and afterwards disappearing again. The crater chain of Laki has only been in eruption once in historic times, namely, the violent and disastrous outbreak of 1783. Iceland, however, possesses no constantly active volcano. There are often long intervals between the successive outbreaks, and many of the volcanoes (and this is especially true of the chains of craters) have only vented themselves in a solitary outburst.

Earthquakes are frequent, especially in the districts which are peculiarly volcanic. Historical evidence goes to show that they are closely associated with three naturally defined regions: (1) the region between Skjálfandi and Axarfjörðr in the north, where violent earth tremblings are extremely common; (2) at Faxaflói, where minor vibrations are frequent; (3) the southern lowlands, between Reykjanes and Mýrdalsjökull, have frequently been devastated by violent earthquake shocks, with great loss of property and life, e.g. on the 14th-16th of August 1784, when 92 farmsteads were totally destroyed, and 372 farmsteads and 11 churches were seriously damaged; and again in August and September 1896, when another terrible earthquake destroyed 161 farmsteads and damaged 155 others. Hot springs are found in every part of Iceland, both singly and in groups; they are particularly numerous in the western portion of the southern lowlands, where amongst others is the famous Geyser (q.v.). Sulphur springs and boiling mud lakes are also general in the volcanic districts; and in places there are carbonic acid springs, these more especially on the peninsula of Snæfellsnes, north of Faxaflói.

Geology.-Iceland is built up almost entirely of volcanic rocks, none of them older, however, than the middle of the Tertiary period. The earlier flows were probably contemporaneous with those of Greenland, the Færoes, the western islands of Scotland and the north-east of Ireland. The principal varieties are basalt and palagonitic breccias, the former covering two-thirds of the entire area, the latter the remaining one-third. Compared with these two systems, all other formations have an insignificant development. The palagonitic breccias, which stretch in an irregular belt across the island, are younger than the basalt. In the north-west, north and east the coasts are formed of basalt, and rise in steep, gloomy walls of rock to altitudes of 3000 ft . and more above sea-level. Deposits of clay, with remains of plants of the Tertiary period, lignite and tree-trunks pressed flat, which the Icelanders call surtarbrandur, occur in places in the heart of the basalt formation. These fossiliferous strata are developed in greatest thickness in the north-west peninsula. Indeed, in some few places well-marked impressions of leaves and fruit have been discovered, proving that in Tertiary times Iceland possessed extensive forests, and its annual mean temperature must have been at least $48^{\circ}$ Fahr., whereas the present mean is $35.6^{\circ}$. The palagonitic breccias, which attain their greatest development in the south of the island and on the tableland, consist of reddish, brown or yellowish rocks, tuffs and breccias, belonging to several different groups or divisions, the youngest of which seems to be of a date subsequent to the Glacial epoch. All over Iceland, in both the basalt and breccia formations, there occur small intrusive beds and dikes of liparite, and as this rock is of a lighter colour than the basalt, it is visible from a distance. In the south-east of the island, in the parish of Lón, there exist a few mountains of gabbro, a rock which does not occur in any other part of Iceland. Near Húsavik in the north there have been found marine deposits containing a number of marine shells; they belong to the Red Crag division of the Pliocene. In the middle of Iceland, where the geological foundation is tuff and breccias, large areas are buried under ancient outflows of lava, which bear evidences of glacial scratching. These lava streams, which are of a doleritic character, flowed before the Glacial age, or during its continuance, out of lava cones with gigantic crater openings, such as may be seen at the present day. During the Glacial epoch the whole of Iceland was covered by a vast sheet of inland ice, except for a few small isolated peaks rising along its outer margins. This ice-cap had on the tableland a thickness of 2300 to 2600 ft . Rocks scored by glacial ice and showing plain indications of striation, together with thousands of erratic blocks, are found scattered all over Iceland. Signs of elevation subsequent to the Glacial epoch are common all round the island, especially on the north-west peninsula. There are found strikingly developed marine terraces of gravel, shore lines and surf beaches marked on the solid rock. In several places there are traces of shells; and sometimes skeletal remains of whales and walruses, as well as ancient driftwood, have been discovered at tolerable distances from the present coast. The ancient shore-lines occur at two different altitudes. Along the higher, 230 to 266 ft . above the existing sea-level, shells have been found which are characteristic of high Arctic latitudes and no longer exist in Iceland; whereas on the lower shore-line, 100 to 130 ft ., the shells belong to species which occur amongst the coast fauna of the present day.
The geysers and other hot springs are due to the same causes as the active volcanoes, and the earthquakes are probably manifestations of the same forces. A feature of special interest to geologists in the present conditions of the island is the great power of the wind both as a transporting and denuding agent. The rock sculpture is often very similar to that of a tropical desert. ${ }^{4}$

Climate.-Considering its high latitude and situation, Iceland has a relatively mild climate. The meteorological conditions vary greatly, however, in different parts of the island. In the south and east the weather is generally changeable, stormy and moist; whilst on the north the
rainfall is less. The climate of the interior tableland approximates to the continental type and is often extremely cold. The mean annual temperature is $37.2^{\circ}$ F. in Stykkishólmr on Breiðifjörðr, $38.5^{\circ}$ at Eyrārbakki in the south of Iceland, $41^{\circ}$ at Vestmannæyjar, $36^{\circ}$ at Akureyri in the north, $36.7^{\circ}$ on Berufjörðr in the east, and $30.6^{\circ}$ at Mödrudalr on the central tableland. The range is great not only from year to year, but also from month to month. For instance, at Stykkishólmr the highest annual mean for March was $39.7^{\circ}$, and the lowest $8^{\circ}$, during a period of thirty-eight years. Iceland lies contiguous to that part of the north Atlantic in which the shifting areas of low pressure prevail, so that storms are frequent and the barometer is seldom firm. The barometric pressure at sea-level in the south-west of Iceland during the period 1878-1900 varied between 30.8 and 27.1 in . The climate of the coasts is relatively mild in summer, but tolerably cold in winter. The winter means of the north and east coasts average $31.7^{\circ}$ and $31.3^{\circ}$ F. respectively; the summer means, $42.8^{\circ}$ and $44.6^{\circ}$; and the means of the year, $33.1^{\circ}$ and $35.6^{\circ}$. The winter means of the south and west coasts average $32^{\circ}$ and $31.7^{\circ}$ respectively; the summer means, $48.2^{\circ}$ and $50^{\circ}$; the annual means, $37.4^{\circ}$ and $39.2^{\circ}$. The rainfall on the south and east coasts is considerable, e.g. at Vestmannæyjar, 49.4 in . in the year; at Berufjörðr, 43.6 in. On the west coast it is less, e.g. 24.3 in . at Stykkishólmr; but least of all on the north coast, being only 14.6 in . on the island of Grimsey, which lies off that coast. Mist is commonly prevalent on the east coast; at Berufjörðr there is mist on no fewer than 212 days in the year. The south and west coasts are washed by the Gulf Stream, and the north coast by an Arctic current, which frequently brings with it a quantity of drift-ice, and thus exercises a considerable effect upon the climate of the island; sometimes it blocks the north coast in the summer months. On the whole, during the 19th century, the north coast was free from ice on an average of one year in every four or five. The clearness of the atmosphere has been frequently remarked. Thunderstorms occur mostly in winter.

Flora.-The vegetation presents the characteristics of an Arctic European type, and is tolerably uniform throughout the island, the differences even on the tableland being slight. At present 435 species of phanerogams and vascular cryptogams are known; the lower orders have been little investigated. The grasses are of the greatest importance to the inhabitants, for upon them they are dependent for the keep of their live stock. Heather covers large tracts, and also affords pasture for sheep. The development of forest trees is insignificant. Birch woods exist in a good many places, especially in the warmer valleys; but the trees are very short, scarcely attaining more than 3 to 10 ft . in height. In a few places, however, they reach 13 to 20 ft . and occasionally more. A few mountain ash or rowan trees (Sorbus aucuparia) are found singly here and there, and attain to 30 ft . in height. Willows are also pretty general, the highest in growth being Salix phyllicifolia, 7 to 10 ft . The wild flora of Iceland is small and delicate, with bright bloom, the heaths being especially admired. Wild crowberries and bilberries are the only fruit found in the island.
Fauna.-The Icelandic fauna is of a sub-Arctic type. But while the species are few, the individuals are often numerous. The land mammals are very poorly represented; and it is doubtful whether any species is indigenous. The polar bear is an occasional visitant, being brought to the coast by the Greenland drift-ice. Foxes are common, both the white and the blue occurring; mice and the brown rat have been introduced, though one variety of mouse is possibly indigenous. Reindeer were introduced in 1770 . The marine mammalia are numerous. The walrus is now seldom seen, although in prehistoric times it was common. There are numerous species of seals; and the seas abound in whales. Of birds there are over 100 species, more than one-half being aquatic. In the interior the whistling swan is common, and numerous varieties of ducks are found in the lakes. The eider duck, which breeds on the islands of Breiðifjörðr, is a source of livelihood to the inhabitants, as are also the many kinds of sea-fowl which breed on the sea-cliffs. Iceland possesses neither reptiles nor batrachians. The fish fauna is abundant in individuals, some sixty-eight species being found off the coasts. The cod fisheries are amongst the most important in the world. Large quantities of herring, plaice and halibut are also taken. Many of the rivers abound in salmon, and trout are plentiful in the lakes and streams.

Population and Towns.-The census of 1890 gave a total population of 70,927 , and this number had increased by 1901 to 78,489 . The increase during the 19th century was 27,000 , while at least 15,600 Icelanders emigrated to America, chiefly to Manitoba, from 1872 to the close of the century. The largest town is Reykjavik on Faxaflói, with 6700 inhabitants, the capital of the island, and the place of residence of the governor-general and the bishop. Here the Althing meets; and here, further, are the principal public institutions of the island (library, schools, \&c.). The town possesses a statue to Thorvaldsen, the famous sculptor, who was of Icelandic descent. The remaining towns include Isafjörðr (pop. 1000) on the north-west peninsula, Akureyri (1000) on the north and Seydisfjörðr (800) in the east.

Industries.-The principal occupation of the Icelanders is cattle-breeding, and more particularly sheep-breeding, although the fishing industries have come rapidly to the front in modern times. In $1850,82 \%$ of the population were dependent upon cattle-breeding and $7 \%$ upon fishing; in 1890 the numbers were $64 \%$ and $18 \%$ respectively. The culture of grain is not
practised in Iceland; all bread-stuffs are imported. In ancient times barley was grown in some places, but it never paid for the cost of cultivation. Cattle-breeding has declined in importance, while the number of sheep has increased. Formerly gardening was of no importance, but considerable progress has been made in this branch in modern times, as also in the cultivation of potatoes and turnips. Fruit-trees will not thrive; but black and red currants and rhubarb are grown, the last-named doing excellently. Iceland possesses four agricultural schools, one agricultural society, and small agricultural associations in nearly every district. The fisheries give employment to about 12,000 people. For the most part the fishing is carried on from open boats, notwithstanding the dangers of so stormy a coast. But larger decked vessels have come into increasing use. In summer the waters are visited by a great number of foreign fishermen, inclusive of about 300 fishing-boats from French ports, as well as by fishing-boats from the Færoes and Norway, and steam trawlers from England. Excellent profit is made in certain parts of the island from the herring fishery; this is especially the case on the east coast. There are marine insurance societies and a school of navigation at Reykjavik. The export of fish and fish products has greatly increased. In 1849 to 1855 the annual average exported was 1480 tons; whereas at the close of the century (in 1899) it amounted to 11,339 tons and 68,079 barrels of oil, valued at $£ 276,596$.

Commerce.-From the first colonization of the island down to the 14th century the trade was in the hands of native Icelanders and Norsemen; in the 15th century it was chiefly in the hands of the English, in the 16th of Germans from the Hanse towns. From 1602 to 1786 commerce was a monopoly of the Danish government; in the latter year it was declared free to all Danish subjects and in 1854 free to all nations. Since 1874, when Iceland obtained her own administration, commerce has increased considerably. Thus the total value of the imports and exports together in 1849 did not exceed $£ 170,000$; while in 1891-1895 the imports averaged $£ 356,000$ and the exports $£ 340,000$. In 1902 imports were valued at $£ 596,193$ and exports at $£ 511,083$. Trade is almost entirely with Denmark, the United Kingdom, and Norway and Sweden, in this order according to value. The principal native products exported are live sheep, horses, salt meat, wool and hides, to which must be added the fish products-cod, train-oil, herring and salmon-eiderdown and woollen wares. The spinning, weaving and knitting of wool is a widespread industry, and the native tweed (vaðmal) is the principal material for the clothing of the inhabitants. The imports consist principally of cereals and flour, coffee, sugar, ale, wines and spirits, tobacco, manufactured wares, iron and metal wares, timber, salt, coal, \&c. The money, weights and measures in use are the same as in Denmark. The Islands Bank in Reykjavik (1904) is authorized to issue bank-notes up to $£ 133,900$ in total value.

Communications.-All land journeys are made on horseback, and in the remoter parts all goods have to be transported by the same means. Throughout the greater part of the island there exist no proper roads even in the inhabited districts, but only bridle-paths, and in the uninhabited districts not even these. Nevertheless much has been done to improve such paths as there are, and several miles of driving roads have been made, more particularly in the south. Since 1888 many bridges have been built; previous to that year there was none. The larger rivers have been spanned by iron swing-bridges, and the Blanda is crossed by a fixed iron bridge. Postal connexion is maintained with Denmark by steamers, which sail from Copenhagen and call at Leith. Besides, steamers go round the island, touching at nearly every port.

Religion.-The Icelanders are Lutherans. For ecclesiastical purposes the island is divided into 20 deaneries and 142 parishes, and the affairs of each ecclesiastical parish are administered by a parish council, and in each deanery by a district (hjerað) council. When a living falls vacant, the governor-general of the island, after consultation with the bishop, selects three candidates, and from these the congregation chooses one, the election being subsequently confirmed by the governor-general. In the case of certain livings, however, the election requires confirmation by the crown. In 1847 a theological seminary was founded at Reykjavik, and there the majority of the Icelandic ministry are educated; some, however, are graduates of the university of Copenhagen.

Health.-The public health has greatly improved in modern times; the death-rate of young children has especially diminished. This improvement is due to greater cleanliness, better dwellings, better nourishment, and the increase in the number of doctors. There are now doctors in all parts of the country, whereas formerly there were hardly any in the island. There is a modern asylum for leprosy at Laugarnes near Reykjavik, and a medical school at Reykjavik, opened in 1876. The general sanitary affairs of the island are under the control of a chief surgeon (national physician) who lives in Reykjavik, and has superintendence over the doctors and the medical school.

Government.-According to the constitution granted to Iceland in 1874, the king of Denmark shares the legislative power with the Althing, an assembly of 36 members, 30 of whom are elected by household suffrage, and 6 nominated by the king. The Althing meets every second year, and sits in two divisions, the upper and the lower. The upper division consists of the 6
members nominated by the king and 6 elected by the representatives of the people out of their own body. The lower division consists of the remaining 24 representative members. The minister for Iceland, who resided in Copenhagen until 1903, when his office was transferred to Reykjavik, is responsible to the king and the Althing for the maintenance of the constitution, and he submits to the king for confirmation the legislative measures proposed by the Althing. The king appoints a governor-general (landshöfðingi) who is resident in the island and carries on the government on the responsibility of the minister. Formerly Iceland was divided into four quarters, the east, the south, the west and north. Now the north and the east are united under one governor, and the south and the west under another. The island is further divided into 18 sýslur (counties), and these again into 169 hreppur (rapes) or poor-law districts. Responsible to the governors are the sheriffs (sýslumenn), who act as tax gatherers, notaries public and judges of first instance; the sheriff has in every hreppur an assistant, called hreppstjóri. In every hreppur there is also a representative committee, who administer the poor laws, and look after the general concerns of the hreppur. These committees are controlled by the committees of the sýslur (county boards), and these again are under the control of the amtsráð (quarter board), consisting of three members. From the sheriff courts appeals lie to the superior court at Reykjavik, consisting of three judges. Appeals may be taken in all criminal cases and most civil cases to the supreme court at Copenhagen.

Iceland has her own budget, the Althing having, by the constitution of 1874, the right to vote its own supplies. As the Althing only meets every other year, the budget is passed for two years at once. The total income and expenditure are each about $£ 70,000$ per financial period. There is a national reserve fund of about $£ 60,000$, but no public debt; nor is there any contribution for either military or naval purposes. Iceland has her own customs service, but the only import duties levied are upon spirits, tobacco, coffee and sugar, and in each case the duties are fairly low.

Education.-Education is pretty widespread amongst the people. In the towns and fishing villages there are a few elementary schools, but often the children are instructed at home; in some places by peripatetic teachers. It is incumbent upon the clergy to see that all children are taught reading, writing and arithmetic. The people are great readers; considering the number of the inhabitants, books and periodicals have a very extensive circulation. Eighteen newspapers are issued (once and twice a week), besides several journals, and Iceland has always been distinguished for her native literature. At Reykjavik there are a Latin school, a medical school and a theological school; at Mödruvellir and Hafnarfjörðr, modern high schools (Realschulen); and in addition to these there are four agricultural schools, a school of navigation, and three girls' schools. The national library at Reykjavik contains some 40,000 volumes and 3000 MSS. At the same place there is also a valuable archaeological collection. Amongst the learned societies are the Icelandic Literary Society (Bokmentafjelag), the society of the Friends of the People, and the Archaeological Society of Reykjavik.

Authorities.-Among numerous works of Dr Thorvald Thoroddsen, see Geschichte der Islands Geographie (Leipzig, 1898); and the following articles in Geografisk Tidskrift (Copenhagen): "Om Islands geografiske og geologiske Undersögelse" (1893); "Islandske Fjorde og Bugter" (1901); "Geog. og geol. Unders. ved den sydlige Del af Faxaflói paa Island" (1903); "Lavaörkener og Vulkaner paa Islands Höjland" (1905). See also C. S. Forbes, Iceland (London, 1860); S. Baring-Gould, Iceland, its Scenes and Sagas (London, 1863); Sir R. F. Burton, Ultima Thule (Edinburgh, 1875); W. T. McCormick, A Ride across Iceland (London, 1892); J. Coles, Summer Travelling in Iceland (London, 1882); H. J. Johnston Lavis, "Notes on the Geography, Geology, Agriculture and Economics of Iceland," Scott. Geog. Mag. xi. (1895); W. Bisiker, Across Iceland (London, 1902); J. Hann, "Die Anomalien der Witterung auf Island in dem Zeitraume 1851-1900, \&c.," Sitzungsberichte, Vienna Acad. Sci. (1904); P. Hermann, Island in Vergangenheit und Gegenwart (Leipzig, 1907). Also Geografisk Tidskrift, and the Geographical Journal (London), passim. (Th. T.)

## History

Shortly after the discovery of Iceland by the Scandinavian, c. 850 (it had long been inhabited by a small colony of Irish Culdees), a stream of immigration set in towards it, which lasted for sixty years, and resulted in the establishment of some 4000 homesteads. In this immigration three distinct streams can be traced. (1) About 870-890 four great noblemen from Norway, Ingolf, Ketil Hæng, Skalla-Grim and Thorolf, settled with their dependants in the south-west of the new found land. (2) In 890-900 there came from the western Islands Queen Aud, widow of Olaf the White, king of Dublin, preceded and followed by a number of her kinsmen and relations (many like herself being Christians), Helgi Biolan, Biorn the Eastern, Helgi the Lean, Ketil the Foolish, \&c., who settled the best land in the island (west, north-west and north), and founded families who long swayed its destinies. There also came from the Western Islands a fellowship of vikings seeking a free home in the north. They had colonized the west in the
viking times; they had "fought at Hafursfirth," helping their stay-at-home kinsmen against the centralization of the great head-king, who, when he had crushed opposition in Norway, followed up his victory by compelling them to flee or bow to his rule. Such were Ingimund the Old, Geirmund Hellskin, Thord Beardie (who had wed St. Edmund's granddaughter,) Audun Shackle, Bryniulf the Old, Uni, to whom Harold promised the earldom of the new land if he could make the settlers acknowledge him as king (a hopeless project), and others by whom the north-west, north and east were almost completely "claimed." (3) In 900-930 a few more incomers direct from Norway completed the settlement of the south, north-east and south-east. Among them were Earl Hrollaug (half-brother of Hrolf Ganger and of the first earl of Orkney), Hialti, Hrafnkell Frey's priest, and the sons of Asbiorn. Fully three-quarters of the land was settled from the west, and among these immigrants there was no small proportion of Irish blood. In 1100 there were 4500 franklins, i.e. about 50,000 souls.

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The unit of Icelandic politics was the homestead with its franklin-owner (buendi), its primal organization the hundred-moot (thing), its tie the goðorð (godar) or chieftainship. The chief who had led a band of kinsmen and dependants to the new land, taken a Organization. "claim" there, and parcelled it out among them, naturally became their leader, presiding as priest at the temple feasts and sacrifices of heathen times, acting as speaker of their moot, and as their representative towards the neighbouring chiefs. He was not a feudal lord nor a local sheriff, for any franklin could change his goðorð when he would, and the rights of "judgment by peers" were in full use; moreover, the office could be bequeathed, sold, divided or pledged by the possessor; still the goði had considerable power as
long as the commonwealth lasted.
Disputes between neighbouring chiefs and their clients, and uncertainty as to the law, brought about the Constitution of Ulfliot (c. 930), which appointed a central moot for the whole island, the Althing, and a speaker to speak a single "law" (principally that followed by the Gulamoot in Norway); the Reforms of Thord Gellir (964), settling a fixed number of moots and chieftaincies, dividing the island into four quarters (thus characterized by Ari: north, thickest settled, most famous; east, first completely settled; south, best land and greatest chiefs; west, remarkable for noble families), to each of which a head-court, the "quarter-court," was assigned; and the Innovations of Skapti (ascribed in the saga to Nial) the Law-Speaker (d. 1030), who set up a "fifth court" as the ultimate tribunal in criminal matters, and strengthened the community against the chiefs. But here constitutional growth ceased: the law-making body made few and unimportant modifications of custom; the courts were still too weak for the chiefs who misused and defied them; the speaker's power was not sufficiently supported to enable him to be any more than a highly respected lord chief justice, whereas he ought to have become a justiza if anarchy was to be avoided; even the ecclesiastical innovations, while they secured peace for a time, provoked in the end the struggles which put an end to the commonwealth.

Christianity was introduced c. 1000. Tithes were established in 1096, and an ecclesiastical code made c. 1125. The first disputes about the jurisdiction of the clergy were moved by Gudmund in the 13th century, bringing on a civil war, while the questions of patronage and rights over glebe and mortmainland occupied Bishop Arni and his adversaries fifty years afterwards, when the land was under Norwegian viceroys and Norwegian law. For the civil wars broke down the great houses who had monopolized the chieftaincies; and after violent struggles (in which the Sturlungs of the first generation perished at Orlygstad, 1238, and Reykiaholt, 1241, while of the second generation Thord Kakali was called away by the king in 1250, and Thorgils Skardi slain in 1258) the submission of the island to Norway quarter after quarter, took place in 1262-1264, under Gizur's auspices, and the old Common Law was replaced by the New Norse Code "Ironside" in 1271.

The political life and law of the old days is abundantly illustrated in the sagas (especially Eyrbyggia, Hamsa-Thori, Reykdæla, Hrafnkell, and Niala), the two collections of law-scrolls (Codex Regius, c. 1235, and Stadarhol's Book, c. 1271), the Libellus, the Liberfragments, and the Landnamabók of Ari, and the Diplomatarium. K. Maurer has made the subject his own in his Beiträge, Island, Grágás, \&c.

The medieval Icelandic church had two bishoprics, Skalholt (S., W., and E.) 1056, and Holar (N.) 1106, and about 175 parishes (two-thirds of which belonged to the southern bishopric). They belonged to the metropolitan see of Bremen, then to Lund, lastly to Nidaros, 1237. There were several religious foundations: Thingore (founded 1133), Thwera (1155), Hitardale (c. 1166), Kirkby Nunnery (1184), Stad Nunnery (1296), and Saurby (c. 1200) were Benedictine, while Ver (1168), Flatey after Holyfell (1172), Videy (1226), Madderfield Priory (1296), and Skrid Priory (14th century) were Augustinian. The bishops, elected by the people at the Althing till 1237, enjoyed considerable power; two, Thorlak of Skalholt and John of Holar, were publicly voted saints at the Althing, and one, Gudmund, received the title of "Good" by decree of the bishop and chapter. Full details as to ecclesiastical history will be found in the Biskupasögur (edited by Dr Vigfusson).

Iceland was not agricultural but pastoral, depending upon flocks Mode of and herds for subsistence, for, though rye and other grain would grow in favoured localities, the hay, selfsown, was the only regular crop. In some districts the fisheries and fowling Mode of life. were of importance, but nine-tenths of the population lived by their sheep and cattle. Life on each homestead was regularly portioned out: out door occupations-fishing, shepherding, fowling, and the hay-making and fuel-gathering-occupying the summer; while indoor business-weaving, tool-making, \&c.-filled up the long winter. The year was broken by the spring feasts and moots, the great Althing meeting at midsummer, the marriage and arval gatherings after the summer, and the long yule feasts at midwinter. There were but two degrees of men, free and unfree, though only the franklins had any political power; and, from the nature of the life, social intercourse was unrestrained and unfettered; goði and thrall lived the same lives, ate the same food, spoke the same tongue, and differed little in clothing or habits. The thrall had a house of his own and was rather villein or serf than slave, having rights and a legal price by law. During the heathen days many great chiefs passed part of their lives in Norway at the king's court, but after the establishment of Christianity in Iceland they kept more at home, visiting the continent, however, for purposes of state, suits with clergy, \&c. Trade was from the first almost entirely in foreign (Norse) hands.

The introduction of a church system brought little change. The great families put their members into orders, and so continued to enjoy the profits of the land which they had given to
the church; the priests married and otherwise behaved like the franklins around them in everyday matters, farming, trading, going to law like laymen.

Life in the commonwealth was turbulent and anarchic, but free and varied; it produced men of mark, and fostered bravery, adventure and progress. But on the union with Norway all this ceased, and there was left but a low dead level of poor peasant proprietors

## Effects of the Union.

 careless of all save how to live by as little labour as possible, and pay as few taxes as they could to their foreign rulers. The island received a foreign governor (Earl, Hirdstjori or Stiptamtsmadr as he was successively called), and was parcelled out into counties (sýslur), administered by sheriffs (sýslumadr) appointed by the king. A royal court took the place of the Althing courts; the local business of the local things was carried out by the (hreppstjóri) bailiff, a subordinate of the sheriff; and the goðorð, things, quarter-courts, trial by jury, \&c., were swept away by these innovations. The power of the crown was increased by the confiscation of the great Sturlung estates, which were underleased to farmers, while the early falling off of the Norse trade threatened to deprive the island of the means of existence; for the great epidemics and eruptions of the 14th century had gravely attacked its pastoral wealth and ruined much of its pasture and fishery.The union of the Three Crowns transferred the practical rule of Iceland to Denmark in 1280, and the old Treaty of Union, by which the island had reserved its essential rights, was disregarded by the absolute Danish monarchs; but, though new taxation was imposed, it was rather their careless neglect than their too active interference that damaged Iceland's interests. But for an English trade, which sprang up out of the half-smuggling, halfbuccaneering enterprise of the Bristol merchants, the island would have fared badly, for during the whole of the 15th century their trade with England, exporting sulphur, eiderdown (of which the English taught them the value), wool, and salt stock-fish, and importing as before wood, iron, honey, wine, grain and flax goods, was their only link with the outer world. This period of Iceland's existence is eventless: she had got peace but with few of its blessings; all spirit seemed to have died with the commonwealth; even shepherding and such agriculture as there had been sank to a lower stage; wagons, ploughs and carts went out of use and knowledge; architecture in timber became a lost art, and the fine carved and painted halls of the heathen days were replaced by turf-walled barns half sunk in the earth; the large decked luggers of the old days gave way to small undecked fishing-boats.

The Reformation in Iceland wakened men's minds, but it left their circumstances little changed. Though the fires of martyrdom were never lighted in Iceland, the story of the easily accepted Reformation is not altogether a pleasant one. When it was

## The <br> Reformation.

 accomplished, the little knot of able men who came to the front did much in preserving the records of the past, while Odd and Hallgrim exhibit the noblest impulses of their time. While there was this revolution in religion a social and political revolution never came to Iceland. The Hanse trade replaced the English for the worse; and the Danish monopoly which succeeded it when the Danish kings began to act again with vigour was still less profitable. The glebes and hospital lands were a fresh power in the hands of the crown, and the subservient Lutheran clergy became the most powerful class in the island, while the system of under-leasing at rackrent and short lease with unsecured tenant right extended over at least a quarter of the better land.A new plague, that of the English, Gascon and Algerine pirates, marked the close of the 16th century and opening of the 17th, causing widespread panic and some devastation in 1579, 1613-1616 and 1627. Nothing points more to the helplessness of the natives' Decadence. condition than their powerlessness against these foes. But the 18th century is the most gloomy in Iceland's annals. Smallpox, famine, sheep disease, and the eruptions of 1765 and 1783 follow each other in terrible succession. Against such visitations, which reduced the population by about a fourth, little could be done. The few literary men, whose work was done and whose books were published abroad, were only concerned with the past, and Jon Vidalin is the one man of mark, beside Eggert Olafsson, who worked and wrote for his own generation. ${ }^{5}$

Gradually the ideas which were agitating Europe spread through Scandinavia into Iceland, and its claims were more respectfully listened to. The continental system,

## Modern

times. which, by its leading to the blockade of Denmark, threatened to starve Iceland, was neutralized by special action of the British government. Trade and fishery grew a little brisker, and at length the turn came.

The rationalistic movement, headed by Magnus Stephenson, a patriotic, narrow-minded lawyer, did little good as far as church reform went, but was accompanied by a more successful effort to educate the people. A Useful Knowledge Society was formed and did some honest work. Newspapers and periodicals were published, and the very stir which the ecclesiastical disputes encouraged did good. When free trade came, and when the free constitution of

Denmark had produced its legitimate effects, the endeavours of a few patriots such as Jon Sigurdsson were able to push on the next generation a step further. Questions of a modern political complexion arose; the cattle export controversy and the great home rule struggle began. After thirty years' agitation home rule was conceded in 1874 (see above, Government).
(F. Y. P.)

## Ancient Literature

Poetry.-Iceland has always borne a high renown for song, but has never produced a poet of the highest order, the qualities which in other lands were most sought for and admired in poetry being in Iceland lavished on the saga, a prose epic, while Icelandic poetry is to be rated very high for the one quality which its authors have ever aimed at-melody of sound. To these generalizations there are few exceptions, though Icelandic literature includes a group of poems which possess qualities of high imagination, deep pathos, fresh love of nature, passionate dramatic power, and noble simplicity of language which Icelandic poetry lacks. The solution is that these poems do not belong to Iceland at all. They are the poetry of the "Western Islands."

It was among the Scandinavian colonists of the British coasts that in the first generations after the colonization of Iceland therefrom a magnificent school of poetry arose, to which we owe works that for power and beauty can be paralleled in no Teutonic language till centuries after their date. To this school, which is totally distinct from the Icelandic, ran its own course apart and perished before the 13th century, the following works belong (of their authors we have scarcely a name or two; their dates can be rarely exactly fixed, but they lie between the beginning of the 9th and the end of the 10th centuries), classified into groups:-
(a) The Helgi trilogy (last third lost save a few verses, but preserved in prose in Hromund Gripsson's Saga), the Raising of Anganty and Death of Hialmar (in Hervarar Saga), the fragments of a Volsung Lay (Volsungakiraða) (part interpolated in earlier poems, part underlying the prose in Volsunga Saga), all by one poet, to whom Dr Vigfusson would also ascribe Völuspá, Vegtamskviða, Prymskviða, Grötta Song and Völundarkviða.
(b) The Dramatic Poems:-Flyting of Loki, the För Skirnis, the Harbarðslioð and several fragments, all one man's work, to whose school belong, probably, the Lay underlying the story of Ivar's death in Skioldunga Saga.
(c) The Didactic Poetry:-Grímnismál, Vafpruðnismál, Alvíssmal, \&c.
(d) The Genealogical and Mythological Poems:-Hyndluljoð written for one of the Haurda-Kari family, so famous in the Orkneys; Ynglingatal and Haustlong, by Thiodolf of Hvin; Rig's Thul, \&c.
(e) The Dirges and Battle Songs-such as that on Hafur-firth Battle Hrafnsmal, by Thiodolf of Hvin or Thorbjörn Hornklofi, shortly after 870; Eirik's Dirge (Eíríksmál) between 950 and 969; the Dart-Lay on Clontarf Battle (1014); Bíarka-mal (fragments of which we have, and paraphrase of more is found in Hrolf Kraki's Saga and in Saxo).

There are also fragments of poems in Half's Saga, Asmund Kappa-Bana's Saga, in the Latin verses of Saxo, and the Shield Lays (Ragnarsdrapa) by Bragi, \&c., of this school, which closes with the Sun-Song, a powerful Christian Dantesque poem, recalling some of the early compositions of the Irish Church, and with the 12th-century Lay of Ragnar, Lay of Starkad, The Proverb Song (Havamal) and Krakumal, to which we may add those singular Gloss-poems, the Pulur, which also belong to the Western Isles.

To Greenland, Iceland's farthest colony, founded in the 10th century, we owe the two Lays of Atli, and probably Hymiskvtiða, which, though of a weirder, harsher cast, yet belong to the Western Isles school and not to Iceland.

In form all these poems belong to two or three classes:-kviða, an epic "cantilena"; tál, a genealogical poem; drapa, songs of praise, \&c., written in modifications of the old Teutonic metre which we know in Beowulf; galdr and lokkr, spell and charm songs in a more lyric measure; and mál, a dialogue poem, and liod, a lay, in elegiac measure suited to the subject.

The characteristics of this Western school are no doubt the result of the contact of Scandinavian colonists of the viking-tide, living lives of the wildest adventure, with an imaginative and civilized race, that exercised upon them a very strong and lasting influence (the effects of which were also felt in Iceland, but in a different way). The frequent intermarriages which mingled the best families of either race are sufficient proof of the close communion of Northmen and Celts in the 9th and 10th centuries, while there are in the poems themselves traces of Celtic mythology, language and manners. ${ }^{6}$

When one turns to the early poetry of the Scandinavian continent, preserved in the runestaves on the memorial stones of Sweden, Norway and Denmark, in the didactic Havamal, the

Great Volsung Lay (i.e. Sigurd II., Fafnis's Lay, Sigrdrifa's Lay) and Hamdismal, all continental, and all entirely consonant to the remains of Old English poetry in metre, feeling and treatment, one can see that it is with this school that the Icelandic "makers" are in sympathy, and that from it their verse naturally descends. While shrewdness, plain straightforwardness, and a certain stern way of looking at life are common to both, the Icelandic school adds a complexity of structure and ornament, an elaborate mythological and enigmatical phraseology, and a regularity of rhyme, assonance, luxuriance, quantity and syllabification, which it caught from the Latin and Celtic poets, and adapted with exquisite ingenuity to its own main object, that of securing the greatest possible beauty of sound.
The first generations of Icelandic poets resemble in many ways the later troubadours; the books of the kings and the sagas are full of their strange lives. Men of good birth (nearly always, too, of Celtic blood on one side at least), they leave Iceland young and attach themselves to the kings and earls of the north, living in their courts as their henchmen, sharing their adventures in weal and woe, praising their victories, and hymning their deaths if they did not fall by their sides-men of quick passion, unhappy in their loves, jealous of rival poets and of their own fame, ever ready to answer criticism with a satire or with a sword-thrust, but clinging through all to their art, in which they attained most marvellous skill.

Such men were Egil, the foe of Eirik Bloodaxe and the friend of Æthelstan; Kormak, the hotheaded champion; Eyvind, King Haakon's poet, called Skaldaspillir, because he copied in his dirge over that king the older and finer Eíríksmál; Gunnlaug, who sang at Æthelred’s court, and fell at the hands of a brother bard, Hrafn; Hallfred, Olaf Tryggvason's poet, who lies in Iona by the side of Macbeth; Sighvat, Saint Olafs henchman, most prolific of all his comrades; Thormod, Coalbrow's poet, who died singing after Sticklestad battle; Ref, Ottar the Black, Arnor the earls' poet, and, of those whose poetry was almost confined to Iceland, Gretti, Biorn the Hitdale champion, and the two model Icelandic masters, Einar Skulason and Markus the Lawman, both of the 12th century.

It is impossible to do more here than mention the names of the most famous of the long roll of poets which are noted in the works of Snorri and in the two Skalda-tal. They range from the rough and noble pathos of Egil, the mystic obscurity of Kormak, the pride and grief of Hallfred, and the marvellous, fluency of Sighvat, to the florid intricacy of Einar and Markus.

The art of poetry stood to the Icelanders in lieu of music; scarcely any prominent man but knew how to turn a mocking or laudatory stanza, and down to the fall of the commonwealth the accomplishment was in high request. In the literary age the chief poets belong to the great Sturlung family, Snorri and his two nephews, Sturla and Olaf, the White Poet, being the most famous "makers" of their day. Indeed, it is in Snorri's Edda, a poetic grammar of a very perfect kind, that the best examples of the whole of northern poetry are to be found. The last part, Hattatal, a treatise on metre, was written for Earl Skuli about 1222, in imitation of Earl Rognvald and Hall's Hattalykill (Clavis metrica) of 1150. The second part, Skaldskapar-mal, a gradus of synonyms and epithets, which contains over 240 quotations from 65 poets, and 10 anonymous lays-a treasury of verse-was composed c. 1230. The first part, an exquisite sketch of northern mythology, Gylfa-ginning, was probably prefixed to the whole later. There is some of Sturla's poetry in his Islendinga Saga, and verses of Snorri occur in the Grammatical Treatise on figures of speech, \&c., of Olaf, which contains about one hundred and forty quotations from various authors, and was written about 1250.

Besides those sources, the Kings' Lives of Snorri and later authors contain a great deal of verse by Icelandic poets. King Harold Sigurdsson, who fell at Stamford Bridge 1066, was both a good critic and composed himself. Many tales are told of him and his poet visitors and henchmen. The Icelandic sagas also comprise much verse which is partly genuine, partly the work of the 12th and 13th century editors. Thus there are genuine pieces in Nial's Saga (chaps. 34, 78, 103, 126, 146), in Eyrbyggia, Laxdæla, Egil's Saga (part only), Grettla (two and a half stanzas, cf. Landnamabók), Biorn's Saga, Gunnlaug's Saga, Havard's Saga, Kormak's Saga, Viga-Glum's Saga, Erik the Red's Saga and Fostbrædra Saga. In Nial's, Gisli's and Droplaug's Sons' Sagas there is good verse of a later poet, and in many sagas worthless rubbish foisted in as ornamental.

To these may be added two or three works of a semi-literary kind, composed by learned men, not by heroes and warriors. Such are Konunga-tál, Hugsvinnsmál (a paraphrase of Cato's Distichs), Merlin's Prophecy (paraphrased from Geoffrey of Monmouth by Gunnlaug the monk), Jomsvikinga-drapa (by Bishop Ketil), and the Islendinga-drapa, which has preserved brief notices of several lost sagas concerning Icelandic worthies, with which Gudmundar-drapa, though of the 14th century, may be also placed.

Just as the change of law gave the death-blow to an already perishing commonwealth, so the rush of medieval influence, which followed the union with Norway, completed a process which had been in force since the end of the 11th century, when it overthrew the old Icelandic poetry
in favour of the rimur.
The introduction of the danz, ballads (or fornkvædi, as they are now called) for singing, with a burden, usually relating to a love-tale, which were immensely popular with the people and performed by whole companies at weddings, yule feasts and the like, had relegated the regular Icelandic poetry to more serious events or to the more cultivated of the chiefs. But these "jigs," as the Elizabethans would have called them, dissatisfied the popular ear in one way: they were, like old English ballads, which they closely resembled, in rhyme, but void of alliteration, and accordingly they were modified and replaced by the "rimur," the staple literary product of the 15 th century. These were rhymed but also alliterative, in regular form, with prologue or mansong (often the prettiest part of the whole), main portion telling the tale (mostly derived in early days from the French romances of the Carlovingian, Arthurian or Alexandrian cycles, or from the mythic or skrök-sögur), and epilogue. Their chief value to us lies in their having preserved versions of several French poems now lost, and in their evidence as to the feelings and bent of Icelanders in the "Dark Age" of the island's history. The ring and melody which they all possess is their chief beauty.

Of the earliest, Olafsrima, by Einar Gilsson (c. 1350), and the best, the Aristophanic Skídarima (c. 1430), by Einar Fostri, the names may be given. Rimur on sacred subjects was called "diktur"; of these, on the legends of the saints' lives, many remain. The most notable of its class is the Lilia of Eystein Asgrimsson, a monk of Holyfell (c. 1350), a most "sweet sounding song." Later the poems of the famous Jon Arason (b. 1484), last Catholic bishop of Holar (c. 1530), Liomr ("gleam") and Píslargrátr ("passion-tears"), deserve mention. Arason is also celebrated as having introduced printing into Iceland.

Taste has sunk since the old days; but still this rimur poetry is popular and genuine. Moreover, the very prosaic and artificial verse of Sturla and the last of the old school deserved the oblivion which came over them, as a casual perusal of the stanzas scattered through Islendinga will prove. It is interesting to notice that a certain number of kenningar (poetical paraphrases) have survived from the old school even to the present day, though the mass of them have happily perished. The change in the phonesis of the language is well illustrated by the new metres as compared with the old Icelandic drott-kvædi in its varied forms. Most of the older rimur and diktur are as yet unprinted. Many of the fornkvædi are printed in a volume of the old Nordiske Litteralur-Samfund.

The effects of the Reformation was deeply felt in Icelandic literature, both prose and verse. The name of Hallgrim Petursson, whose Passion-hymns, "the flower of all Icelandic poetry," have been the most popular composition in the language, is foremost of all writers since the second change of faith. The gentle sweetness of thought, and the exquisite harmony of wording in his poems, more than justify the popular verdict. His Hymns were finished in 1660 and published in 1666, two great Protestant poets thus being contemporaries. A collection of Reformation hymns, adapted, many of them, from the German, the Holar-book, had preceded them in 1619. There was a good deal of verse-writing of a secular kind, far inferior in every way, during this period. In spite of the many physical distresses that weighed upon the island, ballads (fornkvædi) were still written, ceasing about 1750, rimur composed, and more elaborate compositions published.

The most notable names are those of the improvisatore Stephen the Blind; Thorlak Gudbrandsson, author of Ulfar-Rímur, d. 1707; John Magnusson, who wrote Hristafla, a didactic poem; Stefan Olafsson, composer of psalms, rimur, \&c., d. 1688; Gunnar Pálsson, the author of Gunnarslag, often printed with the Eddic poems, c. 1791; and Eggert Olafsson, traveller, naturalist and patriot, whose untimely death in 1768 was a great loss to his country. His Bunadar-balkr, a Georgic written, like Tusser's Points, with a practical view of raising the state of agriculture, has always been much prized. Paul Vidalin's ditties are very naïve and clever.

Of later poets, down to more recent times, perhaps the best was Sigurd of Broadfirth, many of whose prettiest poems were composed in Greenland like those of Jon Biarnisson before him, c. 1750; John Thorlaksson's translation of Milton's great epic into Eddic verse is praiseworthy in intention, but, as may be imagined, falls far short of its aim. He also turned Pope's Essay on Man and Klopstock's Messiah into Icelandic. Benedikt Gröndal tried the same experiment with Homer in his Ilion's Kvædi, c. 1825. There is a fine prose translation of the Odyssey by Sweinbjörn Egillson, the lexicographer, both faithful and poetic in high degree.

Sagas.-The real strength of ancient Icelandic literature is shown in its most indigenous growth, the "Saga" (see also Saga). This is, in its purest form, the life of a hero, composed in regular form, governed by fixed rules, and intended for oral recitation. It bears the strongest likeness to the epic in all save its unversified form; in both are found, as fixed essentials, simplicity of plot, chronological order of events, set phrases used even in describing the restless play of emotion or the changeful fortunes of a fight or a storm, while in both the
absence of digression, comment or intrusion of the narrator's person is invariably maintained. The saga grew up in the quieter days which followed the change of faith (1002), when the deeds of the great families' heroes were still cherished by their descendants, and the exploits of the great kings of Norway and Denmark handed down with reverence. Telling of stories was a recognized form of entertainment at all feasts and gatherings, and it was the necessity of the reciter which gradually worked them into a regular form, by which the memory was relieved and the artistic features of the story allowed to be more carefully elaborated. That this form was so perfect must be attributed to Irish influence, without which indeed there would have been a saga, but not the same saga. It is to the west that the best sagas belong; it is to the west that nearly every classic writer whose name we know belongs; and it is precisely in the west that the admixture of Irish blood is greatest. In comparing the Irish tales with the saga, there will be felt deep divergencies in matter, style and taste, the richness of one contrasting with the chastened simplicity of the other; the one's half-comic, half-earnest bombast is wholly unlike the other's grim humour; the marvellous, so unearthly in the one, is almost credible in the other; but in both are the keen grasp of character, the biting phrase, the love of action and the delight in blood which almost assumes the garb of a religious passion.

When the saga had been fixed by a generation or two of oral reciters, it was written down; and this stereotyped the form, so that afterwards when literary works were composed by learned men (such as Abbot Karl's Swerri's Saga and Sturla's Islendinga) the same style was adopted.

Taking first the sagas relating to Icelanders, of which some thirty-five or forty remain out of thrice that number, they were first written down between 1140 and 1220, in the generation which succeeded Ari and felt the impulse his books had given to writing, on

## Icelandic sagas.

 separate scrolls, no doubt mainly for the reciter's convenience; they then went through the different phases which such popular compositions have to pass in all lands-editing and compounding (1220-1260), padding and amplifying (1260-1300), and finally collection in large MSS. (14th century). Sagas exist showing all these phases, some primitive and rough, some refined and beautified, some diluted and weakened, according as their copyists have been faithful, artistic or foolish; for the first generation of MSS. have all perished. We have also complex sagas put together in the 13th century out of the scrolls relating to a given locality, such a group as still exists untouched in Vapnfirdinga being fused into such a saga as Niala or Laxdæla. Of the authors nothing is known; we can only guess that some belong to the Sturlung school. According to subject they fall into two classes, those relating to the older generation before Christianity and those telling of St Olaf's contemporaries; only two fall into a third generation.Beginning with the sagas of the west, most perfect in style and form, the earliest in subject is that of Gold-Thori (c. 930), whose adventurous career it relates; Hensa-Porissaga tells of the burning of Blund-Ketil, a noble chief, an event which led to Thord Gelli's reforms next year (c. 964); Gislasaga (960-980) tells of the career and death of that ill-fated outlaw; it is beautifully written, and the verses by the editor (13th century) are good and appropriate; Hord's Saga (980) is the life of a band of outlaws on Whalesfirth, and especially of their leader Hord. Of later subject are the sagas of Havard and his revenge for his son, murdered by a neighbouring chief (997-1002); of the Heiðarirgasaga (990-1014), a typical tale of a great blood feud, written in the most primitive prose; of Gunnlaug and Hrafn (Gunnlaugssaga Ormstungu, 980-1008), the rival poets and their ill-starred love. The verse in this saga is important and interesting. To the west also belong the three great complex sagas Egla, Eyrbyggia and Laxdæla. The first (870980), after noticing the migration of the father and grandfather of the hero poet Egil, and the origin of the feud between them and the kings of Norway, treats fully of Egil's career, his enmity with Eirik Bloodaxe, his service with Æthelstan, and finally, after many adventures abroad, of his latter days in Iceland at Borg, illustrating very clearly what manner of men those great settlers and their descendants were, and the feelings of pride and freedom which led them to Iceland. The style is that of Snorri, who had himself dwelt at Borg. Eyrbyggia (8901031) is the saga of politics, the most loosely woven of all the compound stories. It includes a mass of information on the law, religion, traditions, \&c., of the heathen days in Iceland, and the lives of Eric, the real discoverer of Greenland, Biorn of Broadwick, a famous chief, and Snorri, the greatest statesman of his day. Dr Vigfusson would ascribe its editing and completion to Sturla the Lawman, c. 1250. Laxdæla (910-1026) is the saga of Romance. Its heroine Gudrun is the most famous of all Icelandic ladies. Her love for Kiartan the poet, and his career abroad, his betrayal by his friend Bolli, the sad death of Kiartan at his hands, the revenge taken for him on Bolli, whose slayers are themselves afterwards put to death, and the end of Gudrun, who becomes an anchorite after her stormy life, make up the pith of the story. The contrast of the characters, the rich style and fine dialogue which are so remarkable in this saga, have much in common with the best works of the Sturlung school.

Of the north there are the sagas of Kormak (930-960), most primitive of all, a tale of a wild
poet's love and feuds, containing many notices of the heathen times; of Vatzdælasaga (890980), relating to the settlement and the chief family in Waterdale; of Hallfred the poet (9961014), narrating his fortune at King Olafs court, his love affairs in Iceland, and finally his death and burial at Iona; of Reyk-dæla (990), which preserves the lives of Askell and his son VigaSkuti; of Svarf-dæla (980-990), a cruel, coarse story of the old days, with some good scenes in it, unfortunately imperfect, chapters 1-10 being forged; of Viga-Glum (970-990), a fine story of a heathen hero, brave, crafty and cruel. To the north also belong the sagas of Gretti the Strong (1010-1031), the life and death of the most famous of Icelandic outlaws, the real story of whose career is mixed up with the mythical adventures of Beowulf, here put down to Gretti, and with late romantic episodes and fabulous folk-tales (Dr Vigfusson would ascribe the best parts of this saga to Sturla; its last editor, whose additions would be better away, must have touched it up about 1300), and the stories of the Ljosvetningasaga (1009-1060). Gudmund the Mighty and his family and neighbours are the heroes of these tales, which form a little cycle. The Banda-manna saga (1050-1060), the only comedy among the sagas, is also a northern tale; it relates the struggles of a plebeian who gets a chieftancy against the old families of the neighbourhood, whom he successfully outwits; Öl-kofra pattr is a later imitation of it in the same humorous strain. The sagas of the north are rougher and coarser than those of the west, but have a good deal of individual character.

Of tales relating to the east there survive the Weapon-firth cycle-the tales of Thorstein the White (c. 900), of Thorstein the Staffsmitten (c. 985), of Gunnar Thidrand's Bane (1000-1008) and of the Weapon-firth Men (975-990), all relating to the family of Hof and their friends and kin for several generations-and the story of Hrafnkell Frey's Priest (c. 960), the most idyllic of sagas and best of the eastern tales. Of later times there are Droplaug's Sons' Saga (997-1007), written probably about 1110, and preserved in the uncouth style of the original (a brother's revenge for his brother's death is the substance of it; Brandkrossa Pattr is an appendix to it), and the tales of Thorstein Hall of Side's Son (c. 1014) and his brother Thidrandi (c. 996), which belong to the cycle of Hall o' Side's Saga, unhappily lost; they are weird tales of bloodshed and magic, with idyllic and pathetic episodes.

The sagas of the south are either lost or absorbed in that of Nial (970-1014), a long and complex story into which are woven the tales of Gunnar Nial, and parts of others, as Brian Boroimhe, Hall o' Side, \&c. It is, whether we look at style, contents or legal and historical weight, the foremost of all sagas. It deals especially with law, and contains the pith and the moral of all early Icelandic history. Its hero Nial, type of the good lawyer, is contrasted with its villain Mord, the ensample of cunning, chicane, and legal wrong doing; and a great part of the saga is taken up with the three cases and suits of the divorce, the death of Hoskuld and the burning of Nial, which are given with great minuteness. The number and variety of its dramatis personae give it the liveliest interest throughout. The women Hallgerda, Bergthora and Ragnhild are as sharply contrasted as the men Gunnar, Skarphedin, Flosi and Kari. The pathos of such tragedies as the death of Gunnar and Hoskuld and the burning is interrupted by the humour of the Althing scenes and the intellectual interest of the legal proceedings. The plot dealing first with the life and death of Gunnar, type of the chivalry of his day, then with the burning of Nial by Flosi, and how it came about, and lastly with Kari's revenge on the burners, is the ideal saga-plot. The author must have been of the east, a good lawyer and genealogist, and have composed it about 1250, to judge from internal evidence. It has been overworked by a later editor, $c$. 1300, who inserted many spurious verses.

Relating partly to Iceland, but mostly to Greenland and Vinland (N. America), are the Floamannasaga (985-990), a good story of the adventures of Thorgils and of the struggles of shipwrecked colonists in Greenland, a graphic and terrible picture; and

Of Greenland and North America. Eirikssaga rauða (990-1000), two versions, one northern (Flatey-book), one western, the better (in Hawk's Book, and AM. 557), the story of the discovery of Greenland and Vinland (America) by the Icelanders at the end of the 9th century. Later is the Fostbrædrasaga (1015-1030), a very interesting story, told in a quaint romantic style, of Thorgeir, the reckless henchman of King Olaf, and how his death was revenged in Greenland by his sworn brother the true-hearted Thormod Coalbrow's poet, who afterward dies at Sticklestad. The tale of Einar Sookisson (c. 1125) may also be noticed. The lost saga of Poet Helgi, of which only fragments remain, was also laid in Greenland.

Besides complete sagas there are embedded in the Heimskringla numerous small Pættir or episodes, small tales of Icelanders' adventures, often relating to poets and their lives at the kings' courts; one or two of these seem to be fragments of sagas now lost. Among the more notable are those of Orm Storolfsson, Ogmund Dijtt, Halldor Snorrason, Thorstein Oxfoot, Hromund Halt, Thorwald Tasaldi, Svadi and Arnor Herlingar-nef, Audunn of Westfirth, SnegluHalli, Hrafn of Hrutfiord, Hreidar Heimski, Gisli Illugison, Ivar the poet, Gull-Æsu Thord, Einar Skulason the poet, Mani the poet, \&c.

The forged Icelandic sagas appear as early as the 13 th century. They are very poor, and either worked up on hints given in genuine stories or altogether apocryphal.

History.-About the year of the battle of Hastings was born Ari Froði Thorgilsson (10671148), one of the blood of Queen Aud, who founded the famous historical school of Iceland, and himself produced its greatest monument in a work which can be compared for value with the English Domesday Book. Nearly all that we know of the heathen commonwealth may be traced to the collections of Ari. It was he too that fixed the style in which history should be composed in Iceland. It was he that secured and put into order the vast mass of fragmentary tradition that was already dying out in his day. And perhaps it is the highest praise of all to him that he wrote in his own "Danish tongue," and so ensured the use of that tongue by the cultured of after generations. Ari's great works are Konungabók, or The Book of Kings, relating the history of the kings of Norway from the rise of the Yngling dynasty down to the death of Harald Sigurdsson in the year of his own birth. This book he composed from the dictation of old men such as Odd Kolsson, from the genealogical poems, and from the various dirges, battle-songs and eulogia of the poets. It is most probable that he also compiled shorter Kings' Books relating to Denmark and perhaps to England. The Konungabók is preserved under the Heimskringla of Snorri Sturloson, parts of it almost as they came from Ari's hands, for example Ynglinga and Harald Fairhair's Saga, and the prefaces stating the plan and critical foundations of the work, parts of it only used as a framework for the magnificent superstructure of the lives of the two Olafs, and of Harald Hardrada and his nephew Magnus the Good. The best text of Ari's Konungabók (Ynglinga, and the sagas down to but not including Olaf Tryggvason’s) is that of Frisbók.

The Book of Settlements (Landnamabók) is a wonderful performance, both in its scheme and carrying out. It is divided into five parts, the first of which contains a brief account of the discovery of the island; the other four, one by one taking a quarter of the land, describe the name, pedigree and history of each settler in geographical order, notice the most important facts in the history of his descendants, the names of their homesteads, their courts and temples, thus including mention of 4000 persons, one-third of whom are women, and 2000 places. The mass of information contained in so small a space, the clearness and accuracy of the details, the immense amount of life which is breathed into the whole, astonish the reader, when he reflects that this colossal task was accomplished by one man, for his collaborator Kolsegg merely filled up his plan with regard to part of the east coast, a district with which Ari in his western home at Stad was little familiar. Landnamabók has reached us in two complete editions, one edited by Sturla, who brought down the genealogies to his own grandfather and grandmother, Sturla and Gudny, and one by Hawk, who traces the pedigrees still later to himself.

Ari also wrote a Book of Icelanders (Islendingabók, c. 1127), which has perished as a whole, but fragments of it are embedded in many sagas and Kings' Lives; it seems to have been a complete epitome of his earlier works, together with an account of the constitutional history, ecclesiastical and civil, of Iceland. An abridgment of the latter part of it, the little Libellus Islandorum (to which the title of the bigger Liber-Islendingabók-is often given), was made by the historian for his friends Bishops Ketil and Thorlak, for whom he wrote the Liber (c. 1137). This charming little book is, with the much later collections of laws, our sole authority for the Icelandic constitution of the commonwealth, but, "much as it tells, the lost Liber would have been of still greater importance." Kristni-Saga, the story of the christening of Iceland, is also a work of Ari's, "overlaid" by a later editor, but often preserving Ari's very words. This saga, together with several scattered tales of early Christians in Iceland before the change of faith (1002), may have made up a section of the lost Liber. Of the author of these works little is known. He lived in quiet days a quiet life; but he shows himself in his works, as Snorri describes him, "a man wise, of good memory and a speaker of the truth." If Thucydides is justly accounted the first political historian, Ari may be fitly styled the first of scientific historians.

A famous contemporary and friend of Ari is Sæmund (1056-1131), a great churchman, whose learning so impressed his age that he got the reputation of a magician. He was the friend of Bishop John, the founder of the great Odd-Verjar family, and the author of a Book of Kings from Harald Fairhair to Magnus the Good, in which he seems to have fixed the exact chronology of each reign. It is most probable that he wrote in Latin. The idea that he had anything to do with the poetic Edda in general, or the Sun's Song in particular, is unfounded.

The flame which Ari had kindled was fed by his successors in the 12th century. Eirik Oddsson (c. 1150) wrote the lives of Sigurd Evil-deacon and the sons of Harold Gille, in his HryggiarStykki (Sheldrake), of which parts remain in the MSS. collections of Kings' Lives, Morkinskinna, \&c. Karl Jonsson, abbot of Thingore, the Benedictine minister, wrote (c. 1184) Sverrissaga from the lips of that great king, a fine racy biography, with a style and spirit of its own. Böglunga-Sögur tell the story of the civil wars which followed Sverri's death. They are probably by a contemporary.

The Latin Lives of St Olaf, Odd's in Latin (c. 1175), compiled from original authorities, and the Legendary Life, by another monk whose name is lost, are of the medieval Latin school of Sæmund to which Gunnlaug belonged.

Snorri Sturlason (q.v.) was known to his contemporaries as a statesman and poet; to us he is above all an historian. Snorri (1179-1241) wrote the Lives of the Kings (Heimskringla), from Olaf Tryggvason to Sigurd the Crusader inclusive; and we have them substantially as they came from his hand in the Great King Olaf's Saga; St Olaf's Saga, as in Heimskringla and the Stockholm MS.; and the succeeding Kings' Lives, as in Hulda and Hrokkinskinna, in which, however, a few episodes have been inserted.

These works were indebted for their facts to Ari's labours, and to sagas written since Ari's death; but the style and treatment of them are Snorri's own. The fine Thucydidean speeches, the dramatic power of grasping character, and the pathos and poetry that run through the stories, along with a humour such as is shown in the Edda, and a varied grace of style that never flags or palls, make Snorri one of the greatest of historians.

Here it should be noticed that Heimskringla and its class of MSS. (Eirspennil, Jofraskinna, Gullinskinna, Fris-bok and Kringla) do not give the full text of Snorri's works. They are abridgments made in Norway by Icelanders for their Norwegian patrons, the Life of St Olaf alone being preserved intact, for the great interest of the Norwegians lay in him, but all the other Kings' Lives being more or less mutilated, so that they cannot be trusted for historic purposes; nor do they give a fair idea of Snorri's style.

Agrip is a 12th-century compendium of the Kings' Lives from Harald Fairhair to Sverri, by a scholastic writer of the school of Sæmund. As the only Icelandic abridgment of Norwegian history taken not from Snorri but sources now lost, it is of worth. Its real title is Konunga-tal.

Noregs Konunga-tal, now called Fagrskinna, is a Norse compendium of the Kings' Lives from Halfdan the Black to Sverri's accession, probably written for King Haakon, to whom it was read on his death-bed. It is an original work, and contains much not found elsewhere. As nonIcelandic it is only noticed here for completeness.

Styrmi Karason, a contemporary of Snorri's, dying in 1245, was a distinguished churchman (lawman twice) and scholar. He wrote a Life of St Olaf, now lost; his authority is cited. He also copied out Landnamabók and Sverri's Life from his MSS., of which surviving copies were taken.

Sturla, Snorri's nephew, wrote the Hakonssaga and Magnussaga at the request of King Magnus, finishing the first c. 1265, the latter c. 1280. King Haakon's Life is preserved in full; of the other only fragments remain. These are the last of the series of historic works which Ari's labours began, from which the history of Norway for 500 years must be gathered.

A few books relating the history of other Scandinavian realms will complete this survey. In Skioldunga-bok was told the history of the early kings of Denmark, perhaps derived from Ari's collections, and running parallel to Ynglinga. The earlier part of it has perished save a fragment Sogu-brot, and citations and paraphrases in Saxo, and the mythical Ragnar Lodbrok's and Gongu-Hrolf's Sagas; the latter part, Lives of Harold Bluetooth and the Kings down to Sveyn II., is still in existence and known as Skioldunga.

The Knutssaga is of later origin and separate authorships, parallel to Snorri's Heimskringla, but earlier in date. The Lives of King Valdemar and his Son, written c. 1185, by a contemporary of Abbot Karl's, are the last of this series. The whole were edited and compiled into one book, often quoted as Skioldunga, by a 13th-century editor, possibly Olaf, the White Poet, Sturla's brother, guest and friend of King Valdemar II. Jomsvikinga Saga, the history of the pirates of Jom, down to Knut the Great's days, also relates to Danish history.

The complex work now known as Orkneyinga is made up of the Earls' Saga, lives of the first great earls, Turf-Einar, Thorfinn, \&c.; the Life of St Magnus, founded partly on Abbot Robert's Latin life of him (c. 1150) an Orkney work, partly on Norse or Icelandic biographies; a Miradebook of the same saint; the Lives of Earl Rognwald and Sveyn, the last of the vikings, and a few episodes such as the Burning of Bishop Adam. A scholastic sketch of the rise of the Scandinavian empire, the Foundation of Norway, dating c. 1120, is prefixed to the whole.

Færeyinga tells the tale of the conversion of the Færeys or Faroes, and the lives of its chiefs Sigmund and Leif, composed in the 13th century from their separate sagas by an Icelander of the Sturlung school.

Biographies.-The saga has already been shown in two forms, its original epic shape and its later development applied to the lives of Norwegian and Danish kings and earls, as heroic but deeper and broader subjects than before. In the 13th century it is put to a third use, to tell the plain story of men's lives for their contemporaries, after satisfying which demand it dies away for ever.

These biographies are more literary and medieval and less poetic than the Icelandic sagas and king's lives; their simplicity, truth, realism and purity of style are the same. They run in two parallel streams, some being concerned with chiefs and champions, some with bishops. The former are mostly found embedded in the complex mass of stories known as Sturlunga, from which Dr Vigfusson has extricated them, and for the first time set them in order. Among them are the sagas of Thorgils and Haflidi (1118-1121), the feud and peacemaking of two great chiefs, contemporaries of Ari; of Sturla (1150-1183), the founder of the great Sturlung family, down to the settlement of his great lawsuit by Jon Loptsson, who thereupon took his son Snorri the historian to fosterage,-a humorous story but with traces of the decadence about it, and glimpses of the evil days that were to come; of the Önundar-brennusaga (1185-1200), a tale of feud and fire-raising in the north of the island, the hero of which, Gudmund Dyri, goes at last into a cloister; of Hrafn Sveinbiornsson (1190-1213), the noblest Icelander of his day, warrior, leech, seaman, craftsman, poet and chief, whose life at home, travels and pilgrimages abroad (Hrafn was one of the first to visit Becket's shrine), and death at the hands of a foe whom he had twice spared, are recounted by a loving friend in pious memory of his virtues, c. 1220; of Áron Hiorleifsson (1200-1255), a man whose strength, courage and adventures befit rather a henchman of Olaf Tryggvason than one of King Haakon's thanes (the beginning of the feuds that rise round Bishop Gudmund are told here), of the Svinefell-men (1248-1252), a pitiful story of a family feud in the far east of Iceland.

But the most important works of this class are the Islendinga Saga and Thorgils Saga of Lawman Sturla. Sturla and his brother Olaf were the sons of Thord Sturlason and his mistress Thora. Sturla was born and brought up in prosperous times, but his manhood was passed in the midst of strife, in which his family fell one by one, and he himself, though a peaceful man who cared little for politics, was more than once forced to fly for his life. While in refuge with King Magnus, in Norway, he wrote his two sagas of that king and his father. After his first stay in Norway he came back in 1271, with the new Norse law-book, and served a second time as lawman. The Islendinga must have been the work of his later years, composed at Fairey in Broadfirth, where he died, 30th July 1284, aged about seventy years. The saga of Thorgils Skardi (1252-1261) seems to have been the first of his works on Icelandic contemporary history; it deals with the life of his own nephew, especially his career in Iceland from 1252 to 1258. The second part of Islendinga (1242-1262), which relates to the second part of the civil war, telling of the careers of Thord Kakali, Kolbein the Young, Earl Gizur and Hrafn Oddsson. The end is imperfect, there being a blank of some years before the fragmentary ending to which an editor has affixed a notice of the author's death. The first part of Islendinga (12021242) tells of the beginning and first part of the civil wars, the lives of Snorri and Sighvat, Sturla's uncles, of his cousin and namesake Sturla Sighvatsson, of Bishop Gudmund, and Thorwald Gizursson,-the fall of the Sturlungs, and with them the last hopes of the great houses to maintain the commonwealth, being the climax of the story.

Sturla's power lies in his faithfulness to nature, minute observance of detail and purity of style. The great extent of his subject, and the difficulty of dealing with it in the saga form, are most skilfully overcome; nor does he allow prejudice or favour to stand in the way of the truth. He ranks below Ari in value and below Snorri in power; but no one else can dispute his place in the first rank of Icelandic writers.

Of the ecclesiastical biographers, an anonymous Skalholt clerk is the best. He wrote Hungrvaka, lives of the first five bishops of Skalholt, and biographies of his patron Bishop Paul (Pálssaga) and also of St Thorlak (Thorlakssaga). They are full of interesting notices of social and church life. Thorlak was a learned man, and had studied at Paris and Lincoln, which he left in 1161. These lives cover the years 1056-1193. The life of St John, a great reformer, a contemporary of Thorodd, whom he employed to build a church for him, is by another author (1052-1121). The life of Gudmund (Gudmundar Saga Goda), as priest, recounts the early life of this Icelandic Becket till his election as bishop (1160-1202); his after career must be sought out in Islendinga. It is written by a friend and contemporary. A later life by Arngrim, abbot of Thingore, written $c .1350$, as evidence of his subject's sanctity, tells a good deal about Icelandic life, \&c. The lives of Bishops Arni and Lawrence bring down our knowledge of Icelandic history into the 14th century. The former work, Arna Saga Biskups, is imperfect; it is the record of the struggles of church and state over patronage rights and glebes, written c. 1315; it now covers only the years 1269-1291; a great many documents are given in it, after the modern fashion. The latter, Laurentius Saga Biskups, by his disciple, priest Einar Haflidason, is a charming biography of a good and pious man, whose chequered career in Norway and Iceland is picturesquely told (1324-1331). It is the last of the sagas. Bishop Jon's Table-Talk (1325-1339) is also worth noticing; it contains many popular stories which the good bishop, who had studied at Bologna and Paris, was wont to tell to his friends.

Annals.-The Annals are now almost the sole material for Icelandic history; they had begun earlier, but after 1331 they got fuller and richer, till they end in 1430. The best are Annales

Regii, ending 1306, Einar Haflidason's Annals, known as "Lawman's Annals," reaching to 1392, and preserved with others in Flatey-book, and the New Annals, last of all. The Diplomatarium Islandicum, edited by Jon Sigurdsson, contains what remains of deeds, inventories, letters, \&c., from the old days, completing our scanty material for this dark period of the island's history.

Literature of Foreign Origin.-After the union with Norway and change of law genuine tradition died out with the great houses. The ordinary medieval literature reached Iceland through Norway, and every one began to put it into a vernacular dress, so neglecting their own classics that but for a few collectors like Lawman Hauk they would have perished entirely.

The Norwegian kings, Haakon Haakonson (c. 1225), and Haakon V. (c. 1305), employed Icelanders at their courts in translating the French romances of the Alexander, Arthur and Charlemagne cycles. Some forty or fifty of these Riddara-Sögur (Romances of Chivalry) remain. They reached Iceland and were eagerly read, many Rimur being founded on them. Norse versions of Mary of Brittany's Lays, the stories of Brutus and of Troy, and part of the Pharsalia translated are also found. The Speculum Regale, with its interesting geographical and social information, is also Norse, written c. 1240, by a Halogalander. The computistic and arithmetical treatises of Stiorn-Odd, Biarni the Number-skilled (d. 1173), and Hauk Erlendsson the Lawman (d. 1334), and the geography of Ivar Bardsson, a Norwegian (c. 1340), are of course of foreign origin. A few tracts on geography, \&c., in Hauk's book, and a Guide to the Holy Land, by Nicholas, abbot of Thwera (d. 1158), complete the list of scientific works.

The stories which contain the last lees of the old mythology and pre-history seem to be also non-Icelandic, but amplified by Icelandic editors, who probably got the plots from the Western Islands. Völsunga Saga and Hervarar Saga contain quotations and paraphrases of lays by the Helgi poet, and Half's, Ragnar's and Asmund Kappabana's Sagas all have bits of Western poetry in them. Hrolf Kraki's Saga paraphrases part of Biarkamal; Hromund Gripsson's gives the story of Helgi and Kara (the lost third of the Helgi trilogy); Gautrek's Arrow Odd's, Frithiof's Sagas, \&c., contain shreds of true tradition amidst a mass of later fictitious matter of no worth. With the Riddara-Sögur they enjoyed great popularity in the 15 th century, and gave matter for many Rimur. Thidrek's Saga, a late version of the Völsung story, is of Norse composition (c. 1230), from North German sources.

The medieval religious literature of Western Europe also influenced Iceland, and the Homilies (like the Laws) were, according to Thorodd, the earliest books written in the vernacular, antedating even Ari's histories. The lives of the Virgin, the Apostles and the Saints fill many MSS. (edited in four large volumes by Professor Unger), and are the works of many authors, chiefly of the 13th and 14th centuries; amongst them are the lives of SS. Edward the Confessor, Oswald of Northumbria, Dunstan and Thomas of Canterbury. Of the authors we know Priest Berg Gunsteinsson (d. 1211); Kygri-Biorn, bishop-elect (d. 1237); Bishop Brand (d. 1264); Abbot Runolf (d. 1307); Bishop Lawrence’s son Arni (c. 1330); Abbot Berg (c. 1340), \&c. A paraphrase of the historical books of the Bible was made by Bishop Brand (d. 1264), called Gydinga Sögur. About 1310 King Haakon V. ordered a commentary on the Bible to be made, which was completed down to Exodus xix. To this Brand's work was afterwards affixed, and the whole is known as Stiorn. The Norse version of the famous Barlaam and Josaphat, made for Prince Haakon (c. 1240), must not be forgotten.

Post-classical Literature.-The post-classical literature falls chiefly under three headsreligious, literary and scientific. Under the first comes foremost the noble translation of the New Testament by Odd Gottskalksson, son of the bishop of Hólar. Brought up in Norway, he travelled in Denmark and Germany, and took upon him the new faith before he returned to Iceland, where he became secretary to Bishop Ogmund of Skalholt. Here he began by translating the Gospel of Matthew into his mother-tongue in secret. Having finished the remainder of the New Testament at his own house at Olves, he took it to Denmark, where it was printed at Roskild in 1540 . Odd afterwards translated the Psalms, and several devotional works of the day, Corvinus's Epistles, \&c. He was made lawman of the north and west, and died from a fall in the Laxa in Kios, June 1556. Three years after his death the first press was set up in Iceland by John Matthewson, at Breidabolstad, in Hunafloe, and a Gospel and Epistle Book, according to Odd's version, issued from it in 1562. In 1584 Bishop Gudbrand, who had brought over a splendid fount of type from Denmark in 1575 (which he completed with his own hands), printed a translation of the whole Bible at Hólar, incorporating Odd's versions and some books (Proverbs and the Son of Sirach, 1580) translated by Bishop Gizar, but supplying most of the Old Testament himself. This fine volume was the basis of every Bible issued for Iceland till 1826, when it was replaced by a bad modern version. For beauty of language and faithful simplicity of style the finer parts of this version, especially the New Testament, have never been surpassed.

The most notable theological work Iceland ever produced is the Postill-Book of Bishop John Vidalin (1666-1720), whose bold homely style and stirring eloquence made "John's Book," as it
is lovingly called, a favourite in every household, till in the 19th century it was replaced for the worse by the more sentimental and polished Danish tracts and sermons. Theological literature is very popular, and many works on this subject, chiefly translations, will be found in the lists of Icelandic bibliographers.

The first modern scientific work is the Iter per patriam of Eggert Olafsson and Biarni Paulsson, which gives an account of the physical peculiarities-fauna, flora, \&c.-of the island as far as could be done at the date of its appearance, 1772. The island was first made known to "the world" by this book and by the sketch of Unno von Troil, a Swede, who accompanied Sir Joseph Banks to Iceland in 1772, and afterwards wrote a series of "letters" on the land and its literature, \&c. This tour was the forerunner of an endless series of "travels," of which those of Sir W. J. Hooker, Sir G. S. Mackenzie (1810), Ebenezer Henderson (1818), Joseph Paul Gaimard (1838-1843), Paijkull (1867) and, lastly, that of Sir Richard Burton, an excellent account of the land and people, crammed with information of every kind (1875), are the best.

Iceland is emphatically a land of proverbs, while of folk-tales, those other keys to the people's heart, there is plentiful store. Early work in this direction was done by Jon Gudmundsson, Olaf the Old and John Olafsson in the 17th century, who all put traditions on paper, and their labours were completed by the magnificent collection of Jon Arnason (1862-1864), who was inspired by the example of the Grimms. Many tales are but weak echoes of the sagas; many were family legends, many are old fairy tales in a garb suited to their new northern home; but, besides all these, there are a number of traditions and superstitions of indigenous origin.

The Renaissance of Iceland dates from the beginning of the 17th century, when a school of antiquaries arose. Arngrim Jonsson's Brevis Commentarius (1593), and Crymogaea (1609), were the first-fruits of this movement, of which Bishops Odd, Thorlak and Bryniulf (worthy parallels to Parker and Laud) were the wise and earnest supporters. The first (d. 1630) collected much material for church history. The second (d. 1656) saved Sturlunga and the Bishops'Lives, encouraged John Egilsson to write his New Hungerwaker, lives of the bishops of the Dark Ages and Reformation, and helped Biorn of Skardsa (d. 1655), a bold and patriotic antiquary (whose Annals continue Einar's), in his researches. The last (d. 1675) collected a fine library of MSS., and employed the famous copyist John Erlendsson, to whom and the bishop's brother, John Gizurarsson (d. 1648), we are indebted for transcripts of many lost MSS.

Torfaeus (1636-1719) and Bartholin, a Dane (d. 1690), roused the taste for northern literature in Europe, a taste which has never since flagged; and soon after them Arni Magnusson (1663-1730) transferred all that remained of vellum and good paper MSS. in Iceland to Denmark, and laid the foundations of the famous library and bequest, for which all Icelandic students are so much beholden. For over forty years Arni stuck to his task, rescuing every scrap he could lay hands on from the risks of the Icelandic climate and carelessness, and when he died only one good MSS. remained in the island. Besides his magnificent collection, there are a few MSS. of great value at Upsala, at Stockholm, and in the old royal collection at Copenhagen. Those in the university library in the latter city perished in the fire of 1728. Sagas were printed at Upsala and Copenhagen in the 17th century, and the Arna-Magnaean fund has been working since 1772. In that year appeared also the first volume of Bishop Finn Jonsson's Historia Ecclesiastica Islandiae, a work of high value and much erudition, containing not only ecclesiastical but civil and literary history, illustrated by a well-chosen mass of documents, 8701740. It has been continued by Bishop P. Peterson to modern times, 1740-1840. The results, however, of modern observers and scholars must be sought for in the periodicals, Safn, Felagsrit, Ny Felagsrit and others. John Espolin's Arbækr is very good up to its date, 1821.

A brilliant sketch of Icelandic classic literature is given by Dr Gudbrandr Vigfusson in the Prolegomena to Sturlunga Saga (Oxford, 1879). It replaces much earlier work, especially the Sciagraphia of Halfdan Einarsson (1777), and the Saga-Bibliotek of Müller. The numerous editions of the classics by the Icelandic societies, the Danish Société des Antiquités, Nordiske Litteratur Samfund, and the new Gammel Nordisk Litteratur Samfund, the splendid Norwegian editions of Unger, the labours of the Icelanders Sigurdsson and Gislason, and of those foreign scholars in Scandinavia and Germany who have thrown themselves into the work of illustrating, publishing and editing the sagas and poems (men like P. A. Munch, S. Bugge, F. W. Bergmann, Th. Möbius and K. von Maurer, to name only a few), can only be referred to here. See also Finnur Jónsson, Den Oldnorske og Oldislanske Litteraturs Historie (Copenhagen, 1893-1900); R. B. Anderson's translation (Chicago, 1884) of Winkel Horn's History of the Literature of the Scandinavian North; and W. Morris and E. Magnusson's Saga Library.
(F. Y. P.)

## Recent Literature

The recent literature of Iceland has been in a more flourishing state than ever before since the 13th century. Lyrical poetry is by far the largest and the most interesting portion of it. The
great influence of Jónas Hallgrímsson (1807-1845) is still felt, and his school was the reigning one up to the end of the 19th century, although then a change seemed to be in sight. The most successful poet of this school is Steingrímr Thorsteinsson (b. 1830). He is specially famous for his splendid descriptions of scenery (The Song of Gilsbakki), his love-songs and his sarcastic epigrams. As a translator he has enriched the literature with The Arabian Nights, Sakuntala, King Lear and several other masterpieces of foreign literature. Equal in fame is Matthías Jochumsson (b. 1835), who, following another of Jónas Hallgrímsson's many ways, has successfully revived the old metres of the classical Icelandic poets, whom he resembles in his majestic, but sometimes too gorgeous, language. He is as an artist inferior to Steingrímr Thorsteinsson, but surpasses him in bold flight of imagination. He has successfully treated subjects from Icelandic history Grettisljód, a series of poems about the famous outlaw Grettir. His chief fault is a certain carelessness in writing; he can never write a bad poem, but rarely a poem absolutely flawless. He has translated Tegnér's Frithiofs Saga, several plays of Shakespeare and some other foreign masterpieces. The great religious poet of Iceland, Hallgrímr Pétursson, has found a worthy successor in Valdemar Briem (b. 1848), whose Songs of the Bible are deservedly popular. He is like Matthías Jochumsson in the copious flow of his rhetoric; some of his poems are perfect both as regards form and contents, but he sometimes neglects the latter while polishing the former. An interesting position is occupied by Benedict Gröndal (b. 1826), whose travesties of the old romantic stories, ${ }^{7}$ and his Aristophanic drama Gandreiðin ("The Magic Ride") about contemporary events, are among the best satirical and humorous productions of Icelandic literature.

Influenced by Jónas Hallgrímsson with regard to language and poetic diction, but keeping unbroken the traditions of Icelandic medieval poetry maintained by Sigurðr Breiðfjörð (17981846), is another school of poets, very unlike the first. In the middle of the 19th century this school was best represented by Hjálmar Jónsson from Bóla (1796-1875), a poor farmer with little education, but endowed with great poetical talents, and the author of satirical verses not inferior to those of Juvenal both in force and coarseness. In the last decades of the 19th century this school produced two poets of a very high order, both distinctly original and Icelandic. One is Páll Olafsson (b. 1827). His songs are mostly written in the medieval quatrains (ferskeytla), and are generally of a humorous and satirical character; his convivial songs are known by heart by every modern Icelander; and although some of the poets of the present day are more admired, there is none who is more loved by the people. The other is Porsteinn Erlingsson (b. 1858). His exquisite satirical songs, in an easy and elegant but still manly and splendid language, have raised much discussion. Of his poems may be mentioned The Oath, a series of most beautiful ballads, with a tragical love-story of the 17th century as their base, but with many and happy satirical allusions to modern life; Jörundr, a long poem about the convict king, the Danish pirate Jörgensen, who nearly succeeded in making himself the master of Iceland, and The Fate of the Gods and The Men of the West (the Americans), two poems which, with their anti-clerical and half-socialistic tendencies, have caused strong protests from orthodox Lutheran clergy. Near to this school, but still standing apart, is Grímur Thomsen (b. 1820).

In the beginning of the 'eighties a new school arose-having its origin in the colony of Icelandic students at the University of Copenhagen. They had all attended the lectures of Georg Brandes, the great reformer of Scandinavian literature, and, influenced by his literary theories, they chose their models in the realistic school. This school is very dissimilar from the halfromantic school of Jónas Hallgrímsson; it is nearer the national Icelandic school represented by Páll Olafsson and Porsteinn Erlingsson, but differs from those writers by introducing foreign elements hitherto unknown in Icelandic literature, and-especially in the case of the prose-writers-by imitating closely the style and manner of some of the great Norwegian novelists. Their influence brought the Icelandic literature into new roads, and it is interesting to see how the tough Icelandic element gradually assimilates the foreign. Of the lyrical poets, Hannes Hafsteinn (b. 1861) is by far the most important. In his splendid ballad, The Death of Skarphedinn, and in his beautiful series of songs describing a voyage through some of the most picturesque parts of Iceland, he is entirely original; but in his love-songs, beautiful as many of them are, a strong foreign influence can be observed. Among the innovations of this poet we may note a predilection for new metres, sometimes adopted from foreign languages, sometimes invented by himself, a thing practised rarely and generally with small success by the Icelandic poets.

No Icelandic novelist has as yet equalled Jón Thóroddsen (1819-1868). The influence of the realistic school has of late been predominant. The most distinguished writer of that school has been Gestur Pálsson (1852-1891), whose short stories with their sharp and biting satire have produced many imitations in Iceland. The best are A Home of Love and Captain Sigurd. Jónas Jónasson (b. 1856), a clergyman of northern Iceland, has, in a series of novels and short stories, given accurate, but somewhat dry, descriptions of the more gloomy sides of Icelandic country life. His best novel is Randiðr from Hvassafell, an historical novel of the middle ages. Besides these we may mention Torfhildur Hólm, one of the few women who have distinguished
themselves in Icelandic literature. Her novels are mostly historical. The last decade of the 19th century saw the establishment of a permanent theatre at Reykjavik. The poet Matthías Jochumsson has written several dramas, but their chief merits are lyrical. The most successful of Icelandic dramatists as yet is Indrði Einarsson, whose plays, chiefly historical, in spite of excessive rhetoric, are very interesting and possess a true dramatic spirit.

In geography and geology Porvaldr Thoroddsen has acquired a European fame for his researches and travels in Iceland, especially in the rarely-visited interior. Of his numerous writings in Icelandic, Danish and German, the History of Icelandic Geography is a monumental work. In history Páll Melsteð's (b. 1812) chief work, the large History of the World, belongs to this period, and its pure style has had a beneficial influence upon modern Icelandic prose.

Of the younger historians we may mention Porkell Bjarnason (History of the Reformation in Iceland). Jón Borkelsson (b. 1822), inspector of the archives of Iceland, has rendered great services to the study of Icelandic history and literature by his editions of the Diplomatarium Islandicum and Obituarium Islandicum, and by his Icelandic Poetry in the 15th and 16th Century, written in Danish, an indispensable work for any student of that period. A leading position among Icelandic lexicographers is occupied by Jón Porkelsson, formerly head of the Latin school at Reykjavik, whose Supplement til islandske Ordbøger, an Icelandic-Danish vocabulary (three separate collections), has hardly been equalled in learning and accuracy. Other distinguished philologists are his successor as head of the Latin school, Bjôrn Magnússon Olsen (Researches on Sturlunga, Ari the Wise, The Runes in the Old Icelandic Literature-the last two works in Danish); Finnur Jónsson, professor at the University of Copenhagen (History of the Old Norwegian and Icelandic Literature, in Danish, and excellent editions of many old Icelandic classical works); and Valtýr Guðmundsson, lecturer at the University of Copenhagen (several works on the old architecture of Scandinavia) and editor of the influential Icelandic literary and political review, Eimreiðin ("The Locomotive").

See J. C. Poestion, Islandische Dichter der Neuzeit (Leipzig, 1897); C. Küchler, Geschichte der isländischen Dichtung der Neuzeit (Leipzig, 1896); Ph. Schweitzer, Island; Land und Leute (Leipzig, 1885); Alexander Baumgartner, Island und die Faroer (Freiburg im Breisgau, 1889).

1 Jökull, plural jöklar, Icel. snowfield, glacier.
2 Flói, bay; fjörðr, fjord.
3 Vatn, lake.
4 See Th. Thoroddsen, "Explorations in Iceland during the years 1881-1898," Geographical Journal, vol. xiii. (1899), pp. 251-274, 480-513, with map.
5 For the periods succeeding the union, Danish state papers and the History of Finn Jonsson are the best authority.

6 Many of these poems were Englished in prose by the translator of Mallet, by B. Thorpe in his Sæmund's Edda, and two or three by Messrs Morris and Magnussen, as appendices to their translation of Volsunga Saga. Earlier translations in verse are those in Dryden's Miscellany (vol. vi), A. Cottle's Edda, Mathias's Translations, and W. Herbert's Old Icelandic Poetry. Gray's versions of Darradar-liod and Vegtamskviða are well known.

7 E.g. "The Battle of the Plains of Death," a burlesque on the battle of Solferino.

ICELAND MOSS, a lichen (Cetraria islandica) whose erect or ascending foliaceous habit gives it something of the appearance of a moss, whence probably the name. It is often of a pale chestnut colour, but varies considerably, being sometimes almost entirely greyish white; and grows to a height of from 3 to 4 in ., the branches being channelled or rolled into tubes, which terminate in flattened lobes with fringed edges. It grows abundantly in the mountainous regions of northern countries, and it is specially characteristic of the lava slopes and plains of the west and north of Iceland. It is found on the mountains of north Wales, north England, Scotland and south-west Ireland. As met with in commerce it is a light-grey harsh cartilaginous body, almost destitute of colour, and having a slightly bitter taste. It contains about $70 \%$ of lichenin or lichen-starch, a body isomeric with common starch, but wanting any appearance of structure. It also yields a peculiar modification of chlorophyll, called thallochlor, fumaric acid, licheno-stearic acid and cetraric acid, to which last it owes its bitter taste. It forms a nutritious and easily digested amylaceous food, being used in place of starch in some preparations of cocoa. It is not, however, in great request, and even in Iceland it is only habitually resorted to
in seasons of scarcity. Cetraric acid or cetrarin, a white micro-crystalline powder with a bitter taste, is readily soluble in alcohol, and slightly soluble in water and ether. It has been recommended for medicinal use, in doses of 2 to 4 grains, as a bitter tonic and aperient.

ICE-PLANT, the popular name for Mesembryanthemum crystallinum, a hardy annual most effective for rockwork. It is a low-growing spreading herbaceous plant with the fleshy stem and leaves covered with large glittering papillae which give it the appearance of being coated with ice. It is a dry-country plant, a native of Greece and other parts of the Mediterranean region, the Canary Islands, South Africa and California. Mesembryanthemum is a large genus (containing about 300 species) of erect or prostrate fleshy herbs or low shrubs, mostly natives of South Africa, and rarely hardy in the British Isles where they are mostly grown as greenhouse plants. They bear conspicuous white, yellow or red flowers with many petals inserted in the calyx-tube. The thick fleshy leaves are very variable in shape, and often have spiny rigid hairs on the margin. They are essentially sun-loving plants. The best-known member of the genus is $M$. cordifolium, var. variegatum, with heart-shaped green and silvery leaves and bright rosy-purple flowers. It is extensively used for edging flower-beds and borders during the summer months.

ICE-YACHTING, the sport of sailing and racing ice-boats. It is practised in Great Britain, Norway and Sweden, to some extent, and is very popular in Holland and on the Gulf of Finland, but its highest development is in the United States and Canada. The Dutch ice-yacht is a flatbottomed boat resting crossways upon a planking about three feet wide and sixteen long, to which are affixed four steel runners, one each at bow, stern and each end of the planking. The rudder is a fifth runner fixed to a tiller. Heavy mainsails and jibs are generally used and the boat is built more for safety than for speed. The ice-boat of the Gulf of Finland is a V-shaped frame with a heavy plank running from bow to stern, in which the mast is stepped. The stern or steering runner is worked by a tiller or wheel. The sail is a large lug and the boom and gaff are attached to the mast by travellers. The passengers sit upon planks or rope netting. The Russian boats are faster than the Dutch.

In 1790 ice-yachting was in vogue on the Hudson river, its headquarters being at Poughkeepsie, New York. The type was a square box on three runners, the two forward ones being nailed to the box and the third acting as a rudder operated by a tiller. The sail was a flatheaded sprit. This primitive style generally obtained until 1853, when triangular frames with "boxes" for the crew aft and jib and mainsail rig were introduced. A heavy, hard-riding type soon developed, with short gaffs, low sails, large jibs and booms extending far over the stern. It was over-canvassed and the mast was stepped directly over the runner-plank, bringing the centre of sail-balance so far aft that the boats were apt to run away, and the overcanvassing frequently caused the windward runner to swing up into the air to a dangerous height. The largest and fastest example of this type, which prevailed until 1879, was Commodore J. A. Roosevelt's first "Icicle," which measured 69 ft . over all and carried 1070 sq. ft . of canvas. In 1879 Mr H. Relyea built the "Robert Scott," which had a single backbone and wire guy-ropes, and it became the model for all Hudson river ice-yachts. Masts were now stepped farther forward, jibs were shortened, booms cut down, and the centre of sail-balance was brought more inboard and higher up, causing the centres of effort and resistance to come more in harmony. The shallow steering-box became elliptical. In 1881 occurred the first race for the American Challenge Pennant, which represents the championship of the Hudson river, the clubs competing including the Hudson river, North Shrewsbury, Orange lake, Newburgh and Carthage Ice-Yacht Clubs. The races are usually sailed five times round a triangle of which each leg measures one mile, at least two of the legs being to windward. Ice-yachts are divided into four classes, carrying respectively 600 sq. ft. of canvas or more, between 450 and 600, between 300 and 450, and less than 300 sq. ft. Ice-yachting is very popular on the Great Lakes, both in the United States and Canada, the Kingston (Ontario) Club having a fleet of over 25 sail. Other important centres of the sport are Lakes Minnetonka and White Bear in Minnesota, Lakes Winnebago and Pepin in Wisconsin, Bar Harbor lake in Maine, the St Lawrence river, Quinte Bay and Lake Champlain.

A modern ice-yacht is made of a single-piece backbone the entire length of the boat, and a runner-plank upon which it rests at right angles, the two forming a kite-shaped frame. The best woods for these pieces are basswood, butternut and pine. They are cut from the log in such a way that the heart of the timber expands, giving the planks a permanent curve, which, in the finished boat, is turned upward. The two forward runners, usually made of soft cast iron and about 2 ft .7 in . long and $21 / 2 \mathrm{in}$. high, are set into oak frames a little over 5 ft . long and 5 in . high. The runners have a cutting edge of $90 \%$, though a $\mathbf{V}$-shaped edge is often preferred for racing. The rudder is a runner about 3 ft .7 in . long, worked by a tiller, sometimes made very long, $71 / 2 \mathrm{ft}$. not being uncommon. This enables the helmsman to lie in the box at full length and steer with his feet, leaving his hands free to tend the sheet. Masts and spars are generally made hollow for racing-yachts and the rigging is pliable steel wire. The sails are of $10-\mathrm{oz}$. duck for a boat carrying 400 sq. ft. of canvas. They have very high peaks, short hoists and long booms. The mainsail and jib rig is general, but a double-masted lateen rig has been found advantageous. The foremost ice-yacht builder of America is G. E. Buckhout of Poughkeepsie.

An ice-yacht about 40 ft . in length will carry 6 or 7 passengers or crew, who are distributed in such a manner as to preserve the balance of the boat. In a good breeze the crew lie out on the windward side of the runner-plank to balance the boat and reduce the pressure on the leeward runner. A course of 20 m . with many turns has been sailed on the Hudson in less than 48 minutes, the record for a measured mile with flying start being at the rate of about 72 m . an hour. In a high wind, however, ice-yachts often move at the rate of 85 and even 90 m . an hour.

Several of the laws of ice navigation seem marvellous to the uninitiated. Commodore Irving Grinnell, who has made a scientific study of the sport, says: "The two marked peculiarities of ice-yachting which cause it to differ materially from yachting on the sea are: (1) Sailing faster than the wind. (2) Sheets flat aft under all circumstances." Mr H. A. Buck, in the "Badminton Library," Skating, Curling, Tobogganing, \&c., thus explains these paradoxes. An ice-boat sails faster than the wind because she invariably sails at some angle to it. The momentum is increased by every puff of wind striking the sails obliquely, until it is finally equalled by the increase of friction engendered. Thus the continued bursts of wind against the sails cause a greater accumulation of speed in the ice-yacht than is possessed by the wind itself. When the boat sails directly before the wind she is, like a balloon, at its mercy, and thus does not sail faster than the wind. The ice-yacht always sails with its sheets flat aft, because the greater speed of the boat changes the angle at which the wind strikes the sail from that at which it would strike if the yacht were stationary to such a degree that, in whatever direction the yacht is sailing, the result is always the same as if the yacht were close-hauled to the wind. It follows that the yacht is actually overhauling the wind, and her canvas shivers as if in the wind's eye. When eased off her momentum becomes less and less until it drops to the velocity of the wind, when she can readily be stopped by being spun round and brought head to the wind. The latter method is one way of "coming to," instead of luffing up in the usual way from a beam wind. In beating to windward an ice-boat is handled like a water yacht, though she points more closely.

On the bays near New York a peculiar kind of ice-boat has developed, called scooter, which may be described as a toboggan with a sail. A typical scooter is about 15 ft . long with an extreme beam of 5 ft ., perfectly oval in form and flat. It has mainsail and jib carried on a mast 9 or 10 ft . long and set well aft, and is provided with two long parallel metal runners. There is no rudder, the scooter being steered entirely by trimming the sails, particularly the jib. As the craft is flat and buoyant it sails well in water, and can thus be used on very thin ice without danger. A speed of 50 m . an hour has been attained by a scooter (see Outing for March 1905).

See Ice Sports, in the "Isthmian Library"; Skating, Curling, Tobogganing, \&c. in the "Badminton Library."

I-CH'ANG ( $\mathrm{Y}_{\mathrm{I}}-\mathrm{CH}^{\prime}$ ANG, anciently known as Yi-ling), a town of China in the province of Hu-peh, one of the four ports opened to foreign trade by treaty in 1877 . It is situated in $30^{\circ} 42^{\prime} \mathrm{N}$. and (approximately) $111^{\circ} 20^{\prime}$ E., on the Yangtsze-Kiang, 1000 m . from Shanghai. Built on the left bank of the river where it escapes from the ravines and gorges which for 350 m . have imprisoned its channel, I-ch'ang is exposed to considerable risk of floods; in 1870 the waters rose 20 ft . in one day, and the town had many of its houses and about half of its wall swept away. The first English vessels to ascend the river as far as I-ch'ang were those of Admiral Sir James Hope's expedition in 1861. All cargo to or from Szech’uen is here transhipped from steamer to junk, or vice versâ. About 10 m . above I-ch'ang the famed scenery of the Yangtsze gorges begins. Through these the great river runs in a series of rapids, which make navigation
by vessels of any size extremely difficult. A very large trade, nevertheless, is carried on by this route between Chungk'ing and $\mathbf{I}$-ch'ang. As a local centre of distribution this port is of no great consequence, the transhipment trade with Szech'uen being almost its sole business. The population is estimated at 35,000 . The number of foreign residents is very small, trade being carried on by Chinese agents. Before the anti-opium campaign of 1906 (see China) opium was much grown. The trade of the port amounted in 1899 to $£ 531,229$, and in 1904 to $£ 424,442$, the principal import being cotton yarn and the principal export opium.
 to track out), the common name of the North African representative of a number of small weasel-shaped mammals belonging to the carnivorous family Viverridae; the Indian representatives of the group being known as mongooses. A large number of species of the type genus are known, and range over southern


Egyptian Ichneumon (Herpestes ichneumon). Asia and all Africa, the typical Herpestes ichneumon also occurring in the south of Spain. The latter is an inhabitant of Egypt and the north of Africa, where it is known to foreign residents as "Pharaoh's rat." It is covered with long harsh fur of a tawny-grey colour, darker on the head and along the middle of the back, its legs reddish and its feet and tail black. It lives largely on rats and mice, birds and reptiles, and for this reason it is domesticated. It is, however, fond of poultry and their eggs, and its depredations among fowls detract from its merits as a vermin-killer. During the inundations of the Nile it is said to approach the habitations of man, but at other seasons it keeps to the fields and to the banks of the river. The Indian mongoose (H. mungo) is considerably smaller than the Egyptian animal, with fur of a pale-grey colour, the hairs being largely white-ringed, while the cheeks and throat are more or less reddish. Like the former it is frequently domesticated. It is especially serviceable in India as a serpent-killer, destroying not only the eggs and young of these creatures, but killing the most venomous adult snakes. The fact that it survives those encounters has led to the belief that it either enjoys immunity from the effects of snake poison, or that after being bitten it has recourse, as the Hindus maintain, to the root of a plant as an antidote. It has been found, however, that when actually bitten it falls a victim to the poison as rapidly as other mammals, while there is no evidence of its seeking a vegetable antidote. The truth seems to be that the mongoose, by its exceeding agility and quickness of eye, avoids the fangs of the snake while fixing its own teeth in the back of the reptile's neck. Moreover, when excited, the mongoose erects its long stiff hair, and it must be very difficult for a snake to drive its fangs through this and the thick skin which all the members of the genus possess. The mongoose never hesitates to attack a snake; the moment he sees his enemy, "his whole nature," writes a spectator of one of those fights, "appears to be changed. His fur stands on end, and he presents the incarnation of intense rage. The snake invariably attempts to escape, but, finding it impossible to evade the rapid onslaught of the mongoose, raises his crest and lashes out fiercely at his little persecutor, who seems to delight in dodging out of the way just in time. This goes on until the mongoose sees his opportunity, when like lightning he rushes in and seizes the snake with his teeth by the back of the neck close to the head, shaking him as a terrier does a rat. These tactics are repeated until the snake is killed." The mongoose is equally dexterous in killing rats and other four-footed vermin.

ICHNEUMON-FLY, a general name applied to parasitic insects of the section Ichneumonoidea (or Entomophaga), order Hymenoptera, from the typical genus Ichneumon, belonging to the chief family of that section-itself fancifully so called after the Egyptian mammal (Herpestes). The species of the families (Ichneumonidae, Braconidae, Evaniidae, Proctotrypidae, and Chalcididae) are often indiscriminately called "Ichneumons," but the "super-family" of the Ichneumonoidea in the classification of W. H. Ashmead contains only the Evaniidae, the Stephanidae, and the large assemblage of insects usually included in the two families of the Ichneumonidae and the Braconidae, which are respectively equivalent to the Ichneumones genuini and I. adsciti of older naturalists, chiefly differing in the former having
two recurrent nerves to the anterior wing, whilst the latter has only one such nerve. The Ichneumonidae proper are one of the most extensive groups of insects. Gravenhorst described some 1650 European species, to which considerable subsequent additions have been made. There are 6 sub-families of the Ichneumonidae, viz. the Ichneumoninae, Cryptinae, Agriotypinae, Ophioninae, Tryphoninae and Pimplinae, differing considerably in size and facies, but united in the common attribute of being, in their earlier stages, parasitic upon other insects. They have all long narrow bodies; a small free head with long filiform or setaceous antennae, which are never elbowed, and have always more than sixteen joints; the abdomen attached to the thorax at its hinder extremity between the base of the posterior coxae, and provided in the female with a straight ovipositor often exserted and very long; and the wings veined, with perfect cells on the disk of the front pair. Ashmead proposes to separate the Agriotypidae (which are remarkable for their aquatic habit, being parasitic on caddis-worms) from the Ichneumonidae on account of their firm ventral abdominal segments and spined scutellum. He also separates from the Braconidae the Alysiidae as a distinct family; they have peculiar mandibles with out-turned tips.

Their parasitic habits render these flies of great importance in the economy of nature, as they serve to check any inordinate increase in the numbers of injurious insects. Without their aid it would in many cases be impossible for the agriculturist to hold his own against the ravages of his minute insect foes, whose habits are not sufficiently known to render artificial checks or destroying agents available. The females of all the species are constantly on the alert to discover the proper living food for their own larvae, which are hatched from the eggs they deposit in or on the eggs, larvae or pupae of other insects of all orders, chiefly Lepidoptera, the caterpillars of butterflies and moths being specially attacked (as also are spiders). Any one who has watched insect life during the summer can hardly have failed to notice the busy way in which the parent ichneumon, a small four-winged fly, with constantly vibrating antennae, searches for her prey; and the clusters of minute cocoons round the remains of some cabbagebutterfly caterpillar must also have been observed by many. This is the work of Apanteles (or Microgaster) glomeratus, one of the Braconidae, which in days past was a source of disquietude to naturalists, who believed that the life of the one defunct larva had transmigrated into the numerous smaller flies reared from it. Ichneumon-flies which attack external feeders have a short ovipositor, but those attached to wood-feeding insects have that organ of great length, for the purpose of reaching the haunts of their concealed prey. Thus a species from Japan (Bracon penetrator) has its ovipositor nine times the length of the body; and the large species of Rhyssa and Ephialtes, parasitic on Sirex and large wood-boring beetles in temperate Europe, have very long instruments (with which when handled they will endeavour to sting, sometimes penetrating the skin), in order to get at their secreted victims. A common reddishcoloured species of Ophion (O. obscurum), with a sabre-shaped abdomen, is noteworthy from the fact of its eggs being attached by stalks outside the body of the caterpillar of the puss-moth (Cerura vinula). Lepidopterists wishing to breed the latter cut off the eggs of the parasite with scissors.

The larvae of the ichneumon-flies are white, fleshy, cylindrical, footless grubs; the majority of them spin silk cocoons before pupating, often in a mass (sometimes almost geometrically), and sometimes in layers of different colours and texture.

Authorities.-Among the older works on Ichneumonoidea may be specially mentioned J. L. K. Gravenhorst, Ichneumonologia Europaea (Breslau, 1829); A. H. Haliday (Entom. Mag. i.-v., 1833-1838), and A. Förster (Verhandl. Naturhist. Ver. Rheinl. u. Westph. xix., xxv., 1862, 1868). Full reference to the systematic literature of the group will be found in C. G. de Dalla Torre's Catalogus hymenopterorum, vols, iii., iv. (Leipzig, 1898-1902), and a comprehensive summary in W. H. Ashmead's recent memoir (Proc. U.S. Nat. Mus. xxiii., 1901). For the British species consult C. Morley, Ichneumons of Great Britain (Plymouth, 1903), and T. A. Marshall (Trans. Entom. Soc., 1885-1899).
(G. H. C.)

ICHNOGRAPHY (Gr. ̌̌ $\not \subset \nu \circ \varsigma$, a trace, and $ү \rho \alpha \varphi \eta ́$, description), in architecture, a term defined by Vitruvius (i.2) as "the ground-plan of the work," i.e. the geometrical projection or horizontal section representing the plan of any building, taken at such a level as to show the outer walls, with the doorways, windows, fireplaces, \&c., and the correct thickness of the walls; the position of piers, columns or pilasters, courtyards and other features which constitute the design.

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[^0]:    Dr Neale, in a learned dissertation prefixed to his collection of sequences from medieval Missals, and enlarged in a Latin letter to H. A. Daniel (printed in the fifth volume of Daniel's Thesaurus hymnologicus), investigated the laws of caesura and modulation which are discoverable in these works. Those first brought into use were sent by their author to Pope

[^1]:    1 Summary in E. G. Browne, A Literary History of Persia (London, 1902), pp. 387 f.
    2 The preface was translated into German by Theodor Nöldeke in his Beiträge (Hanover, 1864), pp. 1-51.

