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## A Possible Solution of the Number Series on Pages 51 to 58 of the Dresden Codex

Carl E. Guthe

# A POSSIBLE SOLUTION OF THE NUMBER SERIES ON PAGES 51 TO 58 OF THE DRESDEN CODEX 

BY

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Carl E. Guthe
Andover, Massachusetts
July, 1919

# A POSSIBLE SOLUTION OF THE NUMBER SERIES ON PAGES 51 TO 58 OF THE DRESDEN CODEX 

DESCRIPTION

In the Dresden Codex, one of the three Maya manuscripts in existence, there is found a series of numbers covering eight pages, 51 to 58 (plate I). As early as 1886, Dr. Förstemann recognized this series as an important one, and one which probably referred to the moon in some way. Each page is divided into an upper and a lower half designated, respectively, "a" and "b." Pages 51a and 52a form a unit in themselves, but are clearly associated with the remaining pages. The probable meaning of this group is still so doubtful that it has been deemed best to omit entirely a discussion of it at the present time. The remaining sections of these pages form one long series of numbers which should be read from left to right, beginning at 53a, reading to 58a, continuing on 51 b , and ending the series at 58b.
Each half-page is divided, horizontally, into four sections. The upper section consists of two rows of hieroglyphs. The section just below it contains a series of black numbers which increase in value from left to right. The third section consists of three rows of day glyphs with red numbers attached to them. The interval between the glyphs in successive rows can, of course, be mathematically obtained. The last, and bottom, division of the page is filled with a series of black numbers which are of three values only, namely, 177, 148, and 178, of which the first is the most frequent. At more or less regular intervals a vertical strip is run from the top of the half-page to the bottom. This strip contains, in the upper part, eight or ten glyphs. Below them in all but the first strip is a constellation band, and below that a figure of some kind. These strips divide the number series into groups, and are called "pictures," occurring on ten of the fourteen half-pages. Considered vertically the pages are composed of columns. Each column contains, beginning at the top, two hieroglyphs, a long number, three day glyphs, and their numbers, and finally, at the bottom, a short number. The pictures occur between these columns.
The series covers a period of 11,960 days, although the last number recorded in the upper series is only 11,958 . By means of the columns this period of 11,960 days is divided into 69 unequal parts. Let columns 2, 3, and 4 on page 54 b be taken as examples. Then each column in the series should be read in the following manner: ${ }^{[1]}$ The lower number of column 3 is 8.17 or 177 . Add this number to the upper number of column 2 , which is 1 . 2. 11. 9 or 8149 . The result is 8326 which is expressed correctly as 1 .3.2. 6 in the upper number of column 3. The lower number should also be added to the upper day glyph of column 2, which is 10 Caban, giving 5 Ix , which is the day glyph and number appearing as the first in column 3. The second day glyph and number is that of the day following 5 Ix, namely 6 Men. Similarly, 7 Cib is the day after 6 Men. Going through the same process for column 4,148 , that is, the lower number 7.8 of that column, should be added to 8326 to obtain 8474, which is expressed in the upper number of column 4 as 1. 3. 9. 14. Likewise, 148 days after 5 Ix comes 10 Ik , which is the upper day glyph of column 4, and below which are found the two days immediately following, namely, 11 Akbal and 12 Kan .
In short, then, the ideal arrangement of the series is as follows: Each upper number is the sum of all the lower numbers of the preceding columns and its own column. Each lower number expresses the difference between the upper number of its own column and that of the column immediately preceding it. The day names and numbers are three horizontal series, each starting a day later than the one above it, and recording three sets of day names and numbers which would fit the series formed by the upper numbers. It should be noticed that the mathematical interpretation of the series does not appear to depend in any way upon the hieroglyphs appearing at the top of the columns, or upon the pictures.
This series deals quite clearly with synodical revolutions of the moon. The entire series records 11,960 days, although the last number in the upper series is only 11,958 , a condition that will be explained later. Four hundred and five synodical revolutions of the moon consume, according to modern astronomy, $11,959.889$ days, or about .11 of a day less than the length of the series. Moreover, the difference groups 148, 177, and 178 which separate the upper numbers, also record synodical months, for five months consume 147.65 days, and six months 177.18 days. In fact the correspondence between the numbers in the series and the synodical months is so exact, that nowhere does an error of more than one day exist. ${ }^{[2]}$
Unfortunately the ideal arrangement given above is not followed exactly. The actual series as it occurs in the manuscript appears to be full of errors, a list of which will be found in Table I, p. 4. Most of these errors have been pointed out and discussed repeatedly. ${ }^{[3]}$ There still exists some doubt as to which numbers should be considered errors of the original writer and which should be taken at their face value. For this reason the errors are here discussed in some detail, for in some cases the errors, or supposed errors, affect theories in regard to the series.

Table I
APPARENT IRREGULARITIES
Lower number series:
Absence of all 178's that occur in upper series.
Column 23. Presence of 178.
Column 26. 177 instead of 148.
Column 50. 157 " " 177.

Upper number series:

| Column 1. | 157 | instead of | 177. |
| :---: | :---: | :---: | :---: |
| Column 2. | 353 | " " | 354. |
| Column 4. | 674 | " | 679. |
| Column 10. | 1748 | " ${ }^{\prime}$ | 1742. |
| Column 12. | 2016 | " " | 2096. |
| Column 14. | 3142 | " | 242 |
| Column 15. | 2598 |  | 259 |
| Column 24. | 4164 |  | 416 |

Day series:
Column 5. 4 Chicchan instead of 11 Chicchan.
Column 11. Omission of $1 / 2$ tonalamatl.
Column 17. 1 Ik instead of 2 Ik .
Column 36. 4 Ben instead of 4 Ahau.
Column 47. 10 Eznab instead of 11 Eznab.
Column 49. 11 Kan instead of 12 Kan.
Columns:
Columns 6 and 7 are reversed.
Columns 58 and 59 are reversed.
Totals:
Upper number series totals 11,958 instead of 11,960.
Day series totals 11,959 instead of 11,960.

In Table II, pp. 6, 7, both the corrected and the uncorrected series are given. In the centre of the table are three columns containing the actual table. The third column contains the uncorrected upper number; the fourth the lower number; and the fifth the first day sign and its number. Since the other two day series agree, except in a very few cases which will be mentioned later, with the first series, they have been omitted from the table. The sixth column contains the day signs as they probably should occur, and the second contains the corrected upper number. The first column gives the pages of the manuscript and the number of the columns on each in order to facilitate reference to the manuscript. Each column of Table II, with the exception of the first and fourth, is composed of two series of numbers, since each interval between the numbers of the manuscript has been placed in parentheses after the last of the pair of numbers it deals with, in order to facilitate comparison with the lower numbers. The names and numbers in the fifth column which have parentheses have been obliterated in the manuscript, but are easily inferred from the other two rows of day signs and numbers.
The most prominent irregularity is the absence of the number 178 in the lower numbers when the differences in both the day series and the upper numbers show that 178 should be the difference. This occurs in columns 7, 14, 29, 37, 52 and 60 of the manuscript. The only place in which 178 does occur in the lower number is in column 23, when it agrees with the difference in the day series, but not with that of the upper number. In other words, the six occurrences of the 178-day group in the upper numbers are neglected in the lower numbers, and the only occurrence of 178 in the lower numbers does not agree with the upper numbers. This implies that it is of deeper significance than a mere error. There is another disagreement between the upper and lower numbers which could very well be the result of carelessness. In column 26, the lower number is 177, while both the upper number and the day series give a difference of 148. This is the only case in which the differences of 148 are not found at the same place in all series, and, consequently, is probably an error of the scribe. Again in the lower number of column 50, the careless omission of one dot in the Uinal place has resulted in the record of 157 instead of the correct number, 177.

With one exception all of the errors in the upper numbers occur in the first third of the series. That exception, i.e., the writing of 4164 for 4163 in the column 24 , may be explained by the fact that the writer of the series had just added in column 23 the extra day to the day series, which threw it out of agreement with the upper numbers. For the moment this fact slipped his mind, but he corrected the mistake by subtracting one day from the difference between the upper numbers of columns 24 and 25.

Table II


|  | 856 | (177) | 856 | (182) | 177 | 9 Akbal | (177) | 9 Akbal | (177) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 61034 | (178) | 1033 | (177) | 177 | 4 (Ahau) | (177) | 5 Imix | (178) |
| 54a | 1211 | (177) | 1211 | (178) | 177 | (13) Enzab | (178) | 13 Enzab | (177) |
|  | 1388 | (177) | 1388 | (177) | 177 | 8 Men | (177) | 8 Men | (177) |
|  | 1565 | (177) | 1565 | (177) | 177 | 3 Eb | (177) | 3 Eb | (177) |
|  | 1742 | (177) | 1748 | (183) | 177 | 11 Mulu | (177) | 11 Muluc | (177) |
|  | 11919 | (177) | 1919 | (171) | 177 | 6 Cib | ( 47) | 6 Cimi | (177) |
|  | 22096 | (177) | 2016 | (97) | 177 | 1 Akbal | (307) | 1 Akbal | (177) |
|  | 32244 | (148) | 2244 | (288) | 148 | 6 Chuen | (148) | 6 Chuen | (148) |
| 55a |  |  |  |  | Picture |  |  |  |  |
|  | 42422 | (178) | 3142 | (898) | 177 | 2 Muluc | (178) | 2 Muluc | (178) |
|  | 52599 | (177) | 2598 | (-544) | 177 | 10 Cimi | (177) | 10 Cimi | (177) |
|  | 62776 | (177) | 2776 | (178) | 177 | 5 Akbal | (177) | 5 Akbal | (177) |
|  | 72953 | (177) | 2953 | (177) | 177 | 13 Ahau | (177) | 13 Ahau | (177) |
|  | 83130 | (177) | 3130 | (177) | 177 | 8 ? | ? | 8 Caban | (177) |
| 56a 19 | 93278 | (148) | 3278 | (148) | 148 | ? Chicchan | ? | 13 Chicchan | (148) |
|  |  |  |  |  | Picture |  |  |  |  |
| 20 | 203455 | (177) | 3455 | (177) | 177 | 8 Ik | (177) | 8 Ik | (177) |
| 21 | 13632 | (177) | 3632 | (177) | 177 | 3 Cauac | (177) | 3 Cauac | (177) |
| 22 | 23809 | (177) | 3809 | (177) | 177 | 11 Cib | (177) | 11 Cib | (177) |
| 57a 2 | 33986 | (177) | 3986 | (177) | 178 | 7 Ix | (178) | 7 Ix | (178) |
|  | 44163 | (177) | 4164 | (178) | 177 | 2 Chuen | (177) | 2 Chuen | (177) |
|  | 54340 | (177) | 4340 | (176) | 177 | 10 Lamat | (177) | 10 Lamat | (177) |
|  | 64488 | (148) | 4488 | (148) | 177 | 2 Cib | (148) | 2 Cib | (148) |
|  |  |  |  |  | Picture |  |  |  |  |
| 58a 2 | 74665 | (177) | ? | ? | 177 | 10 Ben | (177) | 10 Ben | (177) |
|  | 88482 | (177) | 4842 | ? | 177 | 5 Oc | (177) | 5 Oc | (177) |
|  | 95020 | (178) | 5020 | (178) | 177 | 1 Lamat | (178) | 1 Lamat | (178) |
|  | 50197 | (177) | 5197 | (177) | 177 | 9 Chicchan | (177) | 9 Chicchan | (177) |
| 51b | 515374 | (177) | 5374 | (177) | 177 | 4 Ik | (177) | 4 Ik | (177) |
|  | 325551 | (177) | 5551 | (177) | 177 | 12 Cauac | (177) | 12 Cauac | (177) |
|  | 335728 | (177) | 5728 | (177) | 177 | 7 Cib | (177) | 7 Cib | (177) |
|  | 345905 | (177) | 5905 | (177) | 177 | 2 Ben | (177) | 2 Ben | (177) |
|  | 556082 | (177) | 6082 | (177) | 177 | 10 Oc | (177) | 10 Oc | (177) |
|  | 66230 | (148) | 6230 | (148) | 148 | 2 Enzab | (148) | 2 Enzab | (148) |
| 52b |  |  |  |  | Picture |  |  |  |  |
|  | 376408 | (178) | 6408 | (178) | 177 | 11 Cib | (178) | 11 Cib | (178) |
|  | 386585 | (177) | 6585 | (177) | 177 | 6 Ben | (177) | 6 Ben | (177) |
|  | 396762 | (177) | 6762 | (177) | 177 | 1 Oc | (177) | 1 Oc | (177) |
|  | 06939 | (177) | 6939 | (177) | 177 | 9 Manik | (177) | 9 Manik | (177) |
| 53b 4 | 17116 | (177) | 7116 | (177) | 177 | 4 Kan | (177) | 4 Kan | (177) |
|  | 27264 | (148) | 7264 | (148) | 148 | 9 Eb | (148) | 9 Eb | (148) |
| 42 |  |  |  |  | Picture |  |  |  |  |
| 43 | 37441 | (177) | 7441 | (177) | 177 | 4 Muluc | (177) | 4 Muluc | (177) |
| 44 | 47618 | (177) | 7618 | (177) | 177 | 12 Cimi | (177) | 12 Cimi | (177) |
| 45 | 57795 | (177) | 7795 | (177) | 177 | 7 Akbal | (177) | 7 Akbal | (177) |
| 54b 46 | 67972 | (177) | 7972 | (177) | 177 | 2 Ahau | (177) | 2 Ahau | (177) |
|  | 78149 | (177) | 8149 | (177) | 177 | 10 Caban | (177) | 10 Caban | (177) |
|  | 88326 | (177) | 8326 | (177) | 177 | 5 Ix | (177) | 5 Ix | (177) |
|  | 98474 | (148) | 8474 | (148) | 148 | 10 Ik | (148) | 10 Ik | (148) |
|  |  |  |  |  | Picture |  |  |  |  |
| 50 | 58651 | (177) | 8651 | (177) | 157 | 5 Cauac | (177) | 5 Cauac | (177) |
| 55b 5 | 18828 | (177) | 8828 | (177) | 177 | 13 Cib | (177) | 13 Cib | (177) |
|  | 59006 | (178) | 9006 | (178) | 177 | 9 Ix | (178) | 9 Ix | (178) |
|  | 539183 | (177) | 9183 | (177) | 177 | 4 Chuen | (177) | 4 Chuen | (177) |
|  | 549360 | (177) | 9360 | (177) | 177 | 12 Lamat | (177) | 12 Lamat | (177) |
|  | 59537 | (177) | 9537 | (177) | 177 | 7 Chicchan | (177) | 7 Chicchan | (177) |
|  | 569714 | (177) | 9714 | (177) | 177 | 2 Ik | (177) | 2 Ik | (177) |
|  | 579891 | (177) | 9891 | (177) | 177 | 10 Cauac | (177) | 10 Cauac | (177) |
|  | 5810039 | (148) | 10039 | (148) | 148 | 2 Manik | (148) | 2 Manik | (148) |
| 56b $\begin{array}{r}\text { 5 } \\ \\ 60 \\ 6 \\ 6 \\ 62\end{array}$ |  |  |  |  | Picture |  |  |  |  |
|  | 510216 | (177) | 10216 | (177) | 177 | 10 Kan | (177) | 10 Kan | (177) |
|  | 610394 | (178) | 10394 | (178) | 177 | 6 Ik | (178) | 6 Ik | (178) |
|  | 110571 | (177) | 10571 | (177) | 177 | 1 Cauac | (177) | 1 Cauac | (177) |
|  | 6210748 | (177) | 10748 | (177) | 177 | 9 Cib | (177) | 9 Cib | (177) |
| 57b 6 | 6310925 | (177) | 10925 | (177) | 177 | 4 Ben | (177) | 4 Ben | (177) |
|  | 6411102 | (177) | 11102 | (177) | 177 | 12 Oc | (177) | 12 Oc | (177) |
|  | 6511250 | (148) | 11250 | (148) | 148 | 4 Eznab | (148) | 4 Eznab | (148) |


| Picture |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 6611427 | (177) | 11427 | (177) | 177 | 12 | Men | (177) |  | Men | (177) |
|  | 6711604 | (177) | 11604 | (177) | 177 | 7 | Eb | (177) | 7 | Eb | (177) |
| 58b | 6811781 | (177) | 11781 | (177) | 177 | 2 | Muluc | (177) | 2 | Muluc | (177) |
|  | 6911958 | (177) | 11958 | (177) | 177 | 10 | Cimi | (177) |  | Cimi | (177) |

The apparent error due to the addition of two dots in the Tun place in the upper number of column 14 is more the result of an error than an error in itself. This number shows a very clear case of erasure. The writer of this section of the manuscript in copying from the older source, at first overlooked column 14, and placed 7.3.18, the upper number in column 15 in this place. Realizing his mistake he erased the three dots in the Uinal place, but utilized two of the bars and the three dots in the Kin place as the 13 needed in the Uinal in column 14, and erased the lower bar of the original 18. This procedure of the writer's threw the upper number of column 14 out of alignment, for the two dots of the Kin appear below the 13, somewhat below the line of Kins of the other columns. The seven in the Tun place should have been a six, so the scribe inserted an extra dot between the two of the original 7, neglecting, however, to erase the other two dots. As a result the upper number of column 14 records the number 3142, which is 720 greater than it should be, namely, 2422.
In column 10, 1748 was recorded instead of 1742 , for a bar and three dots were written in the Kin place instead of only two dots. This is a very peculiar and unexpected form of carelessness, which is, however, corrected in the next column. The remaining irregularities in the upper numbers are all due to the omission of a part of the number. In columns 2 and 15, one dot was omitted in the Kin place, thus recording 353 instead of 354 in the former, and 2598 instead of 2599 in the latter. In column 4 one bar was omitted in the Kin place, making the number 674, five less than it should be, namely, 679. In column 1, one dot was omitted in the Uinal place, and in column 12, four dots of the same denomination, recording, respectively, 157 and 2016 instead of 177 and 2096.

There is only one decided error in the day series. In column 11, 6 Cib, 7 Caban, 8 Eznab were written instead of 6 Cimi, 7 Manik, 8 Lamat. It should be noticed that the number of the day was right. In fact just one-half a tonalamatl, or 130 days, was dropped before the day series of column 11, and added on again immediately afterwards. This is an extremely curious error to make in calculating and may shed some light on the way in which the Mayas reckoned.
The five remaining irregularities in the day series are of two kinds. In column 5, the number preceding the third day, Chicchan, is 4 instead of 11 . Apparently the writer of the manuscript forgot for the moment that the day was added to the one above it and not the one to the left, and wrote 4 because the number associated with the third day sign of column 4 was 3 . The same mistake was made in the third day of column 36 , except that in this case it was the day sign and not the number which was confused. Here, instead of writing Ahau, which followed the Cauac in the second series, Ben was recorded because the sign to the left was Eb. The other three irregularities are all due to carelessness in placing sufficient dots in the number associated with the day sign, for in columns 17, 47 and 49, $1 \mathrm{Ik}, 10$ Eznab and 11 Kan were recorded, respectively, instead of the necessary $2 \mathrm{Ik}, 11$ Eznab and 12 Kan.
There are two places in which columns seem to be misplaced, although the mathematics of the series at these points is correct as it stands. For the sake of uniformity in the arrangement of the difference groups, the 178-day group of column 7 should occur in column 6 , and for the same reason, the 148 -day group of column 58 should occur in column 59. Professor Förstemann calls both of these variations errors, and arranges his version of the table so that each part is just like the other two. He gives no reason for his opinion other than the phrase "for the author [of the manuscript] had confused the differences 178 and $148 \ldots . .{ }^{[4]} \mathrm{Mr}$. Bowditch, on the other hand, allows both of the variations to stand as they appear in the manuscript, and quite rightly holds the opinion that, "It may possibly be that these numbers thus placed are errors of the scribe, but the mere plea for uniformity is not sufficient to lead us to make these changes."[5]
In Table II the apparent mistake in columns 58 and 59 remains as it occurs in the manuscript, for the reason which Mr. Bowditch gives. In the case of columns 6 and 7 there seems to be some evidence that there actually was an error made. The last column on page 53a, which is the one under discussion, contains no day glyph in the first day series. The glyph should have been that of Ahau. There is distinct evidence, altho very faint, that a glyph was once there. Moreover, the smooth coating which covered the material of the manuscript page is not broken. There are other obliterated glyphs in these pages of the manuscript, but few in which the surface, although unbroken, still contains a faint, almost continuous outline of a glyph. The glyph, then, was probably erased. The writer of the manuscript had probably completely finished column 6 and started column 7 before he detected the error. He began to erase the part that was wrong, then realized what an amount of alteration would be necessary, and finally compromised by making the difference come between columns 6 and 7 instead of columns 5 and 6 . This hypothesis in regard to the manner in which the erasure was done may be wrong, but the erasure still stands as a strong evidence to show that the 178 should have occurred in column 6 rather than after it.
Finally there appears to be an error in the totals of the series, for the upper number series records as a total 11,958 days and the day series 11,959 days, although there is strong reason for believing that the series should record 11,960 days. This discrepancy in the totals will be referred to again.

In general, then, the apparent irregularities in the manuscript fall into two great classes, those which are corrected in the next column or are easily detected because of their disagreement with the record in the other two series, and those which are not obviously due to carelessness. The latter will be considered under the solutions. The former may be dismissed as clerical errors not affecting the solution. In this group are two of the irregularities in the lower numbers (columns 26 and 50), and all eight in the upper numbers, seven of which occur in the first third of the manuscript. The six errors in the day series, and the transposition of columns 6 and 7, also belong in this class.
By referring to Table II it will be noticed that the pictures occur after the 148-day groups in each case. The upper numbers immediately preceding the pictures are given in Table III (p. 11), together with the differences between them. By grouping these differences, it becomes apparent that the pictures may be divided into three large groups of 3986 days; two out of the three containing the same difference numbers, 1742, 1034, and 1210. If, in the last group, the number 10,039 were changed to 10,216 by adding 177 , the differences for this group would also read as the others, when the end of the series and the beginning of the series are added together ( $708+$ $502=1210$ ), for the 10 th picture is, in a sense, out of the grouping since it occurs after the last number in the series. The 148-day groups are arranged in the same order for they occur in the same columns as the numbers used above.
By applying the same process to the 178-day groups, it is found that they also can be divided into groups which contain 3986 days. In this case the second and third groups contain the same numbers, 2598 and 1388 (Table IV). If the number 1211 in the first group is changed to 1034 by subtracting 177, the last number of this group would be 1388; and the first number 2598 could be formed by adding the remainder at both ends of the series $(1564+1034=2598)$.

It should be remembered at this point that the only column in which the lower numbers contained 178 is column 23, of which the upper number is 3986 . This gives further grounds for dividing the series as it stands into three parts of 3986 days, each containing 23 columns.

## Table III UPPER NUMBERS OF 148DAY <br> GROUPS

| Number | rence | Group |
| :---: | :---: | :---: |
| 502 |  |  |
| 2244 | 1742 | 3986 |
| 3278 |  | 3986 |
| 4488 |  |  |
| 6230 | 1742 | 3986 |
| 7264 | 1210 | 3986 |
| 8474 | 1565 |  |
| 10039 | 1211 |  |
| 11250 | 708 | 3986 |
| (11958) | (502) |  |

The three parts are not exactly alike, however, as has already been pointed out in considering the probable errors. If the upper numbers and day numbers in column 6 should be altered, so that the difference 178 might occur in that column instead of column 7, and if, by the same process, the difference of 148 could occur in column 59 instead of 58 , then the three parts of the series would be entirely alike. The three facts mentioned are, however, very strong evidence for supposing that the people who used this table considered it as consisting of three equal parts.
This series in the Dresden is very similar to other pages of the Dresden and other manuscripts, two examples of which are given as illustrations. One of the most interesting parallels is the series on pages 46-50 of this same manuscript. This series covers a period of 2920 days which is divided into 20 unequal subdivisions. On page 24 , which just precedes page 46 , this number is used as a unit in multiplication, that is, the numbers occurring on page 24 are separated from each other by 2920 or multiples of this number. On pages 44 b and 45 b the number 78 is divided into four unequal parts, and on pages $43 b$ and $44 b$ it is used as a unit in a series which finally reaches the number $1940 \times 78$.

## SOLUTIONS

The first references to these pages in the manuscript were concerned chiefly with the reading of the numbers without any theories in regard to the probable meaning of the series.
Dr. Förstemann, in 1886, was probably the first to mention these pages specifically. At this time he corrected many of the errors in the series, and related the rows of days to the number series. ${ }^{[6]} \mathrm{He}$ had already recognized a close relation between the difference between the 1 st and 9 th pictures, i.e., 10,748, and the Saturn sidereal period of 10,753 days. Of course, in order to do this he had also identified the various signs in the "constellation bands," assigning them to various planets. ${ }^{[7]}$ These identifications are based on little more than the wish he had that they might be those planets, and for that reason they are seriously open to doubt.
Cyrus Thomas, two years later, also discussed this series at some length, but confined his
considerations entirely to the mathematical side of the work. He also pointed out most of the errors, agreeing in the main with Förstemann. He considered that the series contained 11,960 days. In his conclusion he said "the sum of the series as shown by the numbers over the second column of Plate 58b is 33 years, 3 months, and 18 days. As this includes only the top day of this column ( 10 Cimi ), we must add two days to complete the series, which ends with 12 Lamat."[8]
During the following years, Dr. Förstemann repeatedly referred to these pages in his publications and, in 1898, published an article devoted to these pages alone. ${ }^{[9]}$ The most detailed as well as the final discussion of these pages is that given in his book on the Dresden Codex. ${ }^{[10]}$ In pages $53-58$, and 51 b and 52 b he recognizes the similarity to pages $46-50$, and remarks that the Mayas not only combined the tonalamatl and the Mercury year, but also attempted to bring the lunar revolution into accord with these two. In other words, Förstemann seems to imply that the primary purpose of the series was the counting of the Mercury years, and that the lunar part of the problem was secondary.
He explains the number 11,958 as the result of attempts to make the lunar count agree with 11,960 . "They [the Mayas] found that 405 lunar revolutions amounted approximately to 11,958 days, which is, in fact, the largest number on the second half page of page $58.1{ }^{[11]}$ This will not stand at all as the reason for the 11,958 since 405 lunar revolutions come to $11,959.889$ days, and if the Mayas knew the revolutions accurately enough to know when to intercalate a day, they most certainly would not have intentionally formed the number 11,958 , when they were perfectly well aware of the fact that the time was more than 11,959 days. He recognizes in the numbers 177, 148 and 178 multiples of lunar months of 29 and 30 days.
Dr. Förstemann at this time divides the series into the three equal divisions in which it has since been considered. These are of 3986 days, thus causing the intercalated days to come at the same time in all three. ${ }^{[12]}$ He also divided each of these three divisions into three unequal groups of 1742,1034 , and 1210 days each. He advances theories, based on the positions of the pictures in the series, to show that the series also referred to the siderial periods of Saturn and Jupiter, and discusses the meaning of the glyphs found on these pages.
This detailed discussion by Dr. Förstemann of pages 51-58 of the Dresden has been used as a foundation by many in further studies of these pages. It is highly probable, however, that a careful study of his interpretations will have to be made, in which the proved assumptions must be clearly differentiated from those in which the "wish is father to the thought."
Mr. Bowditch, in 1910, ${ }^{[13]}$ discussed these pages and their relation to the astronomical knowledge of the Mayas. He divided the series into the same groups as Dr. Förstemann, basing his division upon the pictures which occur in every case immediately after the number 148. ${ }^{[14]}$ Mr. Bowditch brought out very clearly that this series is a lunar series, by means of a table which compares the numbers recorded in the manuscript and the multiples of true lunations. ${ }^{[15]}$ There can be no question on this point, for the difference between the recorded days and the true lunations is never more than .9 of a day. He also pointed out a way in which this series could be used over and over again in the form of a cycle, ${ }^{[16]}$ and then discussed the relation of this series to the Saturn and Mercury periods, disagreeing with Förstemann on several points.
Mr. Bowditch also pointed out a peculiar coincidence between the synodical revolutions of Jupiter and the numbers in the series, but based his argument on quite different material from the similar theory of Dr. Förstemann's. The important fact brought out is that the three parts of the series under discussion are almost exactly equal to 10 revolutions of Jupiter, for one revolution of Jupiter consumes 398.867 days. ${ }^{[17]}$ "This would give a reason for the selection of 11,958 to 11,960 days or 405 revolutions, and for the division of this number into three sections of 3986 days each." ${ }^{[18]}$
Dr. Förstemann and Mr. Bowditch differ in regard to some of the corrections which should be made in the manuscript, but on the whole the two discussions of these pages supplement one another. The general conclusion to be drawn from them is that these pages of the Dresden are closely associated with the synodical lunar month, and possibly, with the synodical revolution of Jupiter.
Three years after Mr. Bowditch's discussion, Mr. Meinshausen published an article in which the relation of this series to eclipses was first brought out. ${ }^{[19]}$ He compared, by means of two tables, recorded eclipses of the 18th and 19th centuries with the numbers in the Dresden Codex. Out of the 69 dates in the manuscript all but 15 dates agreed with the first case, and, in the second, all but 13, due to the fact that all the eclipses are not visible at one place on the earth's surface. "Another indication that the numbers in the codex have arisen from the observation of eclipses lies in the fact that the exact grouping of the numbers which is induced by the insertion of pictures in the number periods is also possible in lunar eclipses which are visible at one particular point. ${ }^{[20]}$ In the table given to uphold this statement, the numbers, to be sure, can be grouped in the manner which he suggests; but they can also be grouped in other series. In his opinion the reason for the grouping "lies in the close proximity of a solar eclipse to a lunar eclipse, ${ }^{[21]}$ that is, that at the date at which the pictures are inserted a solar eclipse occurred 15 days either before or after a lunar eclipse. There are two facts which tend to uphold this theory. One is the occurrence of the sun and the moon in shields over nearly all pictures, which he interprets as "signs of solar and lunar eclipses"; the other is the series of dates on pages 51a and 52 a , which are 15 days apart. In a table of recorded eclipses proof is given that such double
eclipses can occur at the intervals which separate the pictures in the manuscript. Since these intervals vary a great deal, Meinshausen believes that they will form the means of identifying the specific eclipses recorded in the manuscript.
His general conclusion is that "the material advanced will prove sufficiently that these numbers are associated in some way with solar and lunar eclipses, and this explanation must remain standing at least until other numbers, corresponding equally remarkably, are found."[22]
Professor R. W. Willson of the Astronomical Department of Harvard University, working on a similar theory at about the same time, had found, however, that no series of solar eclipses corresponding to the intervals of the pictures in the text was visible in Yucatan between the Christian era and the time of the Spanish conquest. ${ }^{[23]}$ This apparently invalidates Meinshausen's theory.
Professor Willson believes that the table in the manuscript indicates the days of ecliptic conjunction (that is, New Moon occurring so near the moon's node that eclipses may occur) and, as Mr. Bowditch has shown, with a high degree of accuracy. Sufficient proof of this, in Professor Willson's opinion, is the close correspondence of the intervals of the codex with the intervals of Schram's lunar table. ${ }^{[24]}$

The similarity between the numbers in the Dresden and Schram's table is so remarkable that it seems advisable to point out some of the most outstanding features. In addition to giving the days of multiples of the lunar synodic months, this table also gives the time of possible occurrences of both solar and lunar eclipses. Eclipses occur in cycles, the best known of which is the Saros, although there are also smaller cycles which are not so accurate. Table V (p.17) gives the occurrences of central solar eclipses according to Schram. It should be noticed that they occur in groups of threes and fours, each set being separated from the preceding one by 29 synodical months. The numbers in each group are only six months apart. Table VI (p. 17) is a corresponding series of lunar eclipses, which also occur in a grouping similar to that of the solar eclipses. It should be noticed in passing that the first numbers of these groups, in both the solar and lunar eclipses are separated by 47 and 41 lunations, the latter occurring after every third group in Table V.
Table VII (p.17) contains the numbers which are in the same columns as the 178-day groups in the Dresden. By comparing Table V and Table VII, it will be found that the numbers in the Dresden are the same as the first numbers in groups $1,2,4,5,7$ and 8 of the solar eclipses. In the last two numbers there is a difference of one day, which is explained by recalling the addition of an extra day in the day series but not in the upper numbers of the Dresden. If 679 days are added to each number in Table VII, which amounts to the same thing as advancing the Dresden table 679 days with respect to Schram's table, it will be found that these numbers will also agree with the first numbers in groups $2,3,5,6$ and 8 and with the second number in group 9 of the lunar eclipses, in Table VI. A similar agreement may be observed for the 148 -day groups (see Table III).

This remarkable agreement between the 178-day groups in the Dresden and the occurrences of eclipses may have several meanings. (1) One possibility, and one which should always be kept in mind, is that this agreement is simply another coincidence, of which there are always many in chronological work. (2) It may be that the numbers refer to dates of prophesied eclipses which the Mayas had learned occurred at more or less regular intervals. (3) Since this table has a place in the calendar of the Mayas (for a date probably occurs on page 52a), it may be that these numbers refer to definite historical eclipses. If they do, they will afford a means by which an absolute correlation between the Maya and the Julian calendars may be obtained. Professor Willson is at present working on this problem.

| Table $V$SOLAR ECLIPSES |  |  | Table VI LUNAR ECLIPSES |  |  | Table VII 178-DAY |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Group | Eclipse | Month | Group | Eclipse | Month |  |  |
|  | [ 1034 | 35 |  | 502 | 17 | Number | Month |
|  | 1211 | 41 | 1 | 679 | 23 | 1034 | 35 |
| 1 | 1388 | 47 |  | 856 | 29 | 2422 | 82 |
|  | 1565 | 53 |  | ¢ 1713 | 58 | 5020 | 170 |
| 2 | [ 2422 | 82 |  | 1890 | 64 | 6408 | 217 |
|  | \{ 2599 | 88 | 2 | 2067 | 70 | 9006 | 305 |
|  | 2776 | 94 |  | 2244 | 76 | 10394 | 352 |
|  | ( 2953 | 100 |  | ¢ 3101 | 105 |  |  |
| 3 | [ 3632 | 123 | 3 | \{ 3278 | 111 |  |  |
|  | \{ 3809 | 129 |  | 3455 | 117 |  |  |
|  | 3987 | 135 |  | ¢ 4311 | 146 |  |  |
|  | 4164 | 141 |  | 4489 | 152 |  |  |
| 4 | [ 5020 | 170 | 4 | 4666 | 158 |  |  |
|  | , 5197 | 176 |  | 4843 | 164 |  |  |
|  | 5375 | 182 |  | [ 5699 | 193 |  |  |
|  | 5552 | 188 |  | 5877 | 199 |  |  |
| 5 | ¢ 6408 | 217 | 5 | ¢ 6054 | 205 |  |  |
|  | \{ 6585 | 223 |  | 6231 | 211 |  |  |
|  | 6762 | 229 |  | ¢ 7087 | 240 |  |  |
|  | [ 7619 | 258 | 6 | \{ 7264 | 246 |  |  |
|  | \{ 7796 | 264 |  | 7442 | 252 |  |  |
|  | 7973 | 270 |  | ¢ 8298 | 281 |  |  |
|  | 8150 | 276 |  | 8475 | 287 |  |  |
|  | [ 9007 | 305 | 7 | 8652 | 293 |  |  |
|  | \{ 9184 | 311 |  | 8830 | 299 |  |  |
|  | 9361 | 317 |  | ¢ 9686 | 328 |  |  |
|  | 9538 | 323 | 8 | \{ 9863 | 334 |  |  |
| 8 | \{ 10395 | 352 |  | 10040 | 340 |  |  |
|  | \{ 10572 | 358 |  | 10896 | 369 |  |  |
|  | 10750 | 364 |  | 11074 | 375 |  |  |
| 9 | $\{11606$ | 393 | 9 | 11251 | 381 |  |  |
|  | $\{11783$ | 399 |  | 11428 | 387 |  |  |
|  | 11960 | 405 |  |  |  |  |  |

In order to determine the exact extent to which the eclipse seasons affect these pages in the Dresden Codex it is necessary to work out in as great detail as possible the calendar represented.
Modern astronomy shows that the synodical revolution of the moon consumes 29.53059 days, about .03 days more than $291 / 2$ days. Since a calendar must be based on whole days the natural method of combining the months would be to alternate one of 29 days with one of 30 days. At the end of two months or 59 days the true synodical month would be in advance of the calendrical month by .06118 days. Every two months this error is doubled so that at the end of 34 months the calendar would have completed 1003 days and the synodical month 1004.04 days. (See Table VIII, p. 19.) One method of correcting this would be to make the last month a 30 -day month instead of one of 29 days as it would be by simple alternation. This 34 -month period could then be repeated as a cycle with an accumulating error of .04 days at every repetition.
Such a series utterly disregards, however, all other phenomena such as eclipses, seasons, etc. As soon as eclipses are considered the arrangement of the months must be altered in order to use the periodicity of eclipses in the calendar. Eclipses occur at regular seasons, approximately six months apart. The average interval between eclipse seasons is 173.310 days, 3.874 days less than six synodical lunar months. In Table IX (p. 20) the eclipse season is compared with the nearest synodical lunar month. It will be noticed that the difference increases between the two series until it is necessary to use five synodical months for one interval instead of six to keep the difference less than half a month. It is necessary to do this three times in 135 synodical months, or 3986.630 days, which exceed 23 eclipse seasons, or 3986.131 days, by practically one half-day. It would be most logical to drop these extra months out of the set of six, during that group in which the difference tends to become most nearly half a month. That would be just before the 23d, 70th, and 117th month, that is, 47 months apart, requiring 41 months to complete the 135month period.
This series of 135 lunar months, or 23 eclipse seasons, can be repeated almost indefinitely, alternating 3986 and 3987 days to the series and still keep the synodical month in accord with the eclipse season. But another factor must also be considered. Months of 29 and 30 days cannot be simply alternated and either conform with the true synodical month or complete the ecliptic series mentioned, for 3986 contains three more days than sixty-eight 30 -day months, and sixtyseven 29 -day months. Therefore in the 3986 series three of the 29 -day months must be changed to 30 -day months, and in the 3987 series four must be changed. The position of these changes is
arbitrary. They can, for example, be the 34th, 68th, and 102d months, and when necessary, the 134th.

| Table VIII |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Number of month | Number of days in month | Elapsed days calendar month | Elapsed days synodical month | Error |
| 1 | 30 | 30 | 29.53 | -0.47 |
| 2 | 29 | 59 | 59.06 | 0.06 |
| 3 | 30 | 89 | 88.59 | -0.41 |
| 4 | 29 | 118 | 118.12 | 0.12 |
| 5 | 30 | 148 | 147.65 | -0.35 |
| 6 | 29 | 177 | 177.18 | 0.18 |
| 7 | 30 | 207 | 206.71 | -0.29 |
| 8 | 29 | 236 | 236.24 | 0.24 |
| 9 | 30 | 266 | 265.78 | -0.22 |
| 10 | 29 | 295 | 295.31 | 0.31 |
| 11 | 30 | 325 | 324.84 | -0.16 |
| 12 | 29 | 354 | 354.37 | 0.37 |
| 13 | 30 | 384 | 383.90 | -0.10 |
| 14 | 29 | 413 | 413.43 | 0.43 |
| 15 | 30 | 443 | 442.96 | -0.04 |
| 16 | 29 | 472 | 472.49 | 0.49 |
| 17 | 30 | 502 | 502.02 | 0.02 |
| 18 | 29 | 531 | 531.55 | 0.55 |
| 19 | 30 | 561 | 561.08 | 0.08 |
| 20 | 29 | 590 | 590.61 | 0.61 |
| 21 | 30 | 620 | 620.14 | 0.14 |
| 22 | 29 | 649 | 649.67 | 0.67 |
| 23 | 30 | 679 | 679.20 | 0.20 |
| 24 | 29 | 708 | 708.73 | 0.73 |
| 25 | 30 | 738 | 738.26 | 0.26 |
| 26 | 29 | 767 | 767.80 | 0.8 |
| 27 | 30 | 797 | 797.33 | 0.33 |
| 28 | 29 | 826 | 826.86 | 0.86 |
| 29 | 30 | 856 | 856.39 | 0.39 |
| 30 | 29 | 885 | 885.92 | 0.92 |
| 31 | 30 | 915 | 915.45 | 0.45 |
| 32 | 29 | 944 | 944.98 | 0.98 |
| 33 | 30 | 974 | 974.51 | 0.51 |
| 34 | 29 | 1003 | 1004.04 | 1.04 |

Table IX
COMPARISON OF SYNODIC MONTHS AND ECLIPSES

| Eclipse season |  |
| :---: | ---: |
| Number | Days |
| 1 | 173.310 |
| 2 | 346.620 |
| 3 | 519.930 |
| 4 | 693.240 |
| 5 | 866.550 |
| 6 | 1039.860 |
| 7 | 1213.170 |
| 8 | 1386.480 |
| 9 | 1559.790 |
| 10 | 1733.100 |
| 11 | 1906.411 |
| 12 | 2079.721 |
| 13 | 2253.031 |
| 14 | 2426.341 |
| 15 | 2599.651 |
| 16 | 2772.961 |
| 17 | 2946.271 |
| 18 | 3119.581 |
| 19 | 3292.891 |
| 20 | 3466.201 |
| 21 | 3639.511 |


| Synodic month |  |  |
| :---: | :---: | ---: |
| Number | Days | Difference |
| 6 | 177.184 | 3.874 |
| 12 | 354.367 | 7.747 |
| 18 | 531.551 | 11.621 |
| 23 | 679.204 | -14.036 |
| 29 | 856.387 | -10.163 |
| 35 | 1033.571 | -6.289 |
| 41 | 1210.754 | -2.416 |
| 47 | 1387.938 | 1.458 |
| 53 | 1565.121 | 5.331 |
| 59 | 1742.305 | 9.205 |
| 65 | 1919.489 | 13.078 |
| 70 | 2067.141 | -12.580 |
| 76 | 2244.325 | -8.706 |
| 82 | 2421.508 | -4.833 |
| 88 | 2598.692 | -0.959 |
| 94 | 2775.875 | 2.914 |
| 100 | 2953.059 | 6.788 |
| 106 | 3130.243 | 10.662 |
| 112 | 3307.426 | 14.535 |
| 117 | 3455.079 | -11.122 |
| 123 | 3632.263 | -7.248 |

## Table X 148-DAY GROUPS

| Upper number | Month number | Interval | Groups |
| :---: | :---: | :---: | :---: |
| 502 | 17 |  |  |
| 2244 | 76 | 59 35 | $94(47+47)$ |
| 3278 | 111 | 35 |  |
| 4488 | 152 | 41 | 41 |
| 6230 | 211 | 59 ) | $94(47+47)$ |
| 7264 | 246 | 35 |  |
| 8474 | 287 | 41 | 41 |
| 10039 | 340 | 53 \} | $94(47+47)$ |
| 11250 | 381 | 41 |  |

The next logical step is a comparison between the theoretical calendars just described and the manuscript. A study of the manuscript reveals that: (1) the series recorded represents 405 lunar months or three times 135 months, and that the series naturally falls into three great subdivisions of 3986 days each; (2) each third consists of 23 columns or unequal subdivisions; (3) the intervals between the 178-day groups are 47 and 88 months; (4) the 148-day groups fall approximately at 47 and 41 month intervals (see Table X); (5) the first 178-day group in each third occurs between the 30th and 35th month inclusive, and the other 178-day group of the third comes 47 months later. Since the number 178 is composed of four 30 -and two 29-day months, an extra day must have been added, that is, a 30 -day month was substituted for one of the 29-day months, if the manuscript represents a regular alternating series.
The obvious conclusions to be drawn from these facts are: (1) that the series was divided into three groups of 3986 days each in order to associate the lunar calendar closely with the ecliptic cycle of the same length; (2) that the 23 columns in each third may represent the twenty-three eclipse seasons in each eclipse period of 3986 days; (3) that groups of 47 and 41 months were used in some way in the series, for the 178 -day groups are separated by 47 and 88 months and 88 is composed of 47 and 41 , the two periods so closely associated with the recurrence of eclipses; (4) that the six months period was changed to one of five months of 148 days approximately every 47 and 41 months, which is the method already advanced in the theoretical ecliptic lunar series for keeping the synodical months and ecliptic seasons together; (5) that one extra day was added to the alternating 29 -and 30 -day months, between the 30 th and 35 th month inclusive of each third, in accordance with the theoretical necessity for so doing already brought out, and that another of the three extra days was added 47 months later.
When the difference groups ${ }^{[25]}$ are divided into months it is found that it is an easy matter to arrange the months in an alternating series. The group of 177 days is composed of three 30 -and three 29-day months, either of which when alternated can begin the group, which then ends with the other, i.e., $29,30,29,30,29,30$, or $30,29,30,29,30,29$. The group of 148 days consists of three 30 -and two 29 -day months, necessitating that it begin and end with a 30 -day month when alternated, thus, $30,29,30,29,30$. In the 178 -day group one of the 29 -day months is replaced by a 30-day month, otherwise the group is exactly like that of 177 days, which it exceeds by one day. It is evident that there will always be three 30-day months in succession in the 178-day group, and that care must be taken in choosing the right sequence of the 177-day groups which fall near those of 148 days in order to avoid having two 30-day months in succession.

There remains simply the substitution of the six or five months, as the case may be, in place of the difference groups in the manuscript. However, if the Mayas considered each third of the table as a unit, it is reasonable to assume that the sequence of the months in each third is identical. Therefore it is necessary to arrange a sequence for only one-third, that is, 135 months, and then, if the assumption is correct, this sequence will fit the other two-thirds of the series.
Each third of the table consists of 135 months covering three more days than would be covered by a simple alternation of 30 -and 29-day months. These three intercalary days were inserted at definite intervals. A clue to the position of two of them is given by the 178-day groups. One was inserted between the 30th and 35th months, another 47 months later, between the 77th and 82d months. Theoretically the extra day should be inserted in the 34 th month after the beginning of a series of alternating 29-and 30-day months, for then the error between the synodical revolution of the moon and the calendrical months becomes more than one day. In the 29-day month preceding the 34 th, namely the 32 d month, the error at the end is also practically one day, i.e., .98 days. The 29-day month most nearly the centre of the first 178-day group is the 32d month of the series, the third in the group. The Mayas may have chosen this month because of its position in the 178-day group, making the sequence of the months $29,30,30,30,29,30$, if the 30 th month was a 29-day month as it would be by simple alternation.
The second time this intercalary day occurs in each third is 47 months later. Obviously, this may be the recurrence of this intercalation in a repetition of a smaller group of months than the 135month group. If 47 months are subtracted from the 79th month which is the third in the second 178 -day group the result is 32 , which implies that the smaller division is 47 months. Two 47-
month periods complete all but 41 of the 135 months in each third. Then, of necessity, if each third of the manuscript is a unit, a 41-month group follows two 47-month groups, an arrangement which also agrees with the eclipse groups in Tables V.
The two 178-day groups account for only two of the three intercalated days, and since no 178-day group occurs in the 41 -month division, the addition of this day must have been accomplished in some more obscure manner. Since both 47 and 41 are odd numbers, each group must contain at least one more month of one kind than the other. Since two synodical revolutions of the moon are slightly longer than two calendrical months it is wisest to start and end each group with a 30-day month. If this is done, the 47-month group will contain twenty-five 30-day and twenty-two 29-day months, and the 41 -month group twenty-one 30 -day and twenty 29 -day months, making for the composition of the 135 months, seventy-one 30 -day and sixty-four 29-day months, that is, seven more of the 30-day months than of those of 29 days, showing that actually three of the sixty-seven 29-day months expected in a normal repetition have become 30-day months. This is caused by the occurrence of two 30 -day months in succession at the end of one series and the beginning of the next. If the 135 months in each third are numbered in succession it will be seen that in the first 47 -month group and in the 41 -month group, the 30 -day months are the odd numbers. In the second 47 -month group they are the even numbers, of which there is one more in this division than odd numbers, thus accounting for the additional one of the three days.
If the period of 3986 days were considered by itself, the arrangement given would be sufficient. As soon, however, as this period is repeated a number of times an error develops, since 135 synodical revolutions of the moon are completed in 3986.63 days. Twice this number gives 7973.26 , or 1.26 days more than twice 3986 . In order to keep the sequence of months in the arrangement given above in accordance with the moon, it becomes necessary to intercalate one more day every two repetitions of the 3986 period. This may be done by changing the last 29-day month in the 41 -month group to a 30 -day month, making the last 177-day group in the third one of 178 days. The Mayas certainly did this in the first third of the series given and arranged for it in the last third in a manner which will be demonstrated later.
Tabulating the solution here advanced will form Table XI (p. 25), in which the 30-and 29-day months in one-third of the manuscript have been arranged in three columns, the first two of which represent the 47 -month groups and the last one the 41 -month group. Before the first column of months are numbers to facilitate locating any given month in the group. The two kinds of months occur in direct alternation in each group, with three exceptions. The 32d month in both of the 47 -month groups is one of 30 days instead of 29, because of the addition of the intercalary day. The 40 th month in the 41 -month group is given as one of 29 days with a 30 in parentheses before it, representing the fact that every other third an extra intercalary day should be inserted in this month. To the right of the month columns are three columns giving the difference groups as found in the manuscript (see Table II), each column giving those numbers found in each third of the manuscript, the first third being the left one of the three. It should be noticed that the misplaced (?) 148-day group in the last third does not interfere with the sequence of the months.
Finally it only remains to review the irregularities of the manuscript in the light of the solution just advanced. Those irregularities which are corrected immediately afterwards, or are at variance with the rest of the column in which they occur, are, in all probability, errors on the part of the writer of the series, such as might have been caused by careless transcription from another copy of the table, and correction in the following column to avoid the task of erasure. Eliminating these irregularities, there remain three to be investigated, namely, (1) the absence of the 178 numbers in the lower number series, with one conspicuous exception; (2) the occurrence of 178 in column 7 instead of 6, and of 148 in column 58 instead of 59; and (3) the discrepancies in the totals of the series.

Table XI.-THE ARRANGEMENT OF LUNAR MONTHS IN THE DRESDEN TABLE

| No. of | Days in | Dresden groups |  |  | Days in | Dresden groups |  |  | Days in month | Dresden groups |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| month | month | 1 | 2 | 3 | month | 1 | 2 | 3 |  | 1 | 2 | 3 |
| 1 | 30 |  |  |  | 307 |  |  |  | 30 |  |  |  |
| 2 | 29 |  |  |  | 29 |  |  |  | 29 |  |  |  |
| 3 | 30 |  | 177 | 177 | 30 \} | 177 | 177 | 177 | 30 | 177 | 177 | 177 |
| 4 | 29 |  | 177 | 177 | 29 | 177 | 177 | 177 | 29 |  | 177 | 177 |
| 5 | 30 |  |  |  | 30 |  |  |  | 30 |  |  |  |
| 6 | 29 |  |  |  | 29 |  |  |  | 29 |  |  |  |
| 7 | 30 ) |  |  |  | 30 ) |  |  |  | 30 |  |  |  |
| 8 | 29 |  |  |  | 29 |  |  |  | 29 |  |  |  |
| 9 | 30 |  | 177 | 177 | 30 | 177 | 177 | 177 | 30 | 177 | 177 | 177 |
| 10 | 29 |  |  |  | 29 | 177 | 177 | 177 | 29 |  | 177 | 177 |
| 11 | 30 |  |  |  | 30 |  |  |  | 30 |  |  |  |
| 12 | 29 |  |  |  | 29 |  |  |  | 29 |  |  |  |
| 13 | 30 |  |  |  | 30 |  |  |  | 30 |  |  |  |
| 14 | 29 |  |  |  | 29 |  |  |  | 29 |  |  |  |
| 15 | 30 | 148 | 148 | 148 | 30 | 7 | 177 | 177 | 30 | 148 | 148 | 148 |
| 16 | 29 |  |  |  | 29 | 177 | 17 | 17 | 29 |  |  |  |

$\left.\begin{array}{ll}17 & 30 \\ 17\end{array}\right\}$

The great bulk of the difference groups as expressed by the lower number series are 177, the only departure from these being the designation of the 148-day groups and the extra 178-day group at the end of the first third. The complete disregard of all of the six normal 178-day groups by the lower numbers seems to imply that no attempt was made to have the latter agree with the actual differences in the upper numbers, a conclusion which is strengthened by the fact that none of the lower numbers shows evidence of the clerical errors in the differences of the upper numbers. It seems most probable that the lower number series was intended merely as a guide to indicate the position of the five month periods and to place emphasis on the extra intercalary day added in the 23d column, without attempting to have this series accurate.
The presence of the 178-day group in column 7 instead of 6 has been discussed at some length under the description of the errors. The scribe, realizing that in neglecting to put in this 178-day group, the first one of the series, a serious error had been committed, may have attempted to erase the incorrect record in column 6; then, realizing that four numbers and three glyphs would have to be altered, decided to correct this mistake-although it was of more importance than the other two errors-as he had the former ones, i.e., by making the correction in the next column.

The very similar irregularity in the last third of the manuscript, the placing of the 148-day group one column ahead of its expected position, cannot be explained in the same manner. It is very evident that this column has been deliberately placed where it is. That it does not have to do with the month sequence is evident, since it does not affect it. It must then affect the ecliptic part of the series, for it causes a short season to occur six months earlier than expected. Upon comparison of Tables VI and X, it will be seen that all of the dates of the 148-day group occur during one of the eclipse groups given in Schram's table. However, had the 148-day group under discussion been placed in the 59th column, as uniformity demands, this number, 10,216, would not have fallen in one of the eclipse groups given in Table VI. This tends to show that there was some reason other than regulating the difference groups to agree with the eclipse seasons, for the position of the 148-day groups. This reason, as yet undetermined, is possibly associated with the pictures, which immediately follow the 148-day groups.
Finally there remain only the totals of the series to be considered. The total of the upper number series records 11,958 days. Sixty-nine eclipse seasons complete $11,958.39$ days, less than half a day more than the recorded number. This close agreement and the failure to add the extra intercalary day to the upper number series at the end of the first third, give rise to the belief that the upper number series is a calendar in itself, and records a means by which dates of probable eclipses may be reckoned. The units of the count were eclipse seasons expressed as lunar months, 69 of which are represented in the calendar recorded on these pages.

The Mayas undoubtedly knew the relation of the eclipses to the moon, at least in a vague way, and felt that it was necessary to associate this eclipse calendar in some way with the lunar calendar, composed of 29 -and 30 -day months. Therefore the day series is found immediately below the upper number series. This series of days constitutes a lunar calendar which coincides as closely as possible with the eclipse calendar. It may be the formal lunar calendar of the Mayas, but it may also be an adaptation of the formal calendar to the eclipse periods. The day series varies from the eclipse series in two places only. At the end of the first third of the series, it was necessary to add one day to the lunar calendar, an addition strongly pointed out in the record, but not to the eclipse calendar, because of the increasing error between the revolutions of the moon and the calendrical lunar months. Therefore, throughout the remaining two-thirds of the series, the lunar calendar was one day in advance of the eclipse calendar. At the end of the series, since 405 of the moon's revolutions complete $11,959.89$ days, and the day series only 11,959 , one more day should be added, in order to keep the error as small as possible. This was accomplished by changing from the middle to the lower line of days.
On page 52 a, immediately preceding the calendar, are four day signs with numbers. One of these, 12 Lamat, is the zero day of the day series, but is associated with the middle line of day glyphs and not the upper line, as might be expected. The series of days which come, calendrically speaking, just before and after the actual series, may have been placed in the record to show that slight variations from the average were to be expected. The entire record is based on the middle line of days until the end of the series. Here the day just below the last day of the middle line is 12 Lamat, the end of 46 tonalamatls (260-day cycle), and the zero day of the recorded series. The tonalamatl was probably as easily used by the Mayas as " 60 days" and " 90 days" are used now. The entire calendrical system of the Mayas is based on the cycle principle. The series recorded in these pages was probably also a cycle, and in order to repeat it, 12 Lamat must again be used as the zero date. If to these arguments is added the fact that an additional day is necessary to keep the calendar in accord with the synodic revolution of the moon, there remains little doubt but that the users of this calendar added the extra day by going from the middle to the lower line of day glyphs, thereby keeping the error between the moon and the calendar as low as possible, completing the 46th tonalamatl, and at the same time making it possible to repeat the recorded series as a cycle. If the series is repeated once, at the end of 810 months, or about $661 / 2$ years, the eclipse calendar will be behind the average eclipse season .78 days, and the lunar calendar will be in advance of the synodical revolutions of the moon only .22 days.
In general, then, the irregularities in the calendar recorded on these pages fall into two groups, those which are clerical errors of the scribe and do not therefore affect the solution advanced, and those which do not appear to be of the clerical type. In the light of the solution advanced, it has been shown that there are perfectly logical reasons for the latter group of apparent irregularities.

## CONCLUSION

On pages 51 to 58 of the Dresden Codex occurs a series of numbers, running continuously through all the pages except the upper halves of the first two. This series records a period of 11,960 days, divided by means of columns into sixty-nine unequal subdivisions, of 177,148 , and 178 days, of which the first is the most frequent.
There are three distinct series. One series of numbers is in the upper part of the record, and consists of totals increased step by step until the final total reached records 11,958 days. Just below this series are three series of day signs and numbers, the middle one of which is the actual series. These dates are separated by the same number of days as the upper number series, except in the 23d group, at which place one extra day is added to the day series and not to the upper number series, causing the former to be in advance of the latter one day throughout the remainder of the record. At the end of the day series another extra day is added by counting in the last day in the lower row of days, thus completing the 11,960-day period.
Below this day series is another number series no term of which exceeds 178. In a general way it records the differences between the dates appearing above each of its numbers. The agreement is however so inaccurate that this lower number series could, at best, have been used only as a general guide to the user of the manuscript, in that it calls attention to the intervals of unusual length.
The series recorded is composed of three equal parts, each composed of 23 subdivisions and covering 3986 days.
The number series on these pages record an eclipse calendar, that is, a series of dates by means of which the occurrence of eclipses was foretold. This calendar is composed of three identical parts, with the exception of one 148-day group which occurs six months earlier in the last third than in the other two. Each third is composed of 23 unequal subdivisions which represent the twenty-three eclipse seasons, expressed in lunar months, in 3986 days. The upper number series records this calendar, and its total of 11,958 days is only .39 days less than 69 eclipse seasons.
In order to make it more intelligible this eclipse calendar is accompanied by a probably more generally known lunar calendar, which may have been altered slightly to conform to the requirements of the eclipse calendar it accompanies. This lunar calendar is contained in the day series just below the eclipse calendar. It also is recorded in three divisions agreeing closely with the eclipse calendar. One hundred and thirty-five lunar months of 30 and 29 days complete 3986 days, .63 days less than 135 synodical revolutions of the moon. This error which amounts to more than one day when repeated once, necessitates the addition of an extra day in the lunar calendar every other third, which was done in the manuscript in the first and last third, making the total
recorded by the lunar calendar 11,960 , two days more than the eclipse calendar, and .11 days more than 405 synodical revolutions of the moon. This period of 11,960 days may have been used as a cycle, the zero day of which is 12 Lamat.
Each third of the lunar calendar consists of 30-and 29-day months arranged in alternating sequence, with intercalary days added by the substitution of a 30-day for a 29-day month when the error arising from the nonconformity of the moon's revolution reaches more than one day. In order that the lunar calendar might agree with the eclipse calendar more closely, these months were recorded in groups of five and six.
The months in each third of the series were divided into three groups, which are the same in each third. The first two groups contained 47 months each, and completed the first sixteen dates of the third. The last group was one of 41 months, which was represented by the last seven dates of the third. An intercalary day was added in the 32d month of each of the 47 -month groups to correct the accumulating error, thereby causing the 6th and 14th subdivisions of the third to be of 178 days. In the first and last third the 40 th month of the 41 -month group also contained an intercalary day for the same reason, making the 23d subdivision 178 days, but in the last column of the record this extra day is added by going from the middle to the lower line of day signs. Each of the 47-and 41-month divisions began and ended with a month of thirty days.
The numerical series of these pages of the Dresden record, then, an eclipse calendar which is referred to a lunar calendar. This solution explains all the irregularities of the series except those which seem clearly to be clerical errors of the scribe.
Only the numerical and calendrical series on these pages have been considered. No attempt has been made to explain the hieroglyphs, the pictures, or the first two pages, which, although showing a close association to the long series, are nevertheless a unit in themselves.

## FOOTNOTES:

[1] For those unacquainted with Maya arithmetic the following points will explain matters: the Mayas used the vigesimal system of enumeration; they counted by twenties instead of tens. A bar represented five, and a dot stood for one. They represented numbers larger than twenty by position, just as we do. However, instead of having the smallest denomination at the right and the largest at the left of a horizontal series of figures, they had the smallest at the bottom and the largest at the top of a column of numbers. Instead of each unit in a given position representing ten times the value of that of the preceding position, it represented twenty times the value, except in the third position where it was only eighteen times as great. Thus each unit of the bottom number represented one (Kin), that of the number above it twenty (Uinal), that of the third number $20 \times 18$ or 360 (Tun), that of the fourth position $20 \times 360$ or 7200 (Katun), etc. For ease in handling, these numbers are written in our script with arabic numerals, the bottom number on the right, and separated by periods. Thus in column three, page 53 a , the upper number is 1.7.2, which means that the kin of this group is 2 , the Uinal $7,(7 \times$ 20) and the Tun $1(1 \times 360)$, making in all $2+140+360$ or 502 .

The Maya calendar, like ours, consisted of a series of numbers and a series of names for each day, each series repeating itself constantly, irrespective of the other. There were twenty different day names, which remained in an unchangeable order, and thirteen numbers. In the pages under discussion these day names appear as glyphs preceded by the necessary number.
For further details consult S. G. Morley, An Introduction to Maya Hieroglyphs, Bulletin 57, Bureau of American Ethnology, Washington, D. C., 1915, and C. P. Bowditch, 1910.
[2] Bowditch, 1910, pp. 222, 223.
[3] By Dr. Förstemann, Dr. Thomas, and Mr. Bowditch.
[4] Förstemann, 1901, p. 123.
[5] Bowditch, 1910, p. 217.
[6] Förstemann, 1886, p. 34.
[7] Ibid., pp. 68-71.
[8] Thomas, 1888, p, 325.
[9] Förstemann, 1898.
[10] Ibid., 1901, pp. 118-133.
[11] Förstemann, 1901, p. 121.
[12] Ibid., p. 123.
[13] Bowditch, 1910, pp. 211-231.
[14] Ibid., p. 218.
[15] Bowditch, 1910, pp. 222, 223.
[16] Ibid., p. 224.
[17] Ibid., pp. 229, 230.
[18] Ibid., p. 231.
[19] Meinshausen, 1913, pp. 221-227.
[20] Ibid., p. 225.
[21] Meinshausen, 1913, p. 225.
[22] Ibid., pp. 226, 227.
[23] Professor Willson's work on the Dresden manuscript has not yet been published. It is referred to here only through his kind permission.
[24] Schram, 1908, pp. 358, 359.
[25] That is, the 177-day, 148-day and 178-day groups.

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Dresden Codex, Pages 51 to 58.

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